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Distribution and Relation of Primes with Tesla Numbers 3, 6, and 9

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Abstract

This work presents a different approach to twin primes, an approach from the perspective of the Tesla numbers and gives a refresh and new observation of twin primes that could lead us to an answer to the Twin Prime Conjecture problem. We expose a peculiar relation between twin primes and the generation of prime numbers with Tesla numbers. Tesla numbers seem to be present in so many domains like time, vibration and frequency [1], and the space between twin primes is not the exception. Let us say that twin primes are more than just prime numbers plus 2 or minus 2, and Tesla numbers are more involved with twin primes than we think, and hopefully, this approach give us a better understanding of the distribution of the twin pairs.

Keywords

Tesla Numbers, Prime Number, Twin Prime Numbers, Twin Pair

1. Introduction

In number theory, Twin Prime Conjecture (TPC) is one of the oldest problems, it says that "There exist infinitely many primes p such that $p + 2$ is a prime" [2] [3]. In other words, twin primes are prime numbers that differ by two, such as 11 and 13, or 41 and 43 and many others [2]. Do not confuse them with consecutive prime numbers for they are numbers that cannot only be close together, but also be far apart [3]. However, it could be said to step towards the famous twin prime conjecture that there are infinitely many prime pairs p and $p + 2$, the gap here being 2 [4]. With that observation established, we present a different perspective that establishes a close relation and interaction with Tesla numbers.

To begin, it is known that aside from the number 3, there is no other prime number with its digital root being a Tesla number (3, 6, 9), [1] that sentence provides a simple beginning to identify prime numbers, we could say that provides a rough method to identify prime numbers that involve Tesla numbers.

2. Historical Context

There is no evidence before the nineteenth century about the awareness of twin primes. There is what we could call an approach of them, mentioned by Polignac in one of the two theorems in his paper in 1849 called "New Research on Prime Numbers" [2] [5]. The theorem says: "Every even number is equal to the difference of two consecutive prime numbers in an infinitude of ways". When changing "every even number" by the number two, there is a reference to the twin primes and their infinitude [2]. It is quite a reach to consider this the origin of the TPC.

In 1879, Glaisher published "An Enumeration of Prime-Pair" where he describes a prime pair being two prime numbers separated only by one number.

After counting all the twin pairs in the first and second million he concluded: "there can be little or no doubt that the number of prime pairs is unlimited; but it would be interesting, though probably not easy, to prove this". Glaisher is seen as the originator of the TPC [2].

2.1. Prime Numbers Identification Methods

2.1.1. Fermat's Primality Test

It is due to mention the Fermat's primality test or Fermat's little theorem that states that, if p is a prime number, then for any integer a, the number $a^p - a$ is an integer multiple of p:

$$\frac{a^p - a}{p} = \text{Integer} \tag{1}$$

This test has its exceptions because there are numbers called pseudo primes, that satisfy the primality test but they are not prime numbers, such as 341, that is a base 2 pseudo prime.

2.1.2. Sieve of Eratosthenes

It is an ancient method that consists in marking in a table the multiples of every known prime number for example the multiples of 2 are 4, 6, 8, 10, 12, 14, 16, 18, etc. The unmark numbers are primes. In **Figure 1**, this method is shown.

The numbers 2, 3, 5, 7, 11, 13, 17, 19, 29, 31, 37, 41, 47, 59, 61, 67, 71, 73, 79, 83, 89, 97 are the prime numbers from 0 to 100.

2.2. Twin Primes' Distribution

As mentioned the infinity of prime numbers also has repercussion in the distribution

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 1. Sieve of Eratosthenes from 0 to 100.

of twin pairs within the infinity where same mathematicians had work in order to achieve the knowledge of how the twin pairs distribute to infinity mathematicians like Brun, Hardy and Littlewood.

2.2.1. Brun's Constant

Brun's constant is defined to be the sum of the reciprocals of all twin primes:

$$B=\left(\frac{1}{3}+\frac{1}{5}\right)+\left(\frac{1}{5}+\frac{1}{7}\right)+\left(\frac{1}{11}+\frac{1}{13}\right)+\cdots \tag{2}$$

Recent calculations of the constant gave us:

$$B=1.9021605831\cdots$$

2.2.2. Hardy and Littlewood Twin Prime Constant

$$\alpha=\prod_{P>2}\left(1-\frac{1}{(P-1)^2}\right)\approx 0.6601618158\cdots \tag{3}$$

Conjecture (Hardy and Littlewood). For every integer $k > 0$, there are infinitely many prime pairs p, $p + 2k$, and the number $\pi_{2k}(x)$ of such pairs less than x is:

$$\pi_{2k}\sim 2\alpha\prod_{\substack{P>2\\ PIk}}\frac{P-1}{P-2}\cdot li_2(x) \tag{4}$$

3. Methodology

From the binary series ($2n$) shown in **Figure 2**, Tesla numbers are obtained by horizontally summing the powers of two on the first level giving us the result to the next level as follows:

1 + 2 = 3, 2 + 4 = 6, 4 + 8 = 12, 8 + 16 = 24, then 3 + 6 = 9, 6 + 12 = 18, 12 + 24 = 36.

From this point and forward the results of the next level are obtained by vertically summing: 12 + 18 = 30, 24 + 36 = 60 and at last 30 + 60 = 90.

The resulting Tesla numbers in **Figure 3** are prime number generators.

These numbers have Tesla numbers 3, 6 and 9 as their digital root $\delta(n)$.

2^0	2^1	2^2	2^3	2^4
1	2	4	8	16
3	6	12	24	
9		18	36	
		30	60	
		90		

Figure 2. Tesla numbers from powers of 2.

12	24
18	36
30	60
90	

Figure 3. Prime numbers generators.

$$\delta(12) = 1 + 2 = 3 \tag{5}$$

$$\delta(24) = 2 + 4 = 6 \tag{6}$$

$$\delta(18) = 1 + 8 = 9 \tag{7}$$

$$\delta(30) = 3 + 0 = 3 \tag{8}$$

$$\delta(36) = 1 + 8 = 9 \tag{9}$$

$$\delta(60) = 6 + 0 = 6 \tag{10}$$

$$\delta(90) = 9 + 0 = 9 \tag{11}$$

Curiously enough, the digital root of the prime vector (12) is 9, a Tesla number (13):

$$11, +13, +17, +19, +23, +29, +31, +37 \tag{12}$$

$$11 + 13 + 17 + 19 + 23 + 29 + 31 + 37 = 180, \delta(180) = 9 \tag{13}$$

To this vector first you add 30, then 60 and continues 90, 120, 150, 180 adding 30 every time. The vector added to the $30n$ Tesla module:

$$30n, +11, +13, +17, +19, +23, +29, +31, +37 \tag{14}$$

This is the Prime Number Generator Vector (PNGV). By adding $30n$ to prime numbers the resulting numbers are prime numbers also.

Twin Prime numbers are generated by Tesla numbers.

As we can see in **Figure 4**, there are five distribution channels each with a different progression.

In each channel, there are 3 columns originated by their progression and the PNGV.

In the first channel, the column in the middle is a Tesla number obtained from its respective progression 6 ($3n - 1$) when $n = 1$, we have: $6 (3 \times 1 - 1) = 12$, in the left column, we have $12 - 1 = 11$ that's a prime number, in the right column we have $12 + 1 = 13$ the twin prime number of 11. Then, we apply the PNGV to the left, middle and right columns to obtain potential twin prime numbers as shown in the table. This means we sum 30 to the columns in first row to obtain the second row, then we sum another 30 to the second row to obtain the third one, and we repeat that process arbitrarily. This method is present in every distribution channel and has a very good approximation on generating prime numbers and twin pairs, but has some exceptions.

Let it be noticed that at the middle of twin prime numbers, there is a Tesla number in the 1st, 2nd and 4th channel.

In the third channel (6 ($3n + 1$)), however, we observe a peculiar situation where even though prime numbers were generated their twin prime numbers were not, due to the numbers of its right column are multiples of five meaning that none of those numbers are prime numbers.

A similar situation occurs on the fifth channel (18 ($n + 1$)), where its left column is empty due to its generated numbers are again multiples of 5.

These channels allow us to perform a quicker identification of prime numbers.

As we can see in **Figure 5**, every channel the column in the middle is a Tesla Number:

1st channel: $\delta[6(3n-1)]$ = Tesla Number.

2nd channel: $\delta(18n)$ = Tesla Number.

3rd channel: $\delta[6(3n+1)]$ = Tesla Number.

4th channel: $\delta(30n)$ = Tesla Number.

5th channel: $\delta(18n+1)$ = Tesla Number.

Also, notice that the digital root of the numbers in the right and left columns has been obtained.

n	30n	6(3n - 1), -1,+1			18n, - 1, + 1			6(3n + 1), -1			30n, -1,+1			18(n + 1),+1		
1		11	12	13	17	18	19	23	24	25	29	30	31	35	36	37
1	30	41	42	43	47	48	49	53	54		59	60	61		66	67
2	60	71	72	73	77	78	79	83	84		89	90	91		96	97
3	90	101	102	103	107	108	109	113	114		119	120	121		126	127
4	120	131	132	133	137	138	139	143	144		149	150	151		156	157
5	150	161	162	163	167	168	169	173	174		179	180	181		186	187
6	180	191	192	193	197	198	199	203	204		209	210	211		216	217
7	210	221	222	223	227	228	229	233	234		239	240	241		246	247
8	240	251	252	253	257	258	259	263	264		269	270	271		276	277
9	270	281	282	283	287	288	289	293	294		299	300	301		306	307
10	300	311	312	313	317	318	319	323	324		329	330	331		336	337
11	330	341	342	343	347	348	349	353	354		359	360	361		366	367
12	360	371	372	373	377	378	379	383	384		389	390	391		396	397
13	390	401	402	403	407	408	409	413	414		419	420	421		426	427
14	420	431	432	433	437	438	439	443	444		449	450	451		456	457
15	450	461	462	463	467	468	469	473	474		479	480	481		486	487
16	480	491	492	493	497	498	499	503	504		509	510	511		516	517
17	510	521	522	523	527	528	529	533	534		539	540	541		546	547
18	540	551	552	553	557	558	559	563	564		569	570	571		576	577
19	570	581	582	583	587	588	589	593	594		599	600	601		606	607
20	600	611	612	613	617	618	619	623	624		629	630	631		636	637
21	630	641	642	643	647	648	649	653	654		659	660	661		666	667
22	660	671	672	673	677	678	679	683	684		689	690	691		696	697
23	690	701	702	703	707	708	709	713	714		719	720	721		726	727
24	720	731	732	733	737	738	739	743	744		749	750	751		756	757
25	750	761	762	763	767	768	769	773	774		779	780	781		786	787
26	780	791	792	793	797	798	799	803	804		809	810	811		816	817
27	810	821	822	823	827	828	829	833	834		839	840	841		846	847
28	840	851	852	853	857	858	859	863	864		869	870	871		876	877
29	870	881	882	883	887	888	889	893	894		899	900	901		906	907
30	900	911	912	913	917	918	919	923	924		929	930	931		936	937
31	930	941	942	943	947	948	949	953	954		959	960	961		966	967
32	960	971	972	973	977	978	979	983	984		989	990	991		996	997

Figure 4. The five distribution channel table.

From **Figure 5**, we could say that "A prime number is an odd number that comes before or after a Tesla number. Also, it is divisible by 1 and by itself" and "Twin Prime numbers are odd numbers that come before and after a Tesla number, also divisible by 1 and themselves", that is why the numbers 2 and 3 are not twin prime numbers.

The numbers in green are prime numbers.

n	30n	6(3n - 1), -1,+1			18n, - 1,+1			6(3n + 1), -1			30n, -1,+1			18(n + 1),+1		
1		2	3	4	8	9	1	5	6	7	2	3	4	8	9	1
1	3	5	6	5	2	3	4	8	9		5	6	5		3	4
2	6	8	9	6	5	6	7	2	3		8	9	6		6	7
3	9	2	3	4	8	9	1	5	6		2	3	4		9	1
4	3	5	6	7	2	3	4	8	9		5	6	7		3	4
5	6	8	9	1	5	6	7	2	3		8	9	1		6	7
6	9	2	3	4	8	9	1	5	6		2	3	4		9	1
7	3	5	6	7	2	3	4	8	9		5	6	7		3	4
8	6	8	9	1	5	6	7	2	3		8	9	1		6	7
9	9	2	3	4	8	9	1	5	6		2	3	4		9	1
10	3	5	6	7	2	3	4	8	9		5	6	7		3	4
11	6	8	9	1	5	6	7	2	3		8	9	1		6	7
12	9	2	3	4	8	9	1	5	6		2	3	4		9	1
13	3	5	6	7	2	3	4	5	9		5	6	7		3	4
14	6	8	9	1	5	6	7	2	3		8	9	1		6	7
15	9	2	3	4	8	9	1	5	6		2	3	4		9	1
16	3	5	6	7	2	3	4	8	9		5	6	7		3	4
17	6	8	9	1	5	6	7	2	3		8	9	1		6	7
18	9	2	3	4	8	9	1	5	6		2	3	4		9	1
19	3	5	6	7	2	3	4	8	9		5	6	7		3	4
20	6	8	9	1	5	6	7	2	3		8	9	1		6	7
21	9	2	3	4	8	9	1	5	6		2	3	4		9	1
22	3	5	6	7	2	3	4	8	9		5	6	7		3	4
23	6	8	9	1	5	6	7	2	3		8	9	1		6	7
24	9	2	3	4	8	9	1	5	6		2	3	4		9	1
25	3	5	6	7	2	3	4	8	9		5	6	7		3	4
26	6	8	9	1	5	6	7	2	3		8	9	1		6	7
27	9	2	3	4	8	9	1	5	6		2	3	4		9	1
28	3	5	6	7	2	3	4	8	9		5	6	7		3	4
29	6	8	9	1	5	6	7	2	3		8	9	1		6	7
30	9	2	3	4	8	9	1	5	6		2	3	4		9	1
31	3	5	6	7	2	3	4	8	9		5	6	7		3	4
32	6	8	9	1	5	6	7	2	3		8	9	1		6	7

Figure 5. Digital roots of the five distribution channels.

x	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	101	103	107	109	113	127	131	137	139
7	49	77	91	119	133	161	203	217	259	287	301	329	371	413	427	469	497	511	553	581	623	679	707	721	749	763	791	889	917	959	973
11		121	141	187	209	253	319	341	407	451	473	517	583	649	671	737	981	803	869	913	979	1067	1111	1133	1177	1199	1243	1397	1441	1507	1397
13			169	221	247	299	377	403	481	533	559	611	689	767	793	871	923	949	1027	1079	1157	1261	1313	1339	1391	1417	1469	1651	1703	1781	1651
17				289	323	391	493	527	626	697	731	799	901	1003	1037	1139	1207	1241	1343	1411	1513	1649	1717	1751	1819	1853	1921	2159	2227	2329	2159
19					361	437	551	589	703	779	817	893	1007	1121	1159	1273	1349	1387	1501	1577	1691	1843	1919	1957	2033	2071	2147	2413	2489	2603	2413
23						529	667	713	851	943	989	1081	1219	1357	1403	1541	1633	1679	1817	1909	2047	2231	2323	2369	2461	2507	2599	2921	3013	3151	2921
29							841	899	1073	1189	1247	1363	1537	1711	1769	1943	2059	2117	2291	2407	2581	2813	2929	2987	3103	3161	3277	3683	3799	3973	3683
31								961	1147	1271	1333	1457	1643	1829	1891	2077	2201	2263	2449	2573	2759	3007	3131	3193	3317	3379	3503	3937	4061	4247	3937

Figure 6. Composite numbers distribution.

x	49	121	169	289	361	529	841	961
7	343	847	1183	2023	2527	3703	5887	6727
11	539	1331	1859	3179	3971	5819	9251	10571
13	637	1573	2197	3757	4693	6877	10933	12493
17	833	2057	2873	4913	6137	8993	14297	16337
19	931	2299	3211	5491	6859	10051	15979	18259
23	1127	2783	3887	6647	8303	12167	19343	22103
29	1421	3509	4901	8381	10469	15341	24389	27869
31	1519	3751	5239	8959	11191	16399	26071	29791

Figure 7. Square and cubic composite numbers distribution.

The numbers in pale yellow are composite numbers. In **Figure 6** and **Figure 7**, we see the distribution of composite numbers.

The numbers in pink are square prime numbers. For example: $7^2 = 49$.

The numbers in blue are square prime numbers times another prime number. For example: $11^2 = 121$.

The numbers in red are cubic prime numbers. For example: $7^3 = 343$.

4. Conclusions

At the center of every twin pair, there is just one number and that number is a Tesla number, which in some cases generates twin primes as we could see in the channels 1^{st}, 2^{nd} and 4^{th}, and others just prime numbers as in the channels 3^{rd} and 5^{th}.

Thanks to the distribution of the channels in the 3^{rd} and 4^{th} channels, we see in a practical way that complete columns of generated numbers are discarded as primes, given their nature of being multiples of 5.

We can state that "between a twin pair there is always one Tesla number".

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Agbanwa, J. (2022) Findings on Tesla's Numbers 3, 6, 9. https://doi.org/10.6084/m9.figshare.19161221

[2] Dunham, W. (2013) A Note on the Origin of the Twin Prime Conjecture. *Notices of the International Consortium of Chinese Mathematicians*, **1**, 63-65. https://doi.org/10.4310/ICCM.2013.v1.n1.a13

[3] Nazardonyavi, S. (2012) Some History about Twin Prime Conjecture. arXiv: 1205.0774. https://doi.org/10.48550/arXiv.1205.0774

[4] Soundararajan, K. (2007) Small Gaps between Prime Numbers: The Work of Goldston-Pintz-Yildirim. *Bulletin of the American Mathematical Society*, **44**, 1-18. https://doi.org/10.1090/S0273-0979-06-01142-6

[5] Baibekov, S.N. and Durmagambetov, A.A. (2016) Infinite Number of Twin Primes. *Advances in Pure Mathematics*, **6**, 954-971. https://doi.org/10.4236/apm.2016.613073