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Integer Linear Program Model for Efficient Bus Scheduling Problem in Transport Management for Akwa Ibom Transport Company

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Integer program was formulated in this work for efficient bus scheduling problem in Transport Company to obtain an optimal bus allocation system for its travelling routes in Nigeria. Integer programs were formulated for both the long and short routes categories. Transportation data from Akwa Ibom Transport Company (AKTC) was used to obtain an optimal bus schedule for the company with increase turnover in daily ticket collection by N2,327,500.00, N825,300.00 and N3,152,800.00 representing 34.78%, 10.60% and 21.78% respectively for the long routes, short routes and the overall routing system, while also meeting daily customers' demand. Hence, the use of integer programming model is recommended for efficient vehicle scheduling and increase turnover in transport companies and personnel management system.

Keywords: Integer program; bus scheduling; branch and bound algorithm; transport company; travelling route.

1 Introduction

Operational planning within the public transit companies has been a major concern. Hence, the need to develop and refine existing models/methodologies for efficient transport system. According to [1], these models include maps, activity charts, balance sheets, PERT network, break-even equation, economic ordering, quantity

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equation, scheduling and simulation, among others. Scheduling was described by [2] as a decision-making process that is used on a regular basis in many manufacturing and services industries, dealing with allocation of resources to tasks over given time period, with the goal of optimizing one or more objectives. While [3-6] defined scheduling as the temporal allocation of activities to resources to achieve some desirable objectives and categorize it into job process arrival, number of machines, the flow pattern and the performance criterion to be optimized.

A scheduling model based on interrelationship between passenger trip demands and bus trip supplies for inter-city carriers was formulated by [7] for efficient passengers' services. Furthermore, [8] presented an efficient optimum solution for a real-life employee days-off scheduling problem to determine the minimum number of workers and feasible days-off assignments. Also, integer linear programming approach was applied by [8,9] and [10] to solve vehicle scheduling problem to cover all tasks and ensure that the number of drivers and costs of shifts is minimized. A real time scheduling method for a variable-route bus to reduce cost and average waiting time of the passengers was considered by [11]. In 2011, [12] underscored the use of buses in public transport technology as the most easily accessible and cheaper means compared to other kinds of public transportation medium. However, [13] showed that there is a declining reliability of bus services with increasing distance between a bus stop and the original terminal. Also, [14] proposed an integer linear programming model for exact solutions, considering skip-stop patterns when designing timetables for express trains and proposed several nonlinear integer programming models with linear constraints. However, [15] built an optimal model to schedule the departure time of each bus service with the objective of minimizing the average waiting time. The model when compared with the existing bus scheduling system helped to reduce the waiting time using experimental results. On this premise, this work will adapt an integer programming modelling approach with days-off as decision variables proposed by [15] to obtain efficient days-off bus scheduling roster. Real time data was obtained from Akwa Ibom Transport Company (AKTC), Uyo, Nigeria.

1.1 Problem definition

Akwa Ibom Transport Company (AKTC), Uyo is one of the major bus operators in Akwa Ibom State with large fleet of buses. The company currently owns 378 active buses with average of 14 passengers travelling on 22 different routes within Nigeria, majorly through the south-south, south-east, north central and south-western regions of the country. These routes are categorized as long or short routes. The routes to the north central, North West and South West are referred to as long routes while routes to South-South and South-East are short routes on an 18-hours (5.00 to 23.00) and 7 days-a-week schedule. A route is said to be long if its trip distance from Uyo terminus is at least 500km. On the other hand, a route is said to be short if its trip distance from Uyo terminus is less than 500km. Buses of long routes, at arrival at specified long route destination, take the next day off and resumes work on the second day after arrival. This implies that, a bus of a long route works 4 days in a week and takes 3 alternating days off. Buses of short routes operate the full 18-hours, only enjoying a 6-hours rest. Each of the company's routes has an average daily passenger demand, daily bus allocation, per ticket price and the required bus allocation.

The company observed that, most times, reserve buses or buses of shorts routes are converted to long routes at emergency to meet the demands of long routes passengers. Thus, the company wishes to explore the lucrative ticket prices of the long routes as well as meet overall passengers' demand. The management, in addition, also needs an allocation system on the remaining buses (resulting from the schedule system of the long route buses) to the short routes, taking into consideration, the per trip ticket collection for each short route trip subject to availability of at least 15 buses at the Uyo terminus for emergency conditions.

Hence, this work seeks to obtain an efficient bus scheduling model that would optimize fleet size and ticket collections for the various routes to ensure optimum profit and guarantees customers satisfaction taking into consideration the 3 alternating days off for the long route drivers and other limiting factors. This will avoid redundancy of buses, optimize man hour of drivers, enhance efficiency and increase productivity of the company.

1.2 Assumptions of the Study

1. Company buses are uniquely assigned to drivers and are not shared among company branches.

2. Company bus travelling on a long route is unavailable for schedule on the next work day: an off day is the day a bus is unavailable, hence, buses have 4 days-on and 3 days-off, irrespective of the alternating nature of this arrangement.
3. Travel time between two consecutive stations is assumed to be constant.
4. All buses have the same capacity of 14-seaters.

2 Methodology

2.1 The branch and bound algorithm for solving integer programming problems

2.1.1 Initialization

Consider the following integer programming problem (ILPP):

$$Maximze:\ Z = \theta x_1 + c\theta_2 x_2 + \ \dots + \theta_q x_q \tag{1}$$

$$subject\ to: \begin{matrix} a_{11}x_1 & + & a_{12}x_2 & + & \dots & a_{1q}x_q & = \tau_1 \\ a_{21}x_1 & + & a_{22}x_2 & + & \dots & a_{2q}x_q & = \tau_2 \\ \vdots & \vdots & \vdots & + & \dots & \vdots & \vdots \\ a_{p1}x_1 & + & a_{p2}x_2 & + & \dots & a_{pq}x_q & = \tau_p \end{matrix} \tag{2}$$

$$x_j \geq 0 \in \square^q\ (for\ \ all\ j = 1,2,3, \dots, q) \tag{3}$$

Eqns (1) - (3) can be expressed in matrix form as;

$$\max Z = \underline{G}_{1xq}\underline{X}_{qx1} \tag{4}$$

$$\text{Subject to:}\quad \underline{A}_{pxq}\underline{X}_{qx1} = \underline{B}_{px1} \tag{5}$$

$$x_j \geq 0 \in \square^q\ (for\ some\ or\ all\ j = 1,2,3, \dots, q)$$

Where:

$$\underline{G}_{1xq} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \dots & \theta_q \end{bmatrix}$$

$$\underline{X}_{qx1}^{T} = \begin{bmatrix} x_1 & x_2 & x_3 & \dots & x_q \end{bmatrix}$$

$$\underline{A}_{pxq} = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & . & . & . & \beta_{1q} \\ \beta_{21} & \beta_{22} & \beta_{23} & . & . & . & \beta_{2q} \\ & & . & & & & \\ & & . & & & & \\ & & . & & & & \\ \beta_{p1} & \beta_{p2} & \beta_{p3} & . & . & . & \beta_{pq} \end{bmatrix} \text{ and } \underline{B}_{px1} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ . \\ . \\ . \\ \tau_p \end{bmatrix}$$

2.1.2 Branching step

Let x_k be one of the basic variables which does not have an integer value and also has the largest fractional value.

Branch (or partition) the LP problem into 2 new LP sub-problems based on integer values of x_k that are immediately above and below its non-integer value. It is partitioned by adding 2 mutually exclusive constraints to the original LP problem. The constraints are; $x_k \leq [x_k]$ and $x_k \geq [x_k] + 1$.

Here $[x_k]$ is the integer portion of the current fractional value of the variable, x_k. This is done to exclude the non-integer value of X_k. The two new LP problems will be:

Maximize: $Z = \sum_{j=1}^{n} c_j x_j$

subject to: $\sum_{j=1}^{n} a_{ij} x_j = b_i$

$x_k \leq [x_k]\ ;\ X_j \geq 0$

and: Maximize $Z = \sum_{j=1}^{n} c_j x_j$

subject to: $\sum_{j=1}^{n} a_{ij} x_j = b_i$

$x_k \geq [x_k] + 1\ ;\ X_j \geq 0$

2.1.3 Bound step

Obtain the optimal solution of the sub-problems. Let the optimal value of the objective functions of the sub-problems be Z_2 and Z_3 respectively. The best integer solution value becomes the lower bound on the integer LP problem. Let the lower bound de denoted by Z_L.

2.1.4 Fathoming step

i. If a sub-problem yields an infeasible solution, terminate the branch.
ii. If a sub-problem yields a feasible non-integer solution, return to step 2.
iii. If a sub-problem yields a feasible integer solution, examine the value of the objective function. If the value is equal to the upper bound, an optimal upper solution is reached. But if it is not equal to the upper bound but exceeds the lower bound, this value is used as the new upper bound, then return to step 2. But, if this value is less than lower bound, terminate the branch.

2.1.5 Termination

The procedure of branching and bounding continues until no further sub-problem remains to be examined. At this stage, the integer solution corresponding to the current lower bound is the optimal all-integer programming problem solution.

2.2 Integer linear programming problem formulation for the long routes

An optimal schedule for the long route buses will take into consideration assurance of these days off for each long-distance bus while meeting the daily bus requirement for all long routes. We let:

x_1 = Number of buses on off on Tuesday, Thursday and Saturday
x_2 = Number of buses on off on Monday, Wednesday and Friday
x_3 = Number of buses on off on Sunday, Tuesday and Thursday
x_4 = Number of buses on off on Saturday, Monday and Wednesday
x_5 = Number of buses on off on Friday, Sunday and Tuesday
x_6 = Number of buses on off on Thursday, Saturday and Monday
x_7 = Number of buses on off on Wednesday, Friday and Sunday

The implication of the definitions of x_1 to x_7 is that, for instance, on Mondays, number of buses on duty will be the sum of x_1, x_3, x_5 and x_7, while on Tuesdays, number of buses on duty will be the sum of x_2, x_4, x_6 and x_7 and so on. Thus, the corresponding integer linear program is given as:

$$Minimize\ Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

subject to:

Monday Constraint: $x_1 \quad + x_3 \quad + x_5 \quad + x_7 \geq r_1$

Tuesday Constraint: $x_2 + x_4 + x_6 + x_7 \geq r_2$
Wednesday Constraint: $x_1 + x_3 + x_5 + x_6 \geq r_3$
Thursday Constraint: $x_2 + x_4 + x_5 + x_7 \geq r_4$
Friday Constraint: $x_1 + x_3 + x_4 + x_6 \geq r_5$
Saturday Constraint: $x_2 + x_3 + x_5 + x_7 \geq r_6$
Sunday Constraint: $x_1 + x_2 + x_4 + x_6 + \geq r_7$
$x_i \geq 0 \in \mathbb{Z} for\ i = 1, 2, 3, \dots, 7$

Where:

$$r_j = daily\ bus\ requirement, j = 1, 2, 3, \dots, 7$$

This implies that $r_1, r_2, \dots, r_7$ represent bus requirement for Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday respectively.

Therefore, a formal integer linear program is formulated as follows:

$$Minimize\ Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \quad (6)$$

subject to:

$$\begin{aligned} x_1 + x_3 + x_5 + x_7 &\geq r_1 \\ x_2 + x_4 + x_6 + x_7 &\geq r_2 \\ x_1 + x_3 + x_5 + x_6 &\geq r_3 \\ x_2 + x_4 + x_5 + x_7 &\geq r_4 \\ x_1 + x_3 + x_4 + x_6 &\geq r_5 \\ x_2 + x_3 + x_5 + x_7 &\geq r_6 \\ x_1 + x_2 + x_4 + x_6 &\geq r_7 \end{aligned} \quad (7)$$

$$x_i \geq 0 \in \Box\ for\ i = 1, 2, 3, \dots, 7 \quad (8)$$

Eqns (6) to (8) can be expressed in matrix form as:

$$Minimize\ Z = [1\ 1\ 1\ 1\ 1\ 1\ 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \quad (9)$$

subject to:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \geq \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \Box \quad (11)$$

Table 1. Travelling routes, passenger demand, bus availability and bus requirement

Travelling route	Average daily passenger demand	Daily bus availability	Daily bus requirement
Long Routes			
Uyo-Ojuelegba	340	21	25
Uyo-Ajah	47	2	4
Uyo-Maza Maza	31	2	3
Uyo-Okota	29	2	3
Uyo-Jabi	153	9	11
Uyo-Nyanya	107	6	8
Uyo-Ibadan	35	2	3
Uyo-Jos	45	3	4
Uyo-Kaduna	25	2	2
Short Routes			
Uyo-Benin	46	3	4
Uyo-Enugu	98	6	7
Uyo-Owerri	158	10	12
Uyo-Warri	95	6	7
Uyo-Bayelsa	42	3	3
Uyo-Asaba	76	5	6
Uyo-Umuahia	471	30	34
Uyo-Calabar	1,104	75	79
Uyo-Port Harcourt	1,188	80	85
Uyo-Awka	74	5	6
Uyo-Abakiliki	72	4	6
Uyo-Onitsha	70	5	5
Uyo-Aba	641	40	46

Table 1 reveals that some long routes' daily bus requirements are similar, implying that, for those that are similar, the solution will be same. Based on this premise, we establish 6 groups: **A, B, C, D, E, F,** respectively. **A** consists Uyo-Maza Maza, Uyo-Okota and Uyo-Ibadan routes. **B** consists Uyo-Ajah and Uyo-Jos routes. **C, D, E,** and **F** are the Uyo-Ojuelegba, Uyo-Jabi, Uyo-Nyanya and Uyo-Kaduna routes, respectively. Therefore, Integer Linear Program would be formulated and analyze for each of these 6 groups.

The integer linear problem for group A (G_A) is given as:

$$Minimize\ Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \quad (12)$$

subject to:

$$\begin{aligned}
x_1 \quad + x_3 \quad + x_5 \quad + x_7 &\geq 3 \\
x_2 \quad + x_4 \quad + x_6 + x_7 &\geq 3 \\
x_1 \quad + x_3 \quad + x_5 + x_6 \quad &\geq 3 \\
x_2 \quad + x_4 + x_5 \quad + x_7 &\geq 3 \\
x_1 \quad + x_3 + x_4 \quad + x_6 \quad &\geq 3 \\
x_2 + x_3 \quad + x_5 \quad + x_7 &\geq 3 \\
x_1 + x_2 \quad + x_4 \quad + x_6 \quad &\geq 3 \\
x_i \geq 0 \in \square\ for\ i = 1, 2, 3, \ldots, 7 &
\end{aligned} \quad (13)$$

and can be expressed in matrix form as:

$$Minimize\ Z = [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \quad (14)$$

subject to:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \geq \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \square \tag{16}$$

Similarly, integer linear programming formulation with daily buses requirement for the other routes are: $G_B = 4$, $G_C = 25$, $G_D = 11$, $G_E = 8$ and $G_F = 2$ with values of r_i's on the right-hand side of the constraints as 4, 25, 11, 8 and 2 respectively.

2.3 Problem formulation for short routes

Let Y_p be the number of buses allocated to the *p*th short route. Therefore,

$y_1 = \text{the number of buses allocated to Benin route}$
$y_2 = \text{the number of buses allocated to Enugu route}$
$y_3 = \text{the number of buses allocated to Owerri route}$
$y_4 = \text{the number of buses allocated to Warri route}$
$y_5 = \text{the number of buses allocated to Bayelsa route}$
$y_6 = \text{the number of buses allocated to Asaba route}$
$y_7 = \text{the number of buses allocated to Umuahia route}$
$y_8 = \text{the number of buses allocated to Calabar route}$
$y_9 = \text{the number of buses allocated to Port Harcourt route}$
$y_{10} = \text{the number of buses allocated to Awka routes}$
$y_{11} = \text{the number of buses allocated to Abakiliki route}$
$y_{12} = \text{the number of buses allocated to Onitsha route}$
$y_{13} = \text{the number of buses allocated to Aba route}$

Let T_p be the per bus ticket collection calculated as

$$T_p = Per\ Ticket\ Price\ \times Bus\ Capacity \tag{17}$$

The Per Ticket Price is shown in Table 2 for the various short routes and bus capacity is given as 14 seater. Thus,

$$T_P = 14(Per\ Ticket\ Price) \tag{18}$$

The management's objective for the short routes is to maximize profit (based on the 378 – 67 – 15 buses left in the company), taking into consideration the daily bus requirement. Therefore, the integer linear problem will be formulated as follows:

$$Maximize\ \sum_{p=1}^{13} T_p Y_p = 296 \tag{19}$$

subject to: $\sum_{p=1}^{13} Y_p = 296$

where:

$$\left.\begin{array}{r} y_1 = 4 \\ y_2 \leq 7 \\ y_3 \leq 12 \\ y_4 \leq 7 \\ y_5 \leq 3 \\ y_6 \leq 6 \\ y_7 \leq 34 \\ y_8 \leq 79 \\ y_9 \leq 85 \\ y_{10} \leq 6 \\ y_{11} \leq 6 \\ y_{12} \leq 5 \\ y_{13} \leq 46 \\ y_p \geq 0 \in \square \end{array}\right\} \quad (20)$$

This integer linear problem can be presented in matrix form as:

$$Maximize\ Z = [672\ \ 448\ \ 322\ \ 532\ \ 448\ \ 532\ \ 280\ \ 210\ \ 322\ \ 427\ \ 560\ \ 455\ \ 154] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \end{bmatrix} \quad (21)$$

subject to:

$$[1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \end{bmatrix} = 305 \quad (22)$$

$$\begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \leq \begin{bmatrix} 4 \\ 7 \\ 12 \\ 7 \\ 3 \\ 6 \\ 34 \\ 79 \\ 85 \\ 6 \\ 6 \\ 5 \\ 46 \end{bmatrix} \tag{23}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \square \tag{24}$$

In general, the optimal integer solution is reached when a feasible integer solution is achieved at a node that has an upper bound greater than or equal to the upper bound at any other ending node. For the purpose of this work, the TORA Optimization Software would be used for the analysis to ease computation, ensure efficiency and accuracy of results.

3 Analysis and Results

The various steps in obtaining optimal solution and the sub problems involved are depicted in a branch and bound diagram. Fig. 1 shows a branch and bound diagram for the long route's bus scheduling problem in group A but with optimal solution at iteration 2. Similar network diagrams were also obtained for problems group B to F for the long routes problem with 1 iteration for group B, 3 iterations with optimal solution at iteration 2 for group C, 11 iterations with optimal solution at iteration 2 for group D, 1 iteration for group E and 7 iterations with optimal solution at iteration 2 for group F.

Table 2. Summary of optimal solution of bus schedule for long routes in groups A to F

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Z
A	0	0	0	0	3	3	0	6
B	1	1	1	1	1	1	1	7
C	6	6	6	6	6	7	7	44
D	2	2	2	2	2	5	5	20
E	2	2	2	2	2	2	2	14
F	0	0	0	0	2	2	0	4

Table 2 summarizes the optimal solutions obtained for bus schedule of long routes in groups **A** to **F**. It shows the number of buses with respective shift patterns to be assigned to a particular group of routes. Specifically, a total of 6, 7, 44, 20, 14 and 4 buses are to be assigned to the routes in groups A, B, C, D, E and F, respectively. The values of $x_1 \ldots x_7$ for the various groups: A, B, C, D, E and F are the number of buses on respective off days. However, the short routes problem converges at one iteration with optimal solutions in Fig. 2.

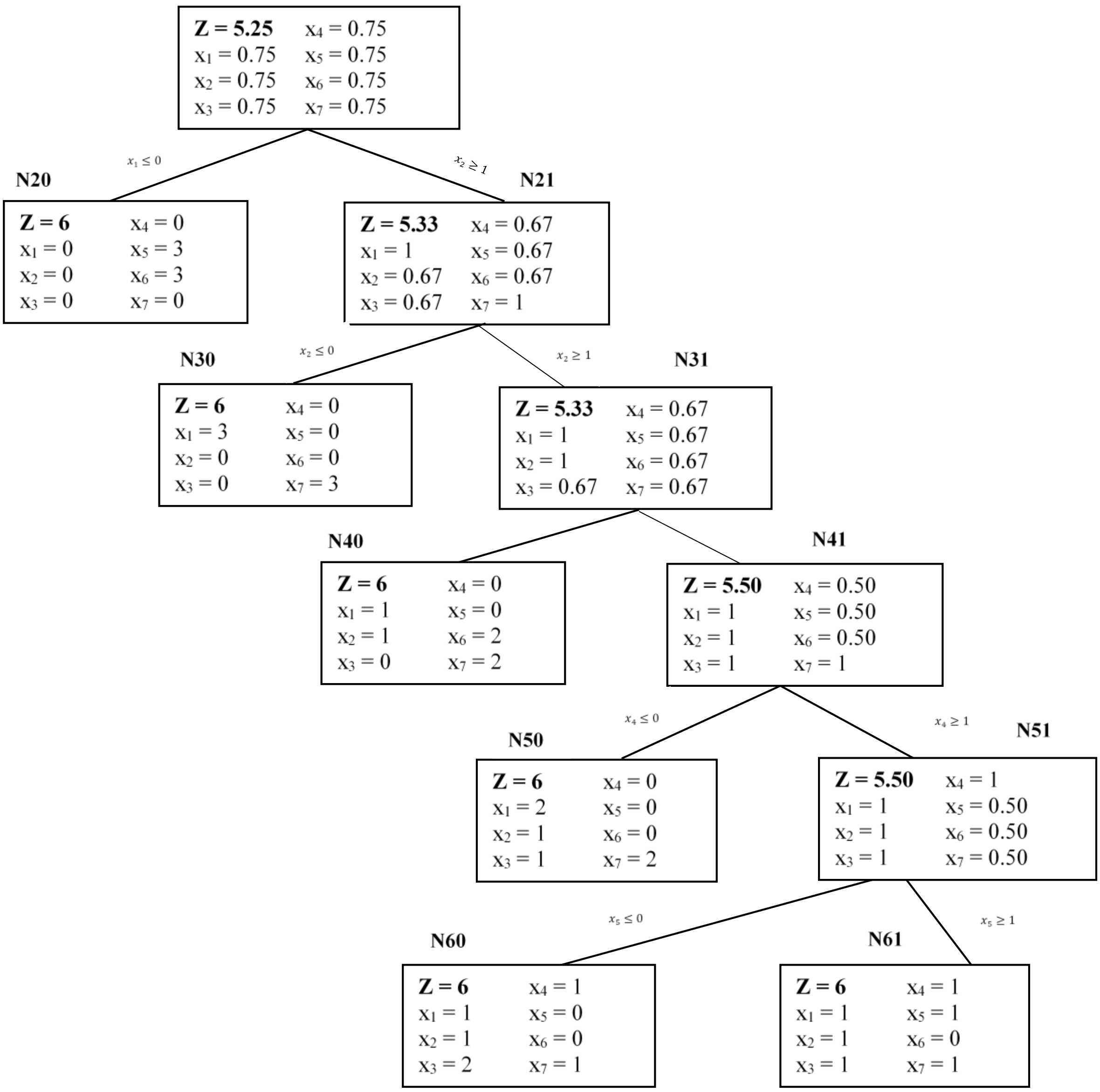

Fig. 1. An 11- iteration Branch and Bound diagram for long route bus scheduling problem in Group A

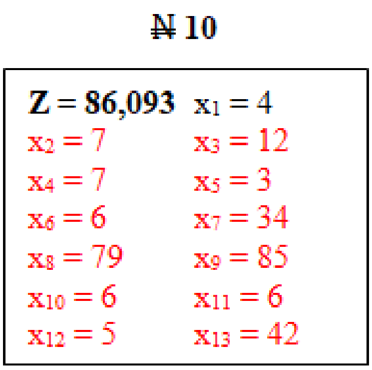

Fig. 2. A one step Branch and Bound diagram for the short routes bus scheduling problem

Table 3. Optimal allocation of buses to short routes in AKTC, Uyo

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}
Optimal allocation	4	7	12	7	3	6	34	79	85	6	6	5	42
Prior allocation	3	6	10	6	3	5	30	75	80	5	4	5	40
Deficit	1	1	2	1	0	1	4	4	5	1	2	0	2

Table 3 shows the optimal allocation, prior allocation and the deficit allocation of buses to the various short routes. The results imply that 4, 7, 12, 7, 3, 6, 34, 79, 85, 6, 6, 5 and 42 buses should be allocated to Uyo-Benin, Uyo-Enugu, Uyo-Owerri, Uyo-Warri, Uyo-Bayelsa, Uyo-Asaba, Uyo-Umuahia, Uyo-Calabar, Uyo-Port Harcourt, Uyo-Awka, Uyo-Abakiliki, Uyo-Onitsha and Uyo-Aba routes, respectively to amass respective profit of ₦67,200.00, ₦44,800.00, ₦64,400.00, ₦53,200.00, ₦0, ₦53,200.00, ₦112,000.00, ₦84,000.00, ₦161,000.00, ₦42,700.00, ₦112,000.00, ₦0 and ₦30,800.00 representing 33.33%, 16.67%, 20%, 16.67%, 0%, 20%, 13.33%, 5.33%, 6.25%, 20%, 50%, 0% and 5% in Table 4. The results further show that the daily bus requirements for each short route have been met, exception of the Aba route, where the optimum allocation of 42 buses is less than the required 46 by 4. This situation, however, will have minimal impact on the overall profit of the company, as the Aba route has the least contribution to the profit from sales of ticket, also see Table 4.

Table 4. Routes and their respective daily allocations (prior and optimal), per ticket price and prior ticket collections

S/N	Routes	Daily allocation (buses)			Per ticket price (tp) (₦)	Prior ticket collection (a_1 x tp x 14) (₦) (p_1)
		Prior daily allocation (a_1)	Optimal daily allocation			
			Upper Limit (a_2)	Lower Limit (a_3)		
1	Long routes: Uyo-Maza Maza	2	3	3	9,500.00	266000
2	Uyo-Okota	2	3	3	9,500.00	266000
3	Uyo-Ibadan	2	3	3	9,500.00	266000
4	Uyo-Ajah	2	4	4	9,500.00	266000
5	Uyo-Jos	3	4	4	10,500.00	441000
6	Uyo-Ojuelegba	21	26	25	9,500.00	2793000
7	Uyo-Jabi	9	14	11	10,000.00	1260000
8	Uyo-Nyanya	6	8	8	10,000.00	840000
9	Uyo-Kaduna	2	4	2	10,500.00	294000
						6692000
10	Short Routes: Uyo-Benin	3	4	4	4,800.00	201600
11	Uyo-Enugu	6	7	7	3,200.00	268800
12	Uyo-Owerri	10	12	12	2,300.00	322000
13	Uyo-Warri	6	7	7	3,800.00	319200
14	Uyo-Bayelsa	3	3	3	3,200.00	134400
15	Uyo-Asaba	5	6	6	3,800.00	266000
16	Uyo-Umuahia	30	34	34	2,000.00	840000
17	Uyo-Calabar	75	79	79	1,500.00	1575000
18	Uyo-Port Harcourt	80	85	85	2,300.00	2576000
19	Uyo-Awka	5	6	6	3,050.00	213500
20	Uyo-Abakiliki	4	6	6	4,000.00	224000
21	Uyo-Onitsha	5	5	5	3,250.00	227500
22	Uyo-Aba	40	42	42	1,100.00	616000
						7784000
	Total					**14476000**

Average revenue increase in naira and their corresponding percentage for both the long and short routes are obtained in Table 5.

Table 5. Calculation of average revenue increases in Naira (₦) and Percentages (%)

S/N	Routes	Optimal ticket collections (₦)		Increase (₦)		Average increase (₦) $p6 = (p_4+p_5)/2$	Average percentage(%) increase $p_7 = (p_6/p_1)*100$
		Upper Limit (a_2 x tp x 14)= (p_2)	Lower Limit (a_3 x tp x 14)= (p_3)	Upper Limit $p_4 = p_2 - p_1$	Lower Limit $p_5 = p_3 - p_1$		
1	Uyo-Maza Maza	399000	399000	133000	133000	133000	50.00
2	Uyo-Okota	399000	399000	133000	133000	133000	50.00
3	Uyo-Ibadan	399000	399000	133000	133000	133000	50.00
4	Uyo-Ajah	532000	532000	266000	266000	266000	100.00
5	Uyo-Jos	588000	588000	147000	147000	147000	33.33
6	Uyo-Ojuelegba	3458000	3325000	665000	532000	598500	21.43
7	Uyo-Jabi	1960000	1540000	700000	280000	490000	38.89
8	Uyo-Nyanya	1120000	1120000	280000	280000	280000	33.33
9	Uyo-Kaduna	588000	294000	294000	0	147000	50.00
						2327500	**34.78**
10	Uyo-Benin	268800	268800	67200	67200	67200	33.33
11	Uyo-Enugu	313600	313600	44800	44800	44800	16.67
12	Uyo-Owerri	386400	386400	64400	64400	64400	20.00
13	Uyo-Warri	372400	372400	53200	53200	53200	16.67
14	Uyo-Bayelsa	134400	134400	0	0	0	0.00
15	Uyo-Asaba	319200	319200	53200	53200	53200	20.00
16	Uyo-Umuahia	952000	952000	112000	112000	112000	13.33
17	Uyo-Calabar	1659000	1659000	84000	84000	84000	5.33
18	Uyo-Port Harcourt	2737000	2737000	161000	161000	161000	6.25
19	Uyo-Awka	256200	256200	42700	42700	42700	20.00
20	Uyo-Abakiliki	336000	336000	112000	112000	112000	50.00
21	Uyo-Onitsha	227500	227500	0	0	0	0.00
22	Uyo-Aba	646800	646800	30800	30800	30800	5.00
						825300	**10.60**
	Total					**3152800**	**21.78**

Table 6. Schedule pattern for long routes problem in group A

S/N	SUN	MON	TUE	WED	THU	FRI	SAT
1	Off	On	Off	On **	On	Off *	On
2	On	Off	On	Off	Off **	On	Off *
3	On	Off	On	Off	Off **	On	Off *
4	On	Off	On	Off	Off **	On	Off *
5	Off	On	Off	On **	On	Off *	On
6	Off	On	Off	On **	On	Off *	On
Optimal	3	3	3	3	3	3	3

* *Indicates the start of an Off*

** *Indicates the start of work for the next alternating week*

Table 6 is a sample bus schedule pattern for long routes in group A. Data in Table 1 show daily allocation of 2 buses each to members of G_A: Uyo-Maza Maza, Uyo-Okota and Uyo-Ibadan routes with daily requirement of 3 buses each as against an optimal allocation of 6 buses to each group member in order to satisfy the daily requirement in Table 2. Similarly, optimal allocation of buses was obtained for groups B to F as contained in Table 2.

The values of the decision variables in Table 2, for instance $x_i = 0$; $i = 2,2,3,4,7$ imply that zero number of idle buses is associated with Tuesday-Thursday-Saturday, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday-Wednesday and Wednesday-Friday-Sunday off shifts, respectively and as a result no bus should be assigned to the Uyo-Maza Maza, Uyo-Okota and Uyo-Ibadan routes. Also, $x_j = 0$; $j = 5$ and 6 implies that 3 idle buses are associated with Friday-Sunday-Tuesday and Thursday-Saturday-Monday off shifts, respectively and same should be assigned to these routes, and so on. This arrangement would lead to an average increase in daily ticket collection for these routes by ₦133,000.00 which represents a 50% increase as shown in Table 5.

Similarly, data in Table 1 show that 2 and 3 buses were respectively allocated to Uyo-Ajah and Uyo-Jos routes with daily requirements of 4 buses as against an optimal allocation of 7 buses required daily for each of these routes to satisfy the daily requirement in Table 2. The values of $x_i =1;\ i=1,2,3,4,5,6,7$ implies that one bus should be assigned to the Uyo-Ajah and Uyo-Jos routes on Tuesday-Thursday-Saturday, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday-Wednesday, Friday-Sunday-Tuesday, Thursday-Saturday-Monday and Wednesday-Friday-Sunday, respectively. This assignment would increase daily ticket collection by ₦266,000.00 and ₦147,000.00, representing 100% and 33.33% for the Uyo-Ajah and Uyo-Jos routes respectively as shown in Table 4. Also, the required allocation to Uyo-Ojuelegba route in G_C is 25 buses daily but with actual allocation of 21 buses. This is against an optimal allocation of 44 buses required to satisfy the daily requirement as shown in Table 2. Also, the values of $x_i =6;\ i=1, 2, 3, 4,5$ in Table 2 shows that 6 buses should be assigned to Tuesday-Thursday-Saturday, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday-Wednesday and Friday-Sunday-Tuesday off shift while $x_j =7;\ j=6$ *and* 7 shows that 7 buses should be assigned to Thursday-Saturday-Monday and Wednesday-Friday-Sunday off shift, respectively to the Ojuelegba route. The daily ticket collection for the Ojuelegba route with this optimal allocation would increase by ₦598,500.00, representing 21.43% increase as shown in Table 5.

Actual bus allocation for the Jabi route in G_D is 9 in Table 1 with daily requirement need of 11 buses. However, the optimal allocation is 20 buses to be able to satisfy the daily requirement of 11 buses. The values of the decision variables: $x_i =2;\ i=1,2,3,4,5$ in Table 2 implies that 2 buses should be assigned to Tuesday-Thursday-Saturday, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday-Wednesday and Friday-Sunday-Tuesday off shift while $x_j =5;\ j=6$ *and* 7 shows that 5 buses should be assigned to Thursday-Saturday-Monday and Wednesday-Friday-Sunday off shift, respectively to the Jabi route. The daily ticket collection for this route under this optimal allocation would increase by ₦490,000.00, representing 38.89% increase as shown in Table 5.

In addition, actual bus allocation for the Nyanya route in G_E is 6 buses with daily requirement of 8 buses as shown in Table 1, whereas an optimal allocation of 14 buses is required to satisfy the daily requirement. Also, the values of the decision variables: $x_i =2;\ i=1,2,3,4,5,6,7$ in Table 2 imply that 2 buses should be assigned to Tuesday-Thursday-Saturday, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday-Wednesday, Friday-Sunday-Tuesday, Thursday-Saturday-Monday and Wednesday-Friday-Sunday off shift to Nyanya route. From Table 5, the daily ticket collection for this route would increase by ₦280,000.00, representing 33.33% increase under this optimal arrangement.

Finally for the Kaduna route in G_F the actual allocation is 2 with daily requirement of 2 in Table 1. However, an optimal allocation of 4 buses is required to satisfy its daily requirement. The decision variables' value: $x_i =0;\ i=1,2,3,4,7$ in Table 2 implies that no bus should be assigned to Tuesday-Thursday-Saturday, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday-Wednesday and Wednesday-Friday-Sunday off shift to Kaduna route. Rather, 2 buses should be assigned to this route for Friday-Sunday-Tuesday and Thursday-Saturday-Monday off shift. From Table 5, this arrangement will increase the ticket collection by ₦147,000.00 representing 50% increase. Therefore, this propose bus scheduling system will yield an overall daily increase in ticket collection by ₦3,152,800.00 which represents a 21.78% increase.

4 Conclusion

In this work, a bus scheduling problem was considered with a study of AKTC, Uyo. The problem was formulated as integer linear program with branch and bound network diagrams for long routes in group A as shown in Fig. 1 and short routes as shown in Fig. 2. The optimal solutions are presented in Tables 2, 3 and 5.

These results show that for long routes, an optimal allocation of 6, 7, 44, 20, 14 and 4 buses should be made to the routes in groups **A, B, C, D, E** and **F**. For the short routes, it was shown that an optimal allocation of 4, 7, 12, 7, 3, 6, 34, 79, 85, 6, 6, 5 and 42 buses should be made to Uyo-Benin, Uyo-Enugu, Uyo-Owerri, Uyo-Warri, Uyo-Bayelsa, Uyo-Asaba, Uyo-Umuahia, Uyo-Calabar, Uyo-Port Harcourt, Uyo-Awka, Abakiliki, Uyo-Onitsha and Uyo-Aba travelling routes, respectively to maximize profit from ticket collections. These optimal solution would result in increased ticket sales on the long, short and overall routing by ₦2,327,500.00, ₦825,300.00 and ₦3,152,800.00 with corresponding percentages: 34.78%, 10.60% and 21.78% respectively, while meeting daily passenger demands for the various routes and ensuring proper bus allocation system. These results are in line with [6,9,10] and [16,17], among others who have shown that the use of integer program for vehicle, personnel and other scheduling problems yield efficient schedule, increase profit and overall turnover. Hence, the use of integer program (IP) model is recommended for scheduling purposes in transport companies, hotels, security organizations and power distributing companies, among others. This would ensure optimal allocation of resources for increase turnover and optimum profit.

Disclaimer

Authors have declared that they have no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper.

Competing Interests

Authors have declared that no competing interests exist.

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