

Research Article

Regarding New Traveling Wave Solutions for the Mathematical Model Arising in Telecommunications

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This research paper focuses on the application of the tanh function method to find the soliton solutions of the (2+1)-dimensional nonlinear electrical transmission line model. Materials used to form a transmitting line are very important to transmit electric charge. In this sense, we find some new voltage behaviors such as dark, trigonometric, and complex function solutions. Choosing some suitable values of parameters, we present some various surfaces of results obtained in this paper. These results play an important role in telecommunications lines used to stand for wave propagations.

1. Introduction

Nonlinear partial differential equations (NPDEs) are recently used to investigate the meanings of physical problems such as fluid dynamics, mathematics, physics and quantum field theory, and nonlinear fiber optics. Systems of NPDEs are also considered as a main tool to investigate in chemical and biological experiments. Some methods such as index the extended tanh method [1, 2], the sine cosine method [3], the inverse scattering transform method [4], finite difference method [5], tanh and extended tanh methods [6–8], Jacobi elliptic function expansion method [9], modified expansion method [10], generalized tanh method [11, 12], sine-Gordon expansion method [13], extended mean value theorem [14], and interval-valued fuzzy topsis method [15] to obtain various solutions of such NPDEs have been presented

in the literatures. Cordero et al. have observed the stability analysis of the fourth-order iterative method in [16]. The (3 +1) Dimensional Boiti-Leon-Manna-Pempinelli equation has been deeply studied in [17]. Using time scale calculus, discrete normal vector field approximation has been presented in [18]. A Handy Technique has been handled in [19]. Classical Boussinesq equations have been immensely studied in [20]. Traveling wave solutions of nonlinear Klein Gordon equation were observed in [21] and so on [15, 17, 18, 22–43].

The contents of this paper are as follows. Section 2 presents the general properties of the tanh function method (TFM) [44]. This TFM has been proposed as a strong and creditable method for finding the solutions of the nonlinear models. Section 3 introduces some new complex dark, trigonometric, and hyperbolic soliton solutions to the nonlinear

electrical transmission line. To express physical properties in terms of mathematical dynamics, such as wave propagation of electrical transmission lines, has been presented by equation defined as [9]

$$v_{tt} - \alpha(v^2)_{tt} + \beta(v^3)_{tt} - w_0^2 \delta_1^2 v_{xx} - w_0^2 \frac{\delta_1^4}{12} v_{xxxx} - \omega_0^2 \delta_2^2 v_{yy} - \omega_0^2 \frac{\delta_2^4}{12} v_{yyyy} = 0, \quad (1)$$

where $v = v(x, y, t)$ is used to explain the tightness throughout the electrical line and x and y are interpreted like the promulgation distance. t is the period, and α and β are constants with nonzero. δ_1 is the space between two proximate sections in during longitudinally flank, while δ_2 is the space between two proximate sections in the transversal flank [9]. With the help of some computational programs, we are able to plot in terms of 2D, 3D, and contour surfaces of the results theoretically found. Finally, the main conclusions are given in the last section of the paper.

2. The Tanh Function Method

In this part of the paper, we present the general properties of the tanh function method in a detailed manner [45–48].

$$P(u, u_x, u_y, u_t, u_{xx}, uu_{xyt}, \dots) = 0, \quad (2)$$

where P is a polynomial in the dependent variable u . Considering the traveling wave transformation as $u = u(x, y, t) = U(\xi)$, $\xi = k(x + y - ct)$, we obtain the following nonlinear ordinary differential equation

$$N(U, U^2, U', U'', U''', \dots) = 0, \quad (3)$$

with N is a polynomial of $U = U(\xi)$. Now, finding the traveling wave solutions to Eq. (2) is equivalent to obtain the solution to reduced ordinary differential Eq. (3), and it can be introduced a new independent variable defined as

$$Y(\xi) = \text{Tanh}(\xi). \quad (4)$$

We can find the following for some derivations as

$$\begin{aligned} \frac{d}{d\xi}(\cdot) &= (1 - Y^2) \frac{d}{dY}(\cdot), \\ \frac{d^2}{d\xi^2}(\cdot) &= (1 - Y^2) \left[-2Y \frac{d}{dY}(\cdot) + (1 - Y^2) \frac{d^2}{dY^2}(\cdot) \right], \\ \frac{d^3}{d\xi^3}(\cdot) &= \dots, \\ &\vdots \end{aligned} \quad (5)$$

As the last step, we present the tanh series as being

$$U(\xi) = S(Y) = \sum_{i=0}^m a_i Y^i, \quad (6)$$

where m is a positive integers. The values of m , generally, with the help of the balance principle can be determined.

3. Mathematical Analysis

In this part of the paper, we find some new complex dark, trigonometric, and hyperbolic function solutions of Eq. (1) by using TFM. First of all, we consider the traveling wave transformation defined as

$$v = V(\xi), \quad \xi = k(x + y - ct), \quad (7)$$

where k, c are nonzero constants or complex-valued parameters. When considering Eq. (7) into Eq. (1), we find

$$\begin{aligned} c^2 V_{\xi\xi} - \alpha c^2 (V^2)_{\xi\xi} + \beta c^2 (V^3)_{\xi\xi} - w_0^2 \delta_1^2 V_{\xi\xi} \\ - k^2 w_0^2 \frac{\delta_1^4}{12} V_{\xi\xi\xi\xi} - \omega_0^2 \delta_2^2 V_{\xi\xi} - k^2 \omega_0^2 \frac{\delta_2^4}{12} V_{\xi\xi\xi\xi} = 0. \end{aligned} \quad (8)$$

Integrating Eq. (8) twice with regard to ξ , setting the constants of integrations to zero yields

$$12[c^2 - w_0^2 \delta_1^2 - \omega_0^2 \delta_2^2] V + 12\beta c^2 V^3 - 12\alpha c^2 V^2 - k^2 [w_0^2 \delta_1^4 + \omega_0^2 \delta_2^4] V'' = 0. \quad (9)$$

According to the general properties of TFM, it can be considered as

$$V = S = \sum_{m=0}^M a_m Y^m, \quad (10)$$

Substituting Eqs. (5), (10) into Eq. (9) gives

$$\begin{aligned} [12(c^2 - w_0^2 \delta_1^2 - \omega_0^2 \delta_2^2) S + 12\beta c^2 S^3 - 12\alpha c^2 S^2 \\ - k^2 (w_0^2 \delta_1^4 + \omega_0^2 \delta_2^4) (1 - Y^2) \left(-2Y \frac{dS}{dY} + (1 - Y^2) \frac{d^2 S}{dY^2} \right)] = 0. \end{aligned} \quad (11)$$

Using the balance rule, M can be found as

$$M = 1, \quad (12)$$

which result in

$$S = a_0 + a_1 Y. \quad (13)$$

Substituting Eq. (13) into Eq. (11) by getting necessary derivations presents

$$\begin{aligned} [12(c^2 - w_0^2 \delta_1^2 - \omega_0^2 \delta_2^2) (a_0 + a_1 Y) + 12\beta c^2 (a_0 + a_1 Y)^3 \\ - 12\alpha c^2 (a_0 + a_1 Y)^2 - k^2 (w_0^2 \delta_1^4 + \omega_0^2 \delta_2^4) (1 - Y^2) (-2a_1 Y)] = 0. \end{aligned} \quad (14)$$

After some calculations, it can be obtained as follows:

$$Y^0: 12(c^2 a_0 - w_0^2 \delta_1^2 a_0 - \omega_0^2 \delta_2^2 a_0) + 12\beta c^2 a_0^3 - 12\alpha c^2 a_0^2 = 0,$$

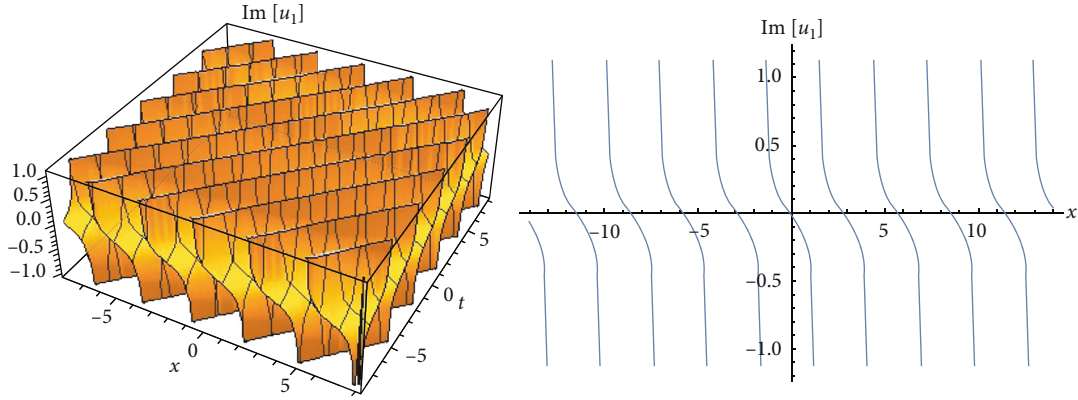


FIGURE 1: 2D and 3D surfaces of imaginary part of Eq. (16).

$$\begin{aligned}
 Y^1: & 12(c^2a_1 - \omega_0^2\delta_1^2a_1 - \omega_0^2\delta_2^2a_1) + 36\beta c^2a_0^2a_1 - 24\alpha c^2a_0a_1 \\
 & + 2k^2\omega_0^2\delta_2^4a_1 + 2k^2\omega_0^2\delta_1^4a_1 = 0, \\
 Y^2: & 36\beta c^2a_0a_1^2 - 12c^2\alpha a_1^2 = 0, \\
 Y^3: & 12\beta c^2a_1^3 - 2k^2\omega_0^2\delta_2^4a_1 - 2k^2\omega_0^2\delta_1^4a_1 = 0.
 \end{aligned}$$

Solving this system yields the following cases:

Case 1. When

$$\begin{aligned}
 a_0 &= \frac{\alpha}{3\beta}; a_1 = \frac{\alpha}{3\beta}; k = \frac{i\sqrt{6}\sqrt{\alpha^2(\omega_0^2\delta_1^2 + \delta_2^2\omega_0^2)}}{\sqrt{(2\alpha^2 - 9\beta)(\delta_1^4\omega_0^2 - \delta_2^4\omega_0^2)}}; c \\
 &= \frac{3i\sqrt{\beta(\omega_0^2\delta_1^2 + \delta_2^2\omega_0^2)}}{\sqrt{2\alpha^2 - 9\beta}},
 \end{aligned} \tag{15}$$

we get the following complex trigonometric function solution

$$v_1(x, y, t) = \frac{\alpha}{3\beta} + \frac{i\alpha}{3\beta} \tan \left[\frac{i\sqrt{6}\sqrt{\alpha^2(\omega_0^2\delta_1^2 + \delta_2^2\omega_0^2)}(x + y - (3i\sqrt{\beta(\omega_0^2\delta_1^2 + \delta_2^2\omega_0^2)}/\sqrt{2\alpha^2 - 9\beta})t)}{\sqrt{(2\alpha^2 - 9\beta)(\omega_0^2\delta_1^4 + \delta_2^4\omega_0^2)}} \right], \tag{16}$$

in which $\alpha, \beta, \delta_1, \delta_2,$ and ω_0 are constant and nonzero. Choosing suitable values of coefficients in Eq. (16), we can observe some (Figures 1 and 2).

Case 2. Choosing as

$$\begin{aligned}
 a_0 &= \frac{\alpha}{3\beta}; a_1 = \frac{\alpha}{3\beta}; c = \frac{3ik\delta_2\omega_0\sqrt{\beta(\delta_1^2 - \delta_2^2)}}{\sqrt{6\alpha^2 + k^2(2\alpha^2 - 9\beta)\delta_1^2}}, \omega_0 \\
 &= \frac{i\delta_2\omega_0\sqrt{6\alpha^2 + k^2\delta_2^2(2\alpha^2 - 9\beta)}}{\sqrt{6\alpha^2\delta_1^2 + k^2(2\alpha^2 - 9\beta)\delta_1^4}},
 \end{aligned} \tag{17}$$

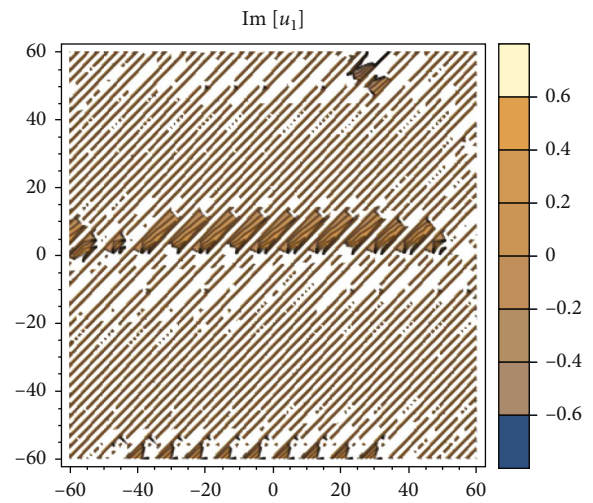


FIGURE 2: Contour surface of imaginary part of Eq. (16).

we get another new complex dark function solution

$$v_2(x, y, t) = \frac{\alpha}{3\beta} + \frac{\alpha}{3\beta} \tanh \left[kx + ky - \frac{3ik^2\delta_2\omega_0\sqrt{\beta(\delta_1^2 - \delta_2^2)}}{\sqrt{6\alpha^2 + k^2(2\alpha^2 - 9\beta)\delta_1^2}} t \right], \tag{18}$$

with strain conditions are $2\alpha^2 - 9\beta > 0, \beta(\delta_1^2 - \delta_2^2) > 0,$ and also $\alpha, \beta, k, \delta_1, \omega_0, \delta_2$ are real constants and nonzero or complex-valued parameters. Considering some values of parameters under the strain conditions, different wave patterns can be observed from (Figures 3 and 4) for Eq. (18).

Case 3. Selecting

$$\begin{aligned}
 a_0 &= \frac{\alpha}{3\beta}; a_1 = \frac{\alpha}{3\beta}; c = \frac{-3ik\delta_2\omega_0\sqrt{\beta(\delta_1^2 - \delta_2^2)}}{\sqrt{6\alpha^2 + k^2(2\alpha^2 - 9\beta)\delta_1^2}}, \omega_0 \\
 &= \frac{-i\delta_2\omega_0\sqrt{6\alpha^2 + k^2\delta_2^2(2\alpha^2 - 9\beta)}}{\sqrt{6\alpha^2\delta_1^2 + k^2(2\alpha^2 - 9\beta)\delta_1^4}},
 \end{aligned} \tag{19}$$

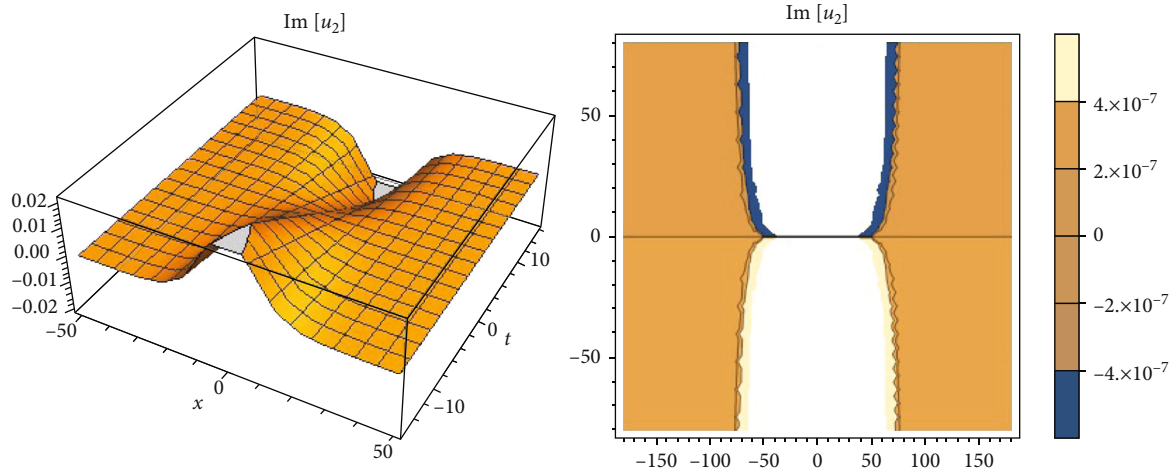


FIGURE 3: 3D and contour graphs of imaginary part of Eq. (18).

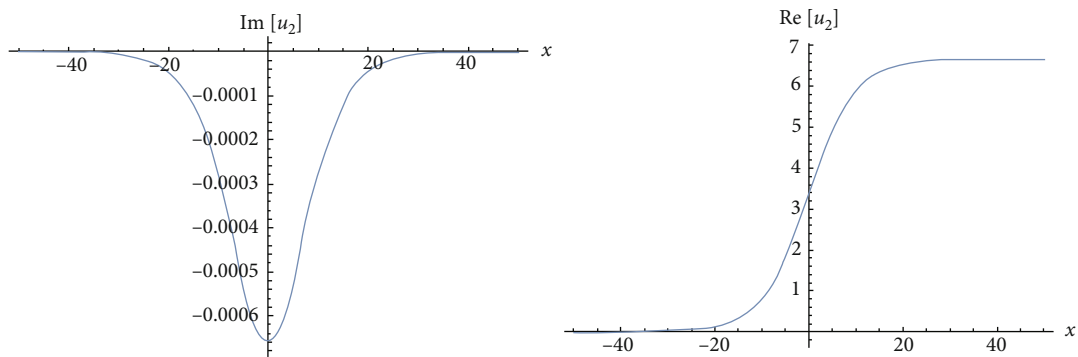


FIGURE 4: 2D figures of imaginary and real parts of Eq. (18).

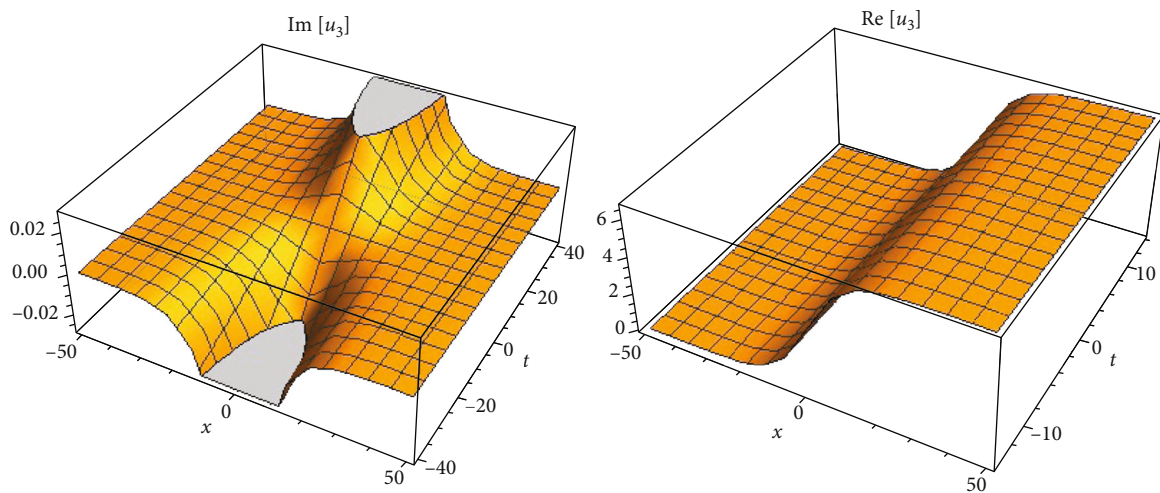


FIGURE 5: 3D graph of imaginary and real parts of Eq. (20).

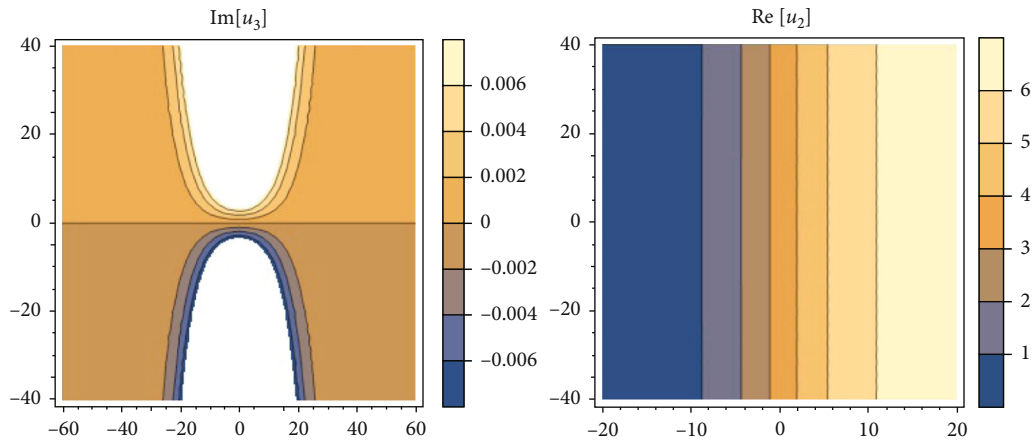


FIGURE 6: Contour figures of imaginary and real parts of Eq. (20).

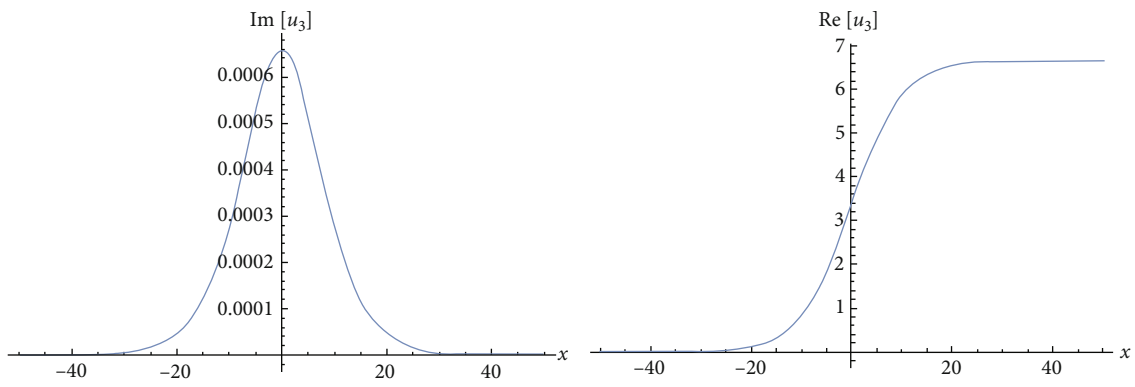


FIGURE 7: 2D figures of imaginary and real parts of Eq. (20).

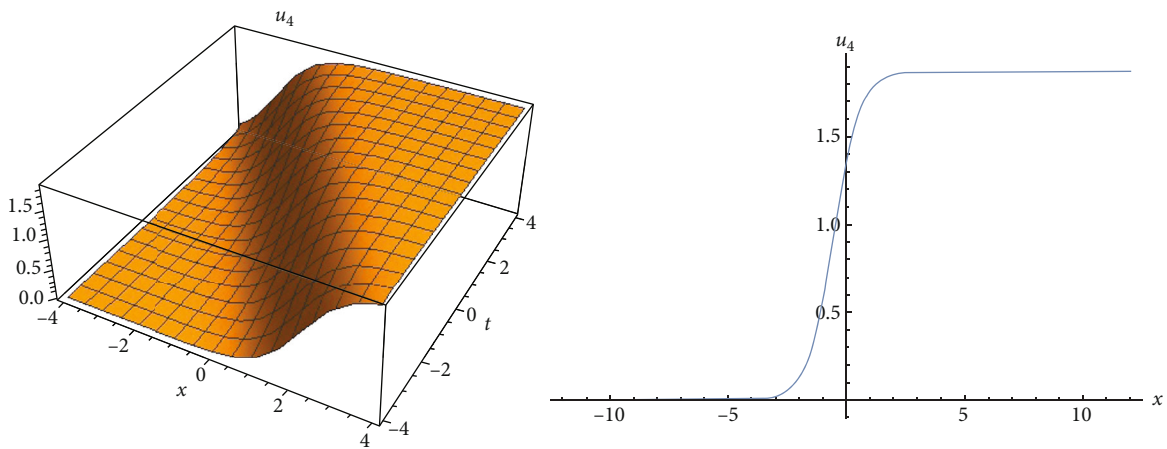


FIGURE 8: 2D and 3D simulations of Eq. (22).

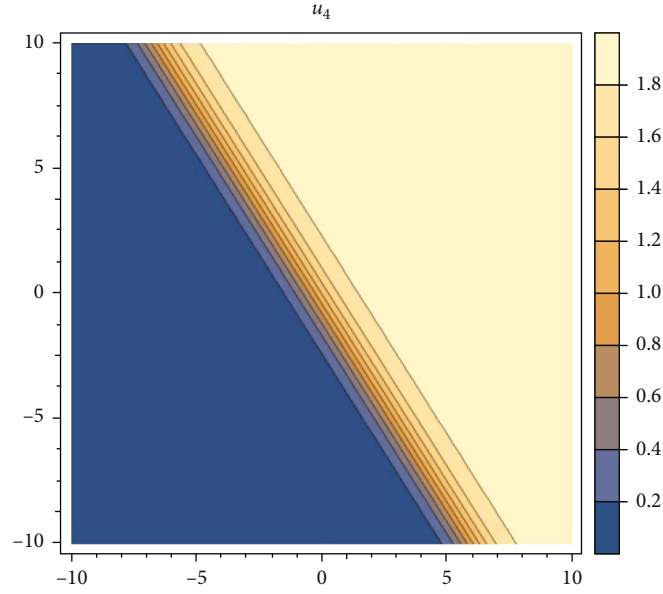


FIGURE 9: Contour graph of Eq. (22).

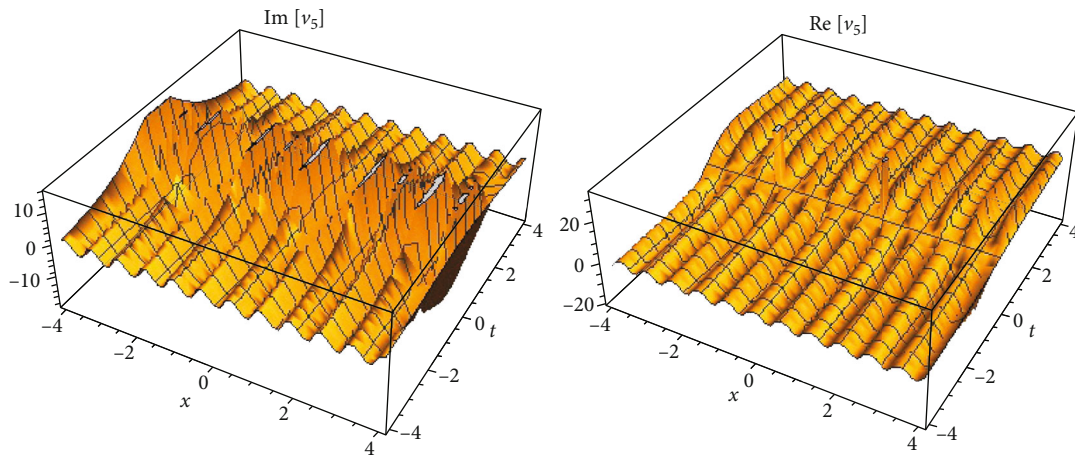


FIGURE 10: 3D figures of imaginary and real parts of Eq. (24).

we get conjugate new complex dark function solution as

$$v_3(x, y, t) = \frac{\alpha}{3\beta} + \frac{\alpha}{3\beta} \tanh \left[kx + ky + \frac{3ik^2\delta_2\omega_0\sqrt{\beta(\delta_1^2 - \delta_2^2)}}{\sqrt{6\alpha^2 + k^2(2\alpha^2 - 9\beta)\delta_1^2}} t \right], \quad (20)$$

with strain conditions are $2\alpha^2 - 9\beta > 0$, $\beta(\delta_1^2 - \delta_2^2) > 0$, and also $\alpha, \beta, k, \delta_1, \omega_0, \delta_2$ are real constants and nonzero or complex-valued parameters. 3D, 2D, and contour surfaces of Eq. (20) can be also seen (Figures 5–7) with the strain conditions.

Case 4. Choosing as

$$a_0 = a_1 = \frac{3k^2(w_0^2\delta_1^4 + \delta_2^4\omega_0^2)}{2\alpha(w_0^2\delta_1^2(3 + k^2\delta_1^2) + \delta_2^2(3 + k^2\delta_2^2))}, c = \frac{-1}{\sqrt{3}} \sqrt{w_0^2\delta_1^2(3 + k^2\delta_1^2) + \delta_2^2\omega_0^2(3 + k^2\delta_2^2)},$$

$$\beta = \frac{2\alpha^2}{9k^2(w_0^2\delta_1^4 + \delta_2^4\omega_0^2)} (w_0^2\delta_1^2(3 + k^2\delta_1^2) + \delta_2^2\omega_0^2(3 + k^2\delta_2^2)), \quad (21)$$

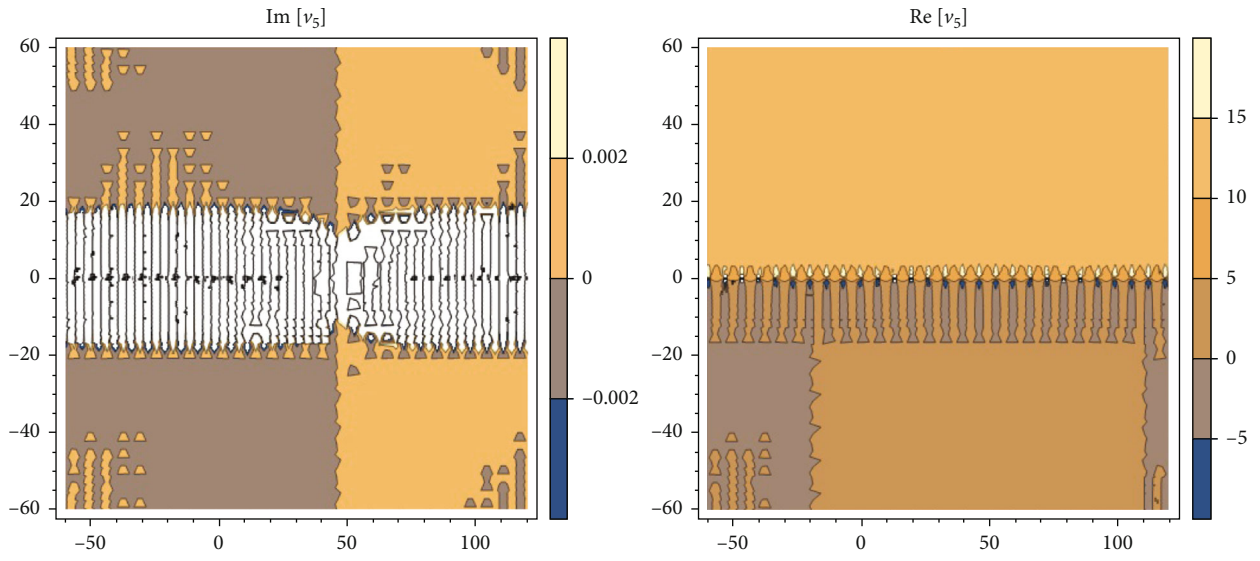


FIGURE 11: Contour figures of imaginary and real parts of Eq. (24).

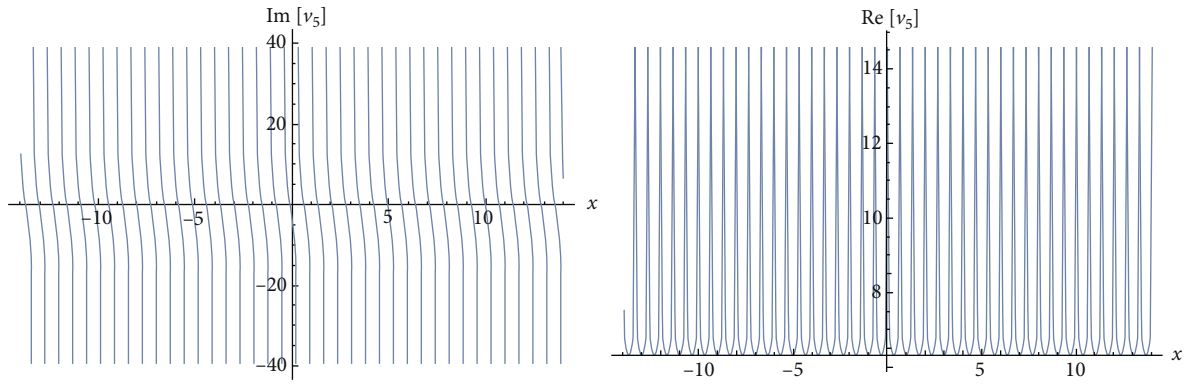


FIGURE 12: 2D figures of imaginary and real parts of Eq. (24).

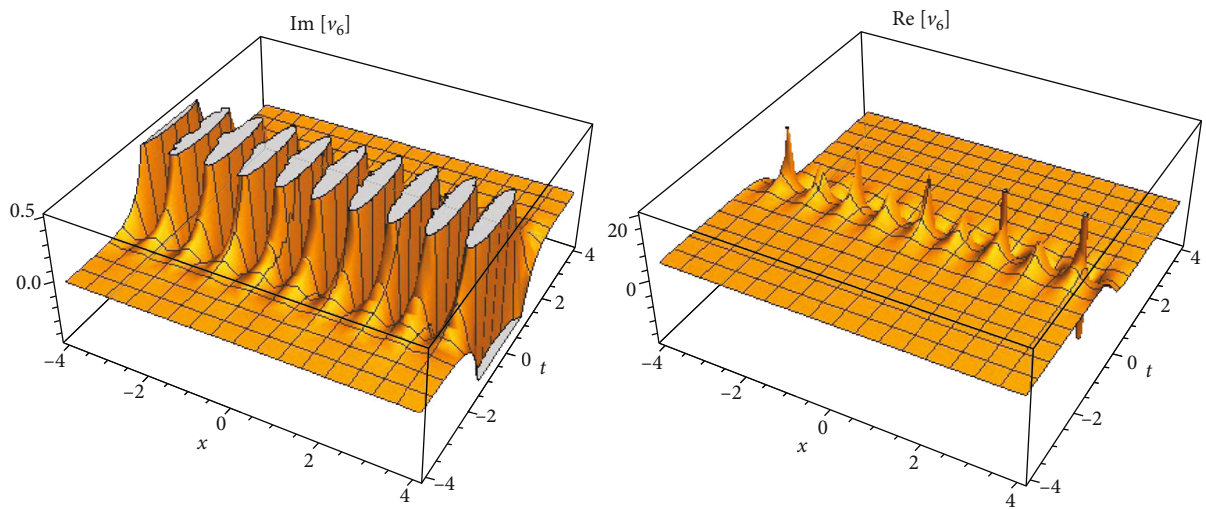


FIGURE 13: 2D and 3D of simulations of imaginary and real parts of Eq. (26).

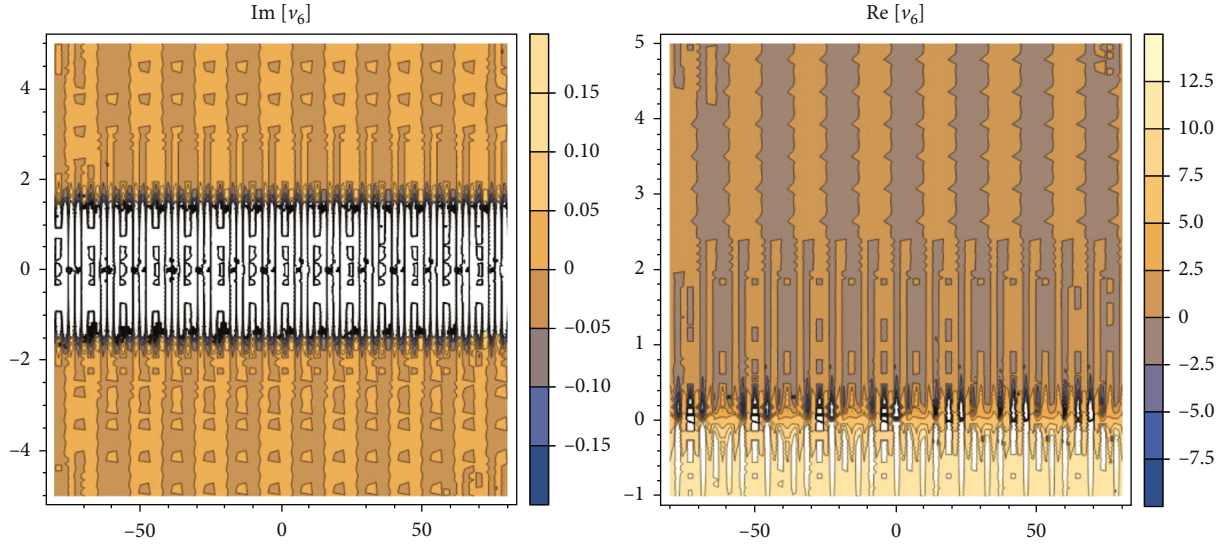


FIGURE 14: Contour surfaces of imaginary and real parts of Eq. (26).

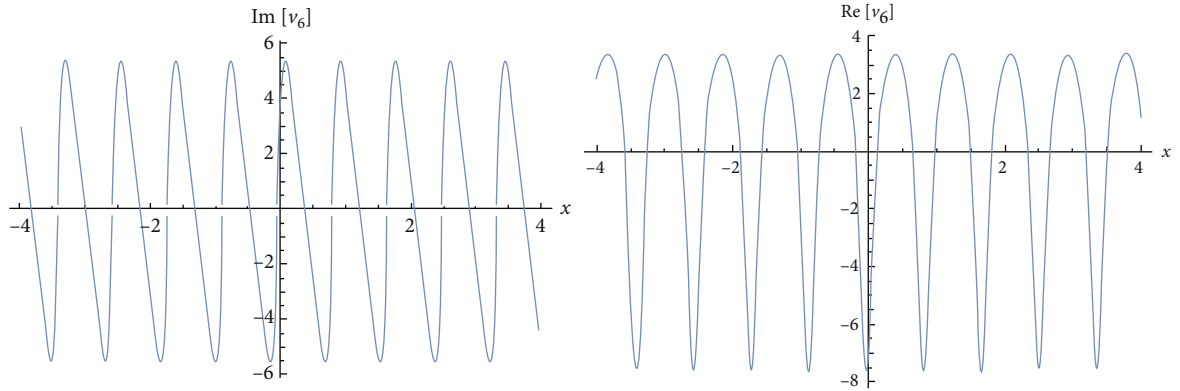


FIGURE 15: 2D figures of imaginary and real parts of Eq. (26).

produces the following dark soliton solution

$$v_4(x, y, t) = \frac{3k^2(\omega_0^2\delta_1^4 + \delta_2^4\omega_0^2)}{2\alpha(\omega_0^2\delta_1^2(3 + k^2\delta_1^2) + \delta_2^2(3 + k^2\delta_2^2))} \cdot \left(1 + \tan h \left[kx + ky + \frac{k}{\sqrt{3}}\omega t \right]\right) \quad (22)$$

in which $\omega = \sqrt{\omega_0^2\delta_1^2(3 + k^2\delta_1^2) + \delta_2^2\omega_0^2(3 + k^2\delta_2^2)}$ and strain conditions are $\alpha, k, \delta_1, \omega_0, \delta_2$ are real constants and nonzero. 3D, 2D, and contour surfaces of Eq. (22) can be also observed (Figures 8 and 9).

Case 5. Taking as

$$a_0 = \frac{\alpha}{3\beta}, a_1 = \frac{\alpha}{3\beta}, k = \frac{-i\sqrt{6}\sqrt{\alpha^2(\omega_0^2\delta_1^2 + \delta_2^2\omega_0^2)}}{\sqrt{(2\alpha^2 - 9\beta)(\delta_1^4\omega_0^2 + \delta_2^4\omega_0^2)}}, c = \frac{-3i\sqrt{\beta(\omega_0^2\delta_1^2 + \delta_2^2\omega_0^2)}}{\sqrt{2\alpha^2 - 9\beta}}, \quad (23)$$

produces another complex dark traveling wave solution to the governing model

$$v_5(x, y, t) = \frac{\alpha}{3\beta} - \frac{i\alpha}{3\beta} \tan \left[\frac{\sqrt{6}\sqrt{\alpha^2(\omega_0^2\delta_1^2 + \delta_2^2\omega_0^2)}(x + y + (3i\sqrt{\beta(\omega_0^2\delta_1^2 + \delta_2^2\omega_0^2)}/\sqrt{2\alpha^2 - 9\beta})t)}{\sqrt{(2\alpha^2 - 9\beta)(\omega_0^2\delta_1^4 + \delta_2^4\omega_0^2)}} \right], \quad (24)$$

with strain conditions $\beta > 0, 2\alpha^2 > 9\beta$ and $\alpha, \beta, \omega_0, \delta_1, \delta_2$ are real constants and nonzero. 3D, 2D, and contour surfaces of Eq. (24) can be also observed in (Figures 10–12).

Case 6. Once we select other coefficients given as

$$a_0 = \frac{\alpha}{3\beta}, a_1 = \frac{\alpha}{3\beta}, k = \frac{-i\sqrt{6}\sqrt{\alpha^2(\omega_0^2\delta_1^2 + \delta_2^2\omega_0^2)}}{\sqrt{(2\alpha^2 - 9\beta)(\delta_1^4\omega_0^2 + \delta_2^4\omega_0^2)}}, c = \frac{3i\sqrt{\beta(\omega_0^2\delta_1^2 + \delta_2^2\omega_0^2)}}{\sqrt{2\alpha^2 - 9\beta}}, \quad (25)$$

results in another complex dark traveling wave solution to the Eq. (1)

$$v_6(x, y, t) = \frac{\alpha}{3\beta} - \frac{i\alpha}{3\beta} \tan \left[\frac{\sqrt{6\sqrt{\alpha^2(\omega_0^2\delta_1^2 + \delta_2^2\omega_0^2)}(x+y - (3i\sqrt{\beta(\omega_0^2\delta_1^2 + \delta_2^2\omega_0^2)/\sqrt{2\alpha^2 - 9\beta}})t)}{\sqrt{(2\alpha^2 - 9\beta)(\omega_0^2\delta_1^4 + \delta_2^4\omega_0^2)}} \right], \quad (26)$$

with strain conditions $\beta > 0$, $2\alpha^2 > 9\beta$ and $\alpha, \beta, \omega_0, \delta_1, \delta_2$ are real constants and nonzero. Various surfaces of Eq. (26) with the considering suitable values of parameters can be also presented as (Figures 13–15).

4. Conclusions

In this manuscript, TFM being one of the powerful techniques has been successfully used to Eq. (1). Many new trigonometric, complex, and hyperbolic function solutions have been extracted, afterwards. The conditions which guarantee the existence of the valid solutions to this model are also given in a detailed manner. Considering the strain conditions of coefficients of results, various simulations have been also plotted by using some computational programs. These solutions are of various physical properties of the electrical transmission line. For example, the tanh function arises in gravitational potential as a dark structure [49]. Hence, it is estimated that the solution of v_4 is of such physical property. From (Figures 1–15), it can be also seen that the results simulate estimated wave behaviors. TFM used in this paper can be considered to solve other nonlinear problems arising in the theory of solitons and other areas of nonlinear science [50–56].

Data Availability

This work is not based on any data.

Conflicts of Interest

The authors declare that there is no conflict of interest.

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