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On the Significance of Density of an Electrically Charged Quantum Particle

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ABSTRACT

Expressions of a conserved 4-current and its density are examined in cases of electrically charged quantum particles. It is proven that the Noether theorem cannot be regarded as a sufficient condition that yields a consistent expression of a conserved 4-current, because electromagnetic interactions pose further constraints. A mathematical analysis shows that the Dirac linear equation yields a consistent expression for a 4-current. In contrast, second order quantum equations, such as the Klein-Gordon equation of an electrically charged scalar particle and the electroweak equation of the $W^{\pm}$ vector particles do not provide a consistent expression for a conserved 4-current that adequately describes electromagnetic interactions.

Keywords: Density; quantum theories; lagrangian function; electromagnetic interactions.

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1 INTRODUCTION

This paper aims to prove the existence of new constraints on the acceptability of an expression for density of an elementary massive quantum particle that carries an electric charge.

The notion of density is known for quite a long time. Charge density is used in the Maxwell equation $\nabla \cdot \mathbf{E} = 4\pi\rho$. Furthermore, the development of quantum mechanics proves that the Schroedinger equation yields an expression for the particle density $\rho = \psi^*\psi$. The present work examines quantum theories of an electrically charged elementary pointlike particle. In this case charge density is proportional to particle density. It is shown in this work that the laws of Maxwellian electrodynamics add new theoretical requirements to the notion of density in quantum theories. This work uses the well known laws of quantum theories and of Maxwellian electrodynamics and provides a comparison between these aspects of density. This point expresses the novelty of this work.

Classical electrodynamics shows that charge density and current density are included in the continuity equation

$$\nabla \cdot \mathbf{j} + \partial\rho/\partial t = 0. \tag{1.1}$$

This equation is an important element of the discussion presented in this work. Maxwellian electrodynamics can be written in a relativistic form [1]. In particular, the charge density and the electric current of (1.1) are components of the 4-vector $(\rho, \mathbf{j})$.

Electrodynamics, quantum mechanics and special relativity are very important physical theories and modern technology relies on them. These issues indicate that the notion of density is a significant element of the study of theoretical physics.

Fundamental physical principles that are used as a basis for the main analysis are mentioned briefly in the second section. The third section describes the crucial role of density in quantum theories. The fourth section compares density expressions derived from the first order Dirac equation with corresponding expressions of second order quantum equations of an electrically charged particle, like the Klein-Gordon (KG) equation and the electroweak $W^{\pm}$ equation. Problems of these issues are discussed in the fifth section. The last section contains concluding remarks.

2 THEORETICAL BACKGROUND

Several theoretical elements that are used in the analysis presented in this work are briefly pointed out below. Units where $\hbar = c = 1$ are used. Greek indices run from 0 to 3. The metric is diag. (1,-1,-1,-1). Relativistic expressions are written in the standard notation. Square brackets [ ] denote the dimension of the enclosed expression. In a system of units where $\hbar = c = 1$ there is just one dimension, and the dimension of length, denoted by $[L]$, is used.

- Accelerators provide abundant data where colliding particles move at a speed which is very close to the speed of light. The design of these machines and the analysis of their data are based on special relativity. Hence, the operation of these accelerators and their data provide a solid experimental basis for the validity of special relativity. For this reason, the analysis carried out below takes a relativistic covariant form. Relativistic expressions are written in the standard notation.
- The correspondence principle states that an appropriate limit of a higher rank theory fits properties of its lower rank theory. For example, the Ehrenfest theorem proves that the classical limit of Quantum Mechanics (QM) fits the laws of classical physics (see e.g. [2], pp. 25-27, 137, 138). In particular, the single particle limit of Quantum Field Theory (QFT) fits Relativistic Quantum Mechanics (RQM) and the nonrelativistic limit of the latter fits QM. The following quotation indicates that this relationship between these quantum theories is already recognized in the literature. "First, some good news: quantum field theory is based on the same quantum mechanics that was invented by Schroedinger, Heisenberg, Pauli, Born, and others in 1925-26, and has been

used ever since in atomic, molecular, nuclear and condensed matter physics" (see [3], p. 49). In this work, these constraints on QFT are called Weinberg correspondence principle. A general discussion of correspondence between physical theories can be found in the literature (see [4], pp. 1-6).

- The de Broglie principle defines the relations between dynamical properties of a massive quantum particle and its phase Φ. It means that every quantum theory of such a particle must provide a consistent expression for the particle's phase. This phase is an argument of an exponential function

$$\exp(i\Phi) = 1 + i\Phi + ... \tag{2.1}$$

Dimensional balance of the two terms that stand on the right hand side of (2.1) and relativistic covariance of these terms mean that the phase Φ must be a dimensionless Lorentz scalar. In the classical limit, the phase of the wave function is written in terms of the action S (see [5], p. 20)

$$\psi = ae^{iS}. \tag{2.2}$$

It follows that in a relativistic theory the action must be a dimensionless Lorentz scalar.

- The electromagnetic interaction term of the Lagrangian density is

$$\mathcal{L}_{int} = -j^\mu A_\mu. \tag{2.3}$$

Here j^μ is the 4-current of the charged particle and A_μ is the electromagnetic 4-potential. As stated after (1.1), charge density is the 0-component of j^μ. Expression (2.3) is a Lorentz scalar whose dimension is $[L^{-4}]$. It is used in calculations of the action S in classical physics and in quantum theories (see [1], p. 75; [3], p. 349).

$$S = S_{Matter} - \int j^\mu A_\mu d^4x - \frac{1}{16\pi}\int F_{\mu\nu}F^{\mu\nu}d^4x. \tag{2.4}$$

Here the last term is the integral of the electromagnetic fields Lagrangian density. It follows that like the electromagnetic fields Lagrangian density, quantum theories of an electrically charged particle should use a Lagrangian density which is a Lorentz scalar whose dimension is $[L^{-4}]$.

- The quantum function which is used in expressions for the Lagrangian density takes the form $\psi(x^\mu)$, where x^μ denotes a *single* set of four space-time coordinates. For this reason, the quantum function $\psi(x^\mu)$ describes an elementary pointlike particle. Indeed, a function that describes a composite particle needs a larger number of independent variables, because it needs four independent variables in order to describe the probability of the existence of the particle at x^μ and other variables that describe the structure of the composite particle. It turns out that standard textbooks use the form $\psi(x^\mu)$ for quantum theories of elementary particles (see e.g. [3, 6, 7, 8]). This approach is also used in the present work.

3 THE ROLE OF DENSITY IN QUANTUM THEORIES

Relying on the Weinberg correspondence principle, one concludes that the Hilbert space is a fundamental element of quantum theories (see e.g. [3], p. 49; [9], pp. 164-166). A well-defined scalar product of two functions is a requirement needed for a Hilbert space of a quantum theory. The form of the scalar product of any two quantum functions of a single particle is

$$< a, b >= \int \psi_a^\dagger \psi_b \, d^3r. \tag{3.1}$$

A Hilbert space has a basis and each of its functions can be written as a linear combination of functions of this basis. A convenient basis of the Hilbert space is made of orthonormal functions. If ψ_a, ψ_b belong to such a basis then

$$< a, b >= \int \psi_a^\dagger \psi_b \, d^3r = \delta_{ab}, \tag{3.2}$$

where δ_{ab} is the Kronecker symbol which equals unity if $a = b$ and is zero otherwise.

The meaning of (3.2) is that the product $\psi_a^\dagger\psi_a$ defines the density of the quantum particle represented by ψ_a. These arguments together with the Weinberg correspondence principle prove that any quantum theory should provide a consistent expression for density.

Another reason for the need of a consistent expression for density applies to theories of an electrically charged quantum particle. The electromagnetic interaction term of a Lagrangian density (2.3) holds in QFT as well as in classical electrodynamics. The 0-component of the 4-current of a particle is its density (see [1], p. 75). Hence, an electrically charged quantum particle should have a consistent expression for density.

The need for density is an example showing powerful properties of the variational principle and of its Lagrangian density. Here the Noether theorem says that if the Lagrangian density is invariant under a given transformation then a corresponding conservation law exists. The Noether theorem has quite a few applications. For the purpose of this work, the following invariance of the Lagrangian density under a global phase transformation

$$\psi(x^\mu) \to \exp(i\alpha)\psi(x^\mu) \tag{3.3}$$

is examined (see [10], p. 314). In this case one obtains the following form of a conserved 4-current

$$j^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi. \tag{3.4}$$

Hereafter, this expression is called the Noether theorem for the 4-current.

This work examines expressions for the 4-current that are obtained from an application of the Noether theorem (3.4) to specific quantum theories of electrically charged particles. Evidently, the Noether theorem for the 4-current (3.4) depends on the quantum function ψ and on its derivatives $\partial_\mu\psi$ but it is independent of the electric charge. The main objective of the present work is to examine how (3.4) fits the requirements of Maxwellian electrodynamics in cases of quantum theories of an electrically charged particle. As stated above, in the case of an elementary pointlike particle the electric 4-current is proportional to the particle's 4-current. Therefore, a transition between expressions of these quantities is quite clear.

4 THE LAGRANGIAN DENSITY OF CHARGED QUANTUM PARTICLES

The Lagrangian density of electrically charged quantum particles and the associated 4-current j^μ are examined below. The Noether theorem (3.4) is used for a derivation of this 4-current. The analysis applies to cases of first order quantum equation (namely, the Dirac equation) and of second order quantum equations, like the KG equation and the electroweak equation of the $W^\pm$. The dimension of physical quantities used in a Lagrangian density plays an important role in this analysis.

The Lagrangian density of a Dirac particle and its interaction with electromagnetic fields is (see [6], p. 84, [7], p. 78)

$$\begin{aligned}\mathcal{L}_D &= \bar\psi[\gamma^\mu(i\partial_\mu) - m]\psi - eA_\mu\bar\psi\gamma^\mu\psi \\ &= \bar\psi[\gamma^\mu(i\partial_\mu - eA_\mu) - m]\psi.\end{aligned} \tag{4.1}$$

Here γ^μ denote the four Dirac γ matrices and $\bar\psi \equiv \psi^\dagger\gamma^0$. An application of the Noether theorem of the 4-current (3.4) to the Dirac Lagrangian density (4.1) yields the following expression for the electric 4-current

$$j^\mu_{Dirac} = -e\bar\psi\gamma^\mu\psi \tag{4.2}$$

(see [6], p. 84, [10], p. 315). This expression for the Dirac 4-current proves that the last term of the first line of (4.1) is consistent with the form of the electromagnetic interaction (2.3). The substitution

$$i\partial_\mu \to i\partial_\mu - eA_\mu \tag{4.3}$$

used in the second line of (4.1) is called the minimal interaction or the minimal substitution (see [3], p. 9; [6], p. 84; [11], p. 198). The minimal interaction (4.3) shows how expressions of a free electrically charged quantum particle transform to expressions that hold for this particle in Maxwellian fields. The equivalence of the two lines of (4.1) means that in the case of Dirac equation, the minimal interaction (4.3) is consistent with the standard form of the electromagnetic interaction (2.3).

An important property of the Dirac equation is that the dimension of each variable enclosed in the square brackets of (4.1) is $[L^{-1}]$. Therefore, the $[L^{-4}]$ dimension of the Lagrangian density means that the dimension of a product of two Dirac function $\bar{\psi}\psi$ is $[L^{-3}]$, which is the dimension of density. This is consistent with the form (4.2) of the Dirac 4-current.

Let us turn to second order quantum equations of electrically charged particles, like that of a charged KG particle and the electroweak equation for the $W^{\pm}$. The Lagrangian density of the KG equation of a free particle is

$$\mathcal{L}_{KG} = g^{\mu\nu}\phi^*_{,\mu}\phi_{,\nu} - m^2\phi^*\phi \qquad (4.4)$$

(see [3], p. 21; [6], p. 38; [11], p. 191). The $[L^{-4}]$ dimension of the Lagrangian density proves that the dimension of the KG function ϕ of (4.4) is $[L^{-1}]$. Hence, the dimension of the product of two KG functions $\phi^*\phi$ is $[L^{-2}]$. The $[L^{-3}]$ dimension of density proves that, unlike the case of a Dirac particle, a KG expression for density must depend on a derivative of its function ϕ with respect to x^μ. Hence, the general form of the 4-current of a KG particle is a function of the following variables

$$j_\mu = f(\phi^*, \phi, \phi^*_{,\mu}, \phi_{,\mu}). \qquad (4.5)$$

Simple dimensional arguments prove that this form applies also to the electroweak 4-current of the $W^{\pm}$. The electromagnetic interaction term $-ej^\mu A_\mu$ of (2.3) means that in the case of second order quantum equations, the form (4.5) *adds derivatives of the quantum function ϕ to the Lagrangian density and alters the 4-current j^μ which is obtained from the Noether theorem for the 4-current (3.4).* Consequences of this problematic point are discussed in the rest of this work.

In the KG case, an application of the Noether theorem for the 4-current (3.4) to the KG Lagrangian density (4.4) yields the following expression

$$j_\mu = i(\phi^*\phi_{,\mu} - \phi^*_{,\mu}\phi) \qquad (4.6)$$

(see [6], p. 40; [11], p. 193). If electromagnetic fields exist at the KG particle's location then one finds that the minimal interaction (4.3) casts the KG Lagrangian density (4.4) into the following form

$$\mathcal{L}_{KG} = g^{\mu\nu}(\phi^*_{,\mu} + eA_\mu)(\phi_{,\nu} - eA_\mu) - m^2\phi^*\phi \qquad (4.7)$$

(see [11], p. 198). This form is unacceptable because Maxwell equations are derived from a Lagrangian density that depends *linearly* on the 4-potential A_μ (see [1], pp. 78-80).

Realizing this discrepancy, let us see what comes out if the minimal interaction (4.3) is ignored. In order to be consistent with Maxwellian electrodynamics, one examines the following expression where the 4-current (4.6) of a free KG particle interacts linearly with an electromagnetic 4-potential

$$\mathcal{L}_{KG\_LIN} = ie(\phi^*\phi_{,\mu} - \phi^*_{,\mu}\phi)A^\mu. \qquad (4.8)$$

This attempt fails because it has already been proved that this kind of interaction yields contradictory results (see [12], pp. 6-8 or the Appendix of this work).

It turns out that the Lagrangian density of the electroweak $W^{\pm}$ contains a product of derivatives of the W function which is analogous to the KG form of (4.4). Unlike the KG function ϕ which is a Lorentz scalar, the electroweak $W^{\pm}$ is a 4-vector which is analogous to the electromagnetic 4-potential A_μ. The relevant term of the W^+ Lagrangian density is

$$\mathcal{L}_W = -\frac{1}{2}|\bar{D}_\mu W^+_\nu - \bar{D}_\nu W^+_\mu|^2 + ..., \qquad (4.9)$$

where D_μ is the electroweak extension of the electromagnetic minimal interaction (4.3) (see [8], p. 518). This D_μ is a sum of three terms: a partial derivative with respect to x^μ, an electromagnetic 4-potential term which is analogous to that of the minimal interaction (4.3) and an electroweak term.

The foregoing analysis proves that unlike the case of the Dirac equation, the Lagrangian density of a second order quantum equation contains a product of derivatives, like that of (4.4). It follows that the minimal substitution (4.3) yields a Lagrangian density that contains *quadratic terms* of the 4-potential A_μ. This property holds for the KG case (4.7) as well as for the electroweak W case (4.9). As pointed out after (4.7), these terms are inconsistent with Maxwellian electrodynamics, where Maxwell equations are derived from a Lagrangian density which depends *linearly* on the 4-potential A_μ (see [1], pp. 78-80). This outcome means that in the case of a second order quantum equation, electromagnetic interactions are inconsistent with the minimal interaction (4.3). Issues pertaining to this dilemma are discussed in the next section.

5 DISCUSSION

The electromagnetic interaction (2.3) couples two kinds of physical objects, an electric charge and an electromagnetic 4-potential. This interaction depends linearly on both the electric charge e and on the 4-potential A_μ. For this reason, the dimension of A_μ must take an integral number. In electromagnetic interactions the dimension of the 4-potential A_μ is $[L^{-1}]$. On the other hand, in quantum theories density (and charge density) depends quadratically on the wave function of an electrically charged quantum particle.

These physical properties explain the fundamental difference between the first order Dirac equation of a spin-1/2 particle on the one hand and the second order KG and the $W^\pm$ equations on the other hand. Some of these differences are quite well known. For example, a Dirac particle has spin-1/2 and several Dirac particles of the same species must be in an antisymmetric state which abides by the Pauli exclusion principle. By contrast, particles that are described by second order quantum equations have an integral spin and several such particles of the same species must be in a symmetric state. The difference between these kinds of particles has a far reaching effect on the world as we know it. For example, the number of electrons of a neutral atom grows together with the number of protons in the atomic nucleus. Due to the Pauli exclusion principle, atomic electrons fill higher and higher energy states. This is the theoretical explanation of the Mendeleev periodic table which is a fundamental element of the richness of chemical states (see e.g. [2], pp. 278-281). By contrast, in a hypothetical world where electrons are replaced by a negatively charged KG particle, atomic states should be symmetric. Therefore, all negatively charged KG particles of such an atom would be in the same lowest energy state (called s-wave), and all chemical elements would behave like a noble gas.

The following discussion addresses some new inherent differences between these kinds of particles which are relevant to this work.

Let us compare the theoretical role of a Dirac function ψ with that of the KG function ϕ and the electroweak $W^\pm$ functions. Concerning interaction terms, a quantum function of an electrically charged Dirac particle plays *one* role – it provides a consistent expression for the electric 4-current (4.2). The case of quantum functions of the KG and the $W^\pm$ particles is different. Here the quantum function plays *two* distinct roles. A KG function ϕ has an integral spin because it carries a Yukawa-like interaction whose form is $g\bar{\psi}\psi\phi$ (see [7], pp. 79, 80). Here ψ is a function of a fermion and the dimension of $\bar{\psi}\psi$ is $[L^{-3}]$. Similarly, the electroweak theory states that the $W^\pm$ particles carry the weak interaction, and their form is analogous to that of the electromagnetic 4-potential A_μ (see [7], p. 701; [13], p. 307). Hence, the dimension of the quantum functions of the KG and the $W^\pm$ particles is $[L^{-1}]$. Moreover, these particles have a *second* task, because they carry an electric charge which interacts with electromagnetic fields. Therefore, theories of these particles must provide an expression for the 4-current of (2.3), whose dimension is $[L^{-3}]$. This is the reason for the use of derivatives in expressions for density in theories of electrically charged KG and $W^\pm$ particles. It is proved in the previous section that this derivative is inconsistent with the well known rule where the minimal interaction (4.3) represents electrodynamics.

The failure of the KG theory and the electroweak $W^\pm$ theory to provide a consistent expression for charge density means that these theories also cannot provide a consistent expression for particle's density. Hence, there is no consistent Hilbert space for these particles. This is yet another contradiction because the Weinberg correspondence principle says that a quantum theory should have a Hilbert space (see [3], p. 49).

Let us examine properties of a well established physical theory. In this case, the scientific literature presents it in consistent and equivalent forms where the differences between various presentations belong to the pedagogical domain. Furthermore, as a rule, textbooks discuss all key element of the given theory. This situation holds in the Dirac equation. In contrast, the scientific literature contains inconsistent treatments of the electromagnetic interaction of second order quantum equations, namely, the cases of an electrically charged KG particle and the electroweak $W^{\pm}$ bosons. Another attribute of an established QFT equation is that it provides an adequate description of physical properties of an experimentally confirmed elementary particle. Here are few examples that illustrate these issues.

1. Pauli and Weisskopf revived the KG equation in 1934 [11]. Their Lagrangian density of a charged KG particle uses the minimal interaction (4.3) (see [11], p. 198). For this reason their Lagrangian density contains terms that depend quadratically on the electromagnetic 4-potential A_μ (see [11], p. 198). It is explained above that this property is incompatible with Maxwellian electrodynamics (see the discussion near the end of section 4).

2. By contrast, the following textbook points out problems that exist with the derivatives included in the interaction term of the KG equation and states that "they appear with a vengeance, since the coupling prescription (15.1) introduces interaction terms containing derivatives" (see [6], p. 87). (Note that (15.1) of this textbook is the above mentioned minimal interaction (4.3).)

3. The primary contradiction of an electrically charged KG particle is the impossibility to write down a consistent interaction term of a 4-current of this field with a Maxwellian 4-potential. This term must take the form of (2.3), where j^μ is a conserved 4-current that depends on the KG function ϕ and A_μ is the electromagnetic 4-potential. It is explained in item 1 above why the minimal interaction (4.3) fails to reach this goal. Furthermore, it has already been proved that a violation of the minimal interaction (4.3) leads to inconsistent results (see the text after eq. (4.8)).

4. There is no experimental support for a KG particle [14]. In particular, pions are known to be composite particles and each of which is a quark-antiquark bound state.

5. The following textbook presents a form of the $W^{\pm}$ electromagnetic interaction term of the electroweak Lagrangian density (see [15], p. 530). The first line of eq. (87.27) of this textbook contains two terms where each of which has a product of two D_μ operators

$$D_\mu = \partial_\mu - ie(A_\mu + ...). \tag{5.1}$$

A product of two such operators (like $D^{\dagger\mu}D_\nu$) contains a term which is proportional to the square of the electric charge e^2 *and* to a quadratic expression of the electromagnetic 4-potential $A^\mu A_\nu$. Either of these results is inconsistent with Maxwellian electrodynamics where the interaction term (2.3) is proportional to the electric charge e and depends linearly on the 4-potential A_μ.

Furthermore, this textbook introduces another electromagnetic interaction term whose form is

$$ieF^{\mu\nu}W^+_\mu W^-_\nu. \tag{5.2}$$

Here the charge carrier function W provides a dimension of $[L^{-2}]$ which contradicts the $[L^{-3}]$ requirement of charge density of Maxwellian electrodynamics.

6. A similar Lagrangian density can be found in another textbook (see [8], p. 522). Hence, the same contradictions persist.

7. A quite different approach is taken by the authors of [16], and their approach is adopted by the ATLAS group at CERN [17]. Eq. (3) of [17] depends linearly on the electromagnetic quantities A_μ and $F^{\mu\nu}$. It means that they do not use the minimal interaction (4.3). Their equation can be reduced to the ordinary electroweak expression of the Standard Model (SM). Here one uses specific values of their coefficients $g_1^V = k_V = 1$ and $\lambda_V = 0$ and finds that the electromagnetic interaction part of the $W^{\pm}$ Lagrangian density is

$$\mathcal{L}_{W\_EM} = -ie(W^\dagger_{\mu\nu}W^\mu A^\nu - W^\dagger_\mu W^{\mu\nu}A_\nu) - ieW^\dagger_\mu W_\nu F^{\mu\nu}. \quad (5.3)$$

Here the commutativity of Maxwellian factors A_μ is used. The definition of $W_{\mu\nu}$ is analogous to that of the electromagnetic field $F_{\mu\nu}$, where $W_{\mu\nu} = W_{\nu,\mu} - W_{\mu,\nu}$. The discussion presented in the Appendix proves that this expression yields an inherent contradiction.
On top of that, it turns out that like the case of the $W^\pm$ Lagrangian density (5.2), the last term of (5.3) contains a product of two W functions and the dimension of this product is $[L^{-2}]$. Hence, the same contradiction exists.

8. Some textbooks simply avoid the dilemma and do not mention explicitly the electromagnetic interaction of the $W^\pm$. For example, it can be found in the literature a statement emphasizing the fact that electromagnetic interactions of the $W^\pm$ "are critical for the internal consistency of the theory" (see [18], p. 79). Another statement of this textbook points out the importance of the Lagrangian, which is "the foundation on which virtually all modern theories are predicated" and that the Lagrangian "concerns the fundamental quantum field theories from which the Feynman rules derive" (see [18], p. 353). In spite of these quite strong statements, this textbook does not show an explicit form of the electromagnetic interactions term of the $W^\pm$ which is included in the Lagrangian density.

The following words summarize the above mentioned arguments. In the case of an electrically charged Dirac particle, all relevant scientific textbooks use equivalent expressions for the charge's 4-current (4.2) (and for its density) and for the associated electromagnetic interaction term (2.3). In contrast, the scientific literature contains contradictory expressions for the electromagnetic interactions of the electroweak $W^\pm$ bosons. All expressions for the $W^\pm$ electromagnetic interactions are inconsistent with Maxwellian electrodynamics. Moreover, some textbooks simply do not discuss this interaction. An analogous situation holds for an electrically charged KG particle.

6 CONCLUSIONS

This work examines density expressions of two sets of quantum equations – linear equations of the Dirac theory and second-order quantum equations. The discussion is restricted to quantum equations of an electrically charged particle. The conserved 4-current

$$j^\mu_{,\mu} = 0, \quad (6.1)$$

which is obtained from the well known Noether theorem (3.4) is examined. It is proved above that the required consistency of electromagnetic interactions *adds further constraints on the acceptability of a quantum expression for density.* For this reason, in the case of an electrically charged quantum particle, the Noether expression (3.4) is not a sufficient condition for a consistent density expression. It is proved above that the Dirac equation meets all requirements, whereas contradictions are obtained from the second order quantum equation of an electrically charged KG particle as well as from the $W^\pm$ electroweak equation. These negative conclusions are compatible with Dirac's lifelong objection to second order quantum equations (see [19], pp. 1-8).

COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX

It is proved here that the electromagnetic part of the effective Lagrangian (5.3) of [16, 17] contains inherent contradictions. The proof is analogous to that of the corresponding linear interaction of the electrically charged KG theory (4.8) (see [12], pp. 6-8).

Let us examine a motionless W^+ particle located at an inner point of a quite large spherical shell. This shell is covered uniformly with electric charge and the same electrostatic potential V holds at all points inside the shell. The uncertainty principle means that due to the macroscopic size of the shell, effects of the W^+ linear momentum can be ignored. Therefore, the phase of this motionless W^+ is

$$W^+ = e^{-i(m+U)t}\Psi, \tag{A.1}$$

where m denotes the mass of the W^+, $U = eV$ is the electrostatic interaction energy and Ψ denotes other elements of the W^+ quantum function.

The second order electroweak equation of the W^+ proves that the contribution of the phase to energy terms of the W^+ equation is

$$E_{PHASE} = (m+U)^2. \tag{A.2}$$

It follows that the phase energy terms that depend on the electrostatic interaction U are

$$E_{PHASE\_U} = 2mU + U^2. \tag{A.3}$$

Let us tern to the electromagnetic interaction of the effective Lagrangian density (5.3) of [16, 17]. Here one finds two terms containing a product of the electromagnetic 4-potential with a first derivative of the W^+. Hence, the energy obtained from these terms is

$$E_{INT\_U} = 2mU + 2U^2. \tag{A.4}$$

A comparison of (A.3) with (A.4) proves that there is an additional U^2 term that cannot be compensated. It means that an imbalance holds and the W^+ equation of motion of [16, 17] contains an inherent contradiction.

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