Current Journal of Applied Science and Technology

40(12): 33-42, 2021; Article no.CJAST.69236
ISSN: 2457-1024
(Past name: British Journal of Applied Science & Technology, Past ISSN: 2231-0843, NLM ID: 101664541)

Stress-Strength Reliability Quantification using M-Transformed Exponential Distributions

Adil H. Khan[1*] and T. R. Jan[2]

[1]*University of Kashmir Srinagar, India.*

Authors' contributions

This work was carried out in collaboration between both authors. Author AHK designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript and Authors TRJ managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/CJAST/2021/v40i1231379
Editor(s):
(1) Dr. Alessandro Buccolieri, Università del Salento, Italy.
Reviewers:
(1) Zakariya Yahya Algamal, University of Mosul, Iraq.
(2) G. Saravanakumar, M. Kumarasamy College of Engineering (Autonomous), India.
(3) M. Parimala, Bannari Amman Institute of Technology, India.
Complete Peer review History: http://www.sdiarticle4.com/review-history/69236

Original Research Article

Received 25 March 2021
Accepted 31 May 2021
Published 04 June 2021

ABSTRACT

The term "Stress-Strength reliability" means $P(X > Y)$, where a system or an equipment with random strength X is subjected to random stress Y in a way that system breaks down, if the stress surpasses the strength. In this paper, a system is considered with standby redundancy, and it is presumed that the distinct components in the system for both stress and strength variables are independent and have different probability distributions viz. M- Transformed Exponential, Exponential, Gamma and Lindley. The expressions for the marginal reliabilities $R(1)$, $R(2)$, $R(3)$ etc. based on its stress-strength models are obtained.

Keywords: Reliability; M-transformed exponential distribution; stress; strength.

1. INTRODUCTION

The reliability of a system is defined as the probability that the system will perform its assigned function successfully under desired environmental conditions. The collocation 'Stress-Strength' was first naturalized by Church and Harris. In 'stress-strength models', strength

**Corresponding author: E-mail: Khanadil_192@yahoo.com;*

X of the system or equipment and stress Y enforced on it by its operational environments are considered as random variables. The expression $R = P(Y < X)$ is the reliability of the system and is defined as the probability that the system or the equipment is strong enough to surpass the stress enforced on it. In a standby system, there are a lot of connected units out of which only one unit works at a time and the rest of units remains standby. When the strike of stress crosses the strength of working unit, it fails and the substitute unit among standbys is activated (if available) and faces the impacting force of stresses of the operating system. The comprehensive system fails when all the units including standbys have failed. The history of this problem is very deep starting with the creative work of Z. W. Birnbaum and R. C. McCarty [1] in which they assumed a problem of procuring a one-sided distribution free confidence interval for $P = P(Y < X)$. The collocation 'Stress-Strength' was first naturalized by Church and Harris [2]. A lot of valuable investigative work has been done since then, both from a parametric and non-parametric viewpoint. Kakati and Sriwastav [3] works with a redundant Stress-Strength Model. The excellent monograph "The Stress–Strength Model and its Generalizations" by Samuel Kotz, Yan Lumelskii, Marianna Pensky [4], offers a complete study of the various Stress-Strength simulations up to 2001. The practical and theoretical findings on the theory and the implementation of Stress-Strength relationships in economic and industrial systems are summarized and published in this book. For the first time, the findings and assumptions that have been spread in the literature over the past forty years have been addressed in a unified way accessible to applied and theoretical statisticians. The estimation of $R = P(Y < X)$ where X and Y are two independent scaled Burr type X distributions with same scale parameters is considered by Raqab and Kundu ([5]). Kundu and Gupta ([6] and [7]) contrasts diverse perspectives of estimating $R = P(Y < X)$ using generalized exponential distribution and Weibull distribution. The Stress vs. Strength problem concerning multi-component devices such as standby redundancy is explored by Gogio and Borah [8]. Adil H. khan and T.R Jan ([9] and [10]) in 2014 obtained expressions for stress-strength reliability for various distributions and also obtained the reliability function of Generalized Possion distribution and Generalized Geometric distribution.

2. MODEL

Consider a v -standby configuration in which there are v components originally, where only one performs during stress impact and the ones left $(v-1)$ are stand-by. However, when the working component fails, other takes its spot from the standby and is exposed to stress impacts and the system works. When the entire part fails, the system fails.

Let $X_1, X_2, \dots, X_v$ be a set of v independent random variables chosen to represent the strengths of v components organized in the process in order of activation and let $Y_1, Y_2, \dots, Y_v$ be just another set of independent random variables chosen to represent the stresses on the v components respectively, then the system reliability R_v of the system is given by

$$R_v = r(1) + R(2) + \cdots R(v) \qquad (2.1)$$

In which the marginal reliability $R(v)$ is the contribution to the reliability of the system by the r^th component and is defined as

$$R(r) = \Pr\left[X_1 < Y_{1,} X_2 < Y_2, \dots, X_{r-1} < Y_{r-1} \geq Y_r\right]$$

and if $f_i(x)$ and $g_i(y)$ are the probability density functions of X_i and $Y_i\,; i = 1,2, \dots, v$ respectively then

$$R(r) = \left[\int_{-\infty}^{\infty} F_1(y)h_1(y)dy\right] \times \left[\int_{-\infty}^{\infty} F_2(y)h_2(y)dy\right] \times \dots \times \left[\int_{-\infty}^{\infty} F_{r-1}(y)h_{r-1}(y)dy\right]\left[\int_{-\infty}^{\infty} \bar{F}_r(y)h_r(y)dy\right] \qquad (2.2)$$

In which $F_i(y)$ is the commutative distribution function of X_i and

$$\bar{F}_i(y) = 1 - F(x)$$

We have assumed strength and stress obey various distributions in this paper, the following instances are regarded.

1) Exponential strength and M-Transformed Exponential stress.
2) Lindley strength and M-Transformed Exponential stress.
3) M-Transformed Exponential strength and M-Transformed Exponential stress.
4) M-Transformed Exponential Strength and Two Parameter Gamma Stress.

Case I: Exponential strength and M-Transformed Exponential stress

Let $f_i(x)$ be the exponential strength with parameter λ_i and $g_i(y)$ be the M-Transformed exponential stress with parameter α_i ; i = $1,2,\ldots,v$, then

$$f_i(x,\lambda_i) = \lambda_i e^{-\lambda_i x} \quad ; x, \lambda_i > 0$$

$$\bar{F}_i(x) = 1 - F_i(x) = e^{-\lambda_i x} \quad ; x, \lambda_i > 0$$

And

$$g_i(y;\alpha_i) = \frac{2e^{\frac{-y}{\alpha}}}{\alpha\left(2 - e^{\frac{-y}{\alpha}}\right)^2} \quad ; y, \alpha_i > 0$$

Then from equation (2.2)

$$R(1) = \int_0^\infty \bar{F}_1(y) g_1(y) dy$$

$$R(1) = \int_0^\infty e^{-\lambda_1 y} \frac{2e^{\frac{-y}{\alpha_1}}}{\alpha_1\left(2 - e^{\frac{-y}{\alpha_1}}\right)^2} dy$$

$$R(1) = \sum_{n=1}^{\infty} \frac{n}{2^n \alpha_1} \int_0^\infty e^{-\left(\lambda_1 + \frac{n}{\alpha_1}\right)y} dy$$

$$R(1) = \sum_{n=1}^{\infty} \frac{n}{2^n(n + \alpha_1\lambda_1)}$$

$$R(2) = \left[\int_0^\infty F_1(y) g_1(y) dy\right]\left[\int_0^\infty \bar{F}_2(y) g_2(y) dy\right]$$

$$R(2) = \left[\int_0^\infty \left(1 - e^{-\lambda_1 y}\right) \frac{2e^{\frac{-y}{\alpha_1}}}{\alpha_1\left(2 - e^{\frac{-y}{\alpha_1}}\right)^2} dy\right]\left[\int_0^\infty e^{-\lambda_2 y} \frac{2e^{\frac{-y}{\alpha_2}}}{\alpha_2\left(2 - e^{\frac{-y}{\alpha_2}}\right)^2} dy\right]$$

$$R(2) = \left[1 - \sum_{n=1}^{\infty} \frac{n}{2^n(n + \alpha_1\lambda_1)}\right]\left[\sum_{n=1}^{\infty} \frac{n}{2^n(n + \alpha_2\lambda_2)}\right]$$

$$R(3) = \left[\int_0^\infty F_1(y) g_1(y) dx\right]\left[\int_0^\infty F_2(y) g_2(y) dy\right]\left[\int_0^\infty \bar{F}_3(y) g_3(y) dy\right]$$

$$R(3) = \left[1 - \sum_{n=1}^{\infty} \frac{n}{2^n(n + \alpha_1\lambda_1)}\right]\left[1 - \sum_{n=1}^{\infty} \frac{n}{2^n(n + \alpha_2\lambda_2)}\right]\left[\sum_{n=1}^{\infty} \frac{n}{2^n(n + \alpha_3\lambda_3)}\right]$$

$$R(v) = \left[1 - \sum_{n=1}^{\infty} \frac{n}{2^n(n + \alpha_1\lambda_1)}\right]\left[1 - \sum_{n=1}^{\infty} \frac{n}{2^n(n + \alpha_2\lambda_2)}\right] \cdots \times \left[1 - \sum_{n=1}^{\infty} \frac{n}{2^n(n + \alpha_{v-1}\lambda_{v-1})}\right]\left[\sum_{n=1}^{\infty} \frac{n}{2^n(n + \alpha_v\lambda_v)}\right]$$

$$R(v) = \left[\sum_{n=1}^{\infty} \frac{n}{2^n(n + \alpha_v\lambda_v)}\right] \prod_{i=1}^{v} \left[1 - \sum_{n=1}^{\infty} \frac{n}{2^n(n + \alpha_{i-1}\lambda_{i-1})}\right]$$

Case II: Lindley strength and M-Transformed Exponential stress

$$f_i(x,\lambda_i) = \frac{{\lambda_i}^2}{1 + \lambda_i}(1 + x)e^{-\lambda_i x} \quad ; x, \lambda_i > 0$$

$$\bar{\bar{F}}_i(x,\lambda_i) = 1 - F_i(x,\lambda_i) = \frac{1 + \lambda_i + \lambda_i x}{1 + \lambda_i} e^{-\lambda_i x} \quad ; x, \lambda_i > 0$$

And

$$g_i(y;\alpha_i) = \frac{2e^{\frac{-y}{\alpha}}}{\alpha\left(2-e^{\frac{-y}{\alpha}}\right)^2} \quad ; y, \alpha_i > 0$$

Then from equation (2.2)

$$R(1) = \int_0^\infty \bar{F}_1(y)g_1(y)dy$$

$$R(1) = \int_0^\infty \frac{1+\lambda_1+\lambda_1 y}{1+\lambda_1} e^{-\lambda_1 y} \frac{2e^{\frac{-y}{\alpha_1}}}{\alpha_1\left(2-e^{\frac{-y}{\alpha_1}}\right)^2} dy$$

$$R(1) = \sum_{n=1}^{\infty} \frac{n}{2^n\alpha_1}\left[\int_0^\infty e^{-\left(\lambda_1+\frac{n}{\alpha_1}\right)y} dy + \frac{\lambda_1}{1+\lambda_1}\int_0^\infty y^2 e^{-\left(\lambda_1+\frac{n}{\alpha_1}\right)y} dy\right]$$

$$R(1) = \sum_{n=1}^{\infty} \frac{n}{2^n}\left[\frac{1}{n+\alpha_1\lambda_1} + \frac{\lambda_1\alpha_1}{(1+\lambda_1)(n+\alpha_1\lambda_1)^2}\right]$$

$$R(2) = \left[\int_0^\infty F_1(y)g_1(y)dy\right]\left[\int_0^\infty \bar{F}_2(y)g_2(y)dy\right]$$

$$R(2) = \left[\int_0^\infty \left(1-\frac{1+\lambda_1+\lambda_1 y}{1+\lambda_1}e^{-\lambda_1 y}\right) \frac{2e^{\frac{-y}{\alpha_1}}}{\alpha_1\left(2-e^{\frac{-y}{\alpha_1}}\right)^2} dy\right]\left[\int_0^\infty \frac{1+\lambda_2+\lambda_2 y}{1+\lambda_2}e^{-\lambda_2 y} \frac{2e^{\frac{-y}{\alpha_2}}}{\alpha_2\left(2-e^{\frac{-y}{\alpha_2}}\right)^2} dy\right]$$

$$R(2) = \left[1-\sum_{n=1}^{\infty}\frac{n}{2^n}\left(\frac{1}{n+\alpha_1\lambda_1}+\frac{\lambda_1\alpha_1}{(1+\lambda_1)(n+\alpha_1\lambda_1)^2}\right)\right]\left[\sum_{n=1}^{\infty}\frac{n}{2^n}\left(\frac{1}{n+\alpha_2\lambda_2}+\frac{\lambda_2\alpha_2}{(1+\lambda_2)(n+\alpha_2\lambda_2)^2}\right)\right]$$

$$R(3) = \left[\int_0^\infty F_1(y)g_1(y)dy\right]\left[\int_0^\infty F_2(y)g_2(y)dy\right]\left[\int_0^\infty \bar{F}_3(y)g_3(y)dy\right]$$

$$R(3) = \left[1-\sum_{n=1}^{\infty}\frac{n}{2^n}\left(\frac{1}{n+\alpha_1\lambda_1}+\frac{\lambda_1\alpha_1}{(1+\lambda_1)(n+\alpha_1\lambda_1)^2}\right)\right]$$

$$\times\left[1-\sum_{n=1}^{\infty}\frac{n}{2^n}\left(\frac{1}{n+\alpha_2\lambda_2}+\frac{\lambda_2\alpha_2}{(1+\lambda_2)(n+\alpha_2\lambda_2)^2}\right)\right]\left[\sum_{n=1}^{\infty}\frac{n}{2^n}\left(\frac{1}{n+\alpha_3\lambda_3}+\frac{\lambda_3\alpha_3}{(1+\lambda_3)(n+\alpha_3\lambda_3)^2}\right)\right]$$

$$R(v) = \left[1-\sum_{n=1}^{\infty}\frac{n}{2^n}\left(\frac{1}{n+\alpha_1\lambda_1}+\frac{\lambda_1\alpha_1}{(1+\lambda_1)(n+\alpha_1\lambda_1)^2}\right)\right]\left[1-\sum_{n=1}^{\infty}\frac{n}{2^n}\left(\frac{1}{n+\alpha_2\lambda_2}+\frac{\lambda_2\alpha_2}{(1+\lambda_2)(n+\alpha_2\lambda_2)^2}\right)\right]\dots$$
$$\times\left[1-\sum_{n=1}^{\infty}\frac{n}{2^n}\left(\frac{1}{n+\alpha_{v-1}\lambda_{v-1}}+\frac{\lambda_{v-1}\alpha_{v-1}}{(1+\lambda_{v-1})(n+\alpha_{v-1}\lambda_{v-1})^2}\right)\right]\left[\sum_{n=1}^{\infty}\frac{n}{2^n}\left(\frac{1}{n+\alpha_v\lambda_v}+\frac{\lambda_v\alpha_v}{(1+\lambda_v)(n+\alpha_v\lambda_v)^2}\right)\right]$$

$$R(v) = \left[\sum_{n=1}^{\infty}\frac{n}{2^n}\left(\frac{1}{n+\alpha_v\lambda_v}+\frac{\lambda_v\alpha_v}{(1+\lambda_v)(n+\alpha_v\lambda_v)^2}\right)\right]$$
$$\times\prod_{i=1}^{v}\left[1-\sum_{n=1}^{\infty}\frac{n}{2^n}\left(\frac{1}{n+\alpha_{i-1}\lambda_{i-1}}+\frac{\lambda_{i-1}\alpha_{i-1}}{(1+\lambda_{i-1})(n+\alpha_{i-1}\lambda_{i-1})^2}\right)\right]$$

Case III: M-Transformed Exponential strength and M-Transformed Exponential stress

$$f_i(x;\alpha_i) = \frac{2e^{\frac{-x}{\alpha_i}}}{\alpha\left(2-e^{\frac{-x}{\alpha_i}}\right)^2} \quad ;x,\alpha_i>0$$

$$\bar{F}_i(x;\alpha_i) = 1-F_i(x;\alpha_i) = 1-\frac{2\left(1-e^{\frac{-x}{\alpha_i}}\right)}{2-e^{\frac{-x}{\alpha_i}}} \quad ;\ x,\alpha_i>0$$

And

$$g_i(y;\alpha_i) = \frac{2e^{\frac{-y}{\alpha}}}{\alpha\left(2-e^{\frac{-y}{\alpha}}\right)^2} \quad ;y,\alpha_i>0$$

Then from equation (2.2)

$$R(1) = \int_0^{\infty}\bar{F}_1(y)g_1(y)dy$$

$$R(1) = \int_0^{\infty}\left[1-\frac{2\left(1-e^{\frac{-y}{\alpha_1}}\right)}{2-e^{\frac{-y}{\alpha_1}}}\right]\left[\frac{2e^{\frac{-y}{\beta_1}}}{\beta_1\left(2-e^{\frac{-y}{\alpha_1}}\right)^2}\right]dy$$

$$R(1) = \int_0^{\infty}\left[\sum_{n=1}^{\infty}\frac{1}{2^n}e^{\frac{-ny}{\alpha_1}}\right]\left[\sum_{m=1}^{\infty}\frac{m}{2^m\beta_1}e^{\frac{-my}{\beta_1}}\right]dy$$

$$R(1) = \frac{\alpha_1}{\alpha_1+\beta_1}$$

Special Case: If $\alpha_1 = \beta_1$, then $R(1) = \frac{1}{2}$

$$R(2) = \left[\int_0^{\infty}F_1(y)g_1(y)dy\right]\left[\int_0^{\infty}\bar{F}_2(y)g_2(y)dy\right]$$

$$R(2) = \left[\int_0^{\infty}\left(1-\sum_{n=1}^{\infty}\frac{1}{2^n}e^{\frac{-ny}{\alpha_1}}\right)\left(\sum_{m=1}^{\infty}\frac{m}{2^m\beta_1}e^{\frac{-my}{\beta_1}}\right)dx\right]\left[\int_0^{\infty}\left(\sum_{n=1}^{\infty}\frac{1}{2^n}e^{\frac{-ny}{\alpha_2}}\right)\left(\sum_{m=1}^{\infty}\frac{m}{2^m\beta_2}e^{\frac{-my}{\beta_2}}\right)dx\right]$$

$$R(2) = \left(\frac{\beta_1}{\alpha_1+\beta_1}\right)\left(\frac{\alpha_2}{\alpha_2+\beta_2}\right)$$

Special Case: If $\alpha_i = \beta_i$; $i = 1,2$ then $R(2) = \frac{1}{2^2}$

$$R(3) = \left[\int_0^\infty F_1(y)g_1(y)dy\right]\left[\int_0^\infty F_2(y)g_2(y)dy\right]\left[\int_0^\infty \bar{F}_3(y)g_3(y)dy\right]$$

$$R(3) = \left[\int_0^\infty \left(1 - \sum_{n=1}^\infty \frac{1}{2^n} e^{\frac{-ny}{\alpha_1}}\right)\left(\sum_{m=1}^\infty \frac{m}{2^m\beta_1} e^{\frac{-my}{\beta_1}}\right)dx\right]\left[\int_0^\infty \left(1 - \sum_{n=1}^\infty \frac{1}{2^n} e^{\frac{-ny}{\alpha_2}}\right)\left(\sum_{m=1}^\infty \frac{m}{2^m\beta_2} e^{\frac{-my}{\beta_2}}\right)dx\right]$$
$$\times \left[\int_0^\infty \left(\sum_{n=1}^\infty \frac{1}{2^n} e^{\frac{-ny}{\alpha_3}}\right)\left(\sum_{m=1}^\infty \frac{m}{2^m\beta_3} e^{\frac{-my}{\beta_3}}\right)dx\right]$$

$$R(2) = \left(\frac{\beta_1}{\alpha_1 + \beta_1}\right)\left(\frac{\beta_2}{\alpha_2 + \beta_2}\right)\left(\frac{\alpha_3}{\alpha_3 + \beta_3}\right)$$

Special Case: If $\alpha_i = \beta_i$; $i = 1,2,3$ then $R(2) = \frac{1}{2^3}$

$$R(v) = \left(\frac{\alpha_v}{\alpha_v + \beta_v}\right)\left(\frac{\beta_2}{\alpha_2 + \beta_2}\right)\left(\frac{\beta_3}{\alpha_3 + \beta_3}\right) \dots \left(\frac{\alpha_v}{\alpha_v + \beta_v}\right)$$

$$R(v) = \left(\frac{\alpha_v}{\alpha_v + \beta_v}\right)\prod_{1=i}^{v}\left(\frac{\alpha_v}{\alpha_{i-1} + \beta_{i-1}}\right)$$

Special Case: If $\alpha_i = \beta_i$; $i = 1,2,\dots,v$ then $R(2) = \frac{1}{2^v}$

Case IV: M-Transformed Exponential Strength and Two Parameter Gamma Stress

$$f_i(x;\alpha_i) = \frac{2e^{\frac{-x}{\alpha_i}}}{\alpha\left(2 - e^{\frac{-x}{\alpha_i}}\right)^2} \quad ; x, \alpha_i > 0$$

$$\bar{F}_i(x;\alpha_i) = 1 - F_i(x;\alpha_i) = 1 - \frac{2\left(1 - e^{\frac{-x}{\alpha_i}}\right)}{2 - e^{\frac{-x}{\alpha_i}}} \quad ; \ x, \alpha_i > 0$$

And

$$g_i(x;\beta,\theta) = \frac{\theta^{\beta_i} x^{\beta_i - 1} e^{-\theta_i x}}{\Gamma(\beta_i)} \quad ; x, \beta_i, \theta_i > 0$$

Then from equation (2.2)

$$R(1) = \int_0^\infty \bar{F}_1(y)g_1(y)dy$$

$$R(1) = \int_0^\infty \left[1 - \frac{2\left(1 - e^{\frac{-y}{\alpha_1}}\right)}{2 - e^{\frac{-y}{\alpha_1}}}\right]\left[\frac{\theta_1^{\beta_1} y^{\beta_1 - 1} e^{-\theta_1 y}}{\Gamma(\beta_1)}\right]dy$$

$$R(1) = \frac{\theta_1^{\beta_1}}{\alpha_1 \Gamma(\beta_1)} \sum_{n=1}^{\infty} \frac{n}{2^n} \int_0^{\infty} y^{\beta_1 - 1} e^{-\left(\frac{n}{\alpha_1} + \theta_1\right) y} dy$$

$$R(1) = \sum_{n=1}^{\infty} \frac{n\theta_1^{\beta_1} \alpha_1^{\beta_1 - 1}}{2^n (n + \alpha_1 \theta_1)^{\beta_1}}$$

$$R(2) = \left[\int_0^{\infty} F_1(y) g_1(y) dy\right] \left[\int_0^{\infty} \bar{F}_2(y) g_2(y) dy\right]$$

$$R(2) = \left[\int_0^{\infty} \left(\frac{2\left(1 - e^{\frac{-y}{\alpha_1}}\right)}{2 - e^{\frac{-y}{\alpha_1}}}\right) \left(\frac{\theta_1^{\beta_1} y^{\beta_1 - 1} e^{-\theta_1 y}}{\Gamma(\beta_1)}\right) dy\right] \left[\int_0^{\infty} \left(1 - \frac{2\left(1 - e^{\frac{-y}{\alpha_2}}\right)}{2 - e^{\frac{-y}{\alpha_2}}}\right) \left(\frac{\theta_2^{\beta_2} y^{\beta_2 - 1} e^{-\theta_2 y}}{\Gamma(\beta_2)}\right) dy\right]$$

$$R(2) = \left[1 - \sum_{n=1}^{\infty} \frac{n\theta_1^{\beta_1} \alpha_1^{\beta_1 - 1}}{2^n (n + \alpha_1 \theta_1)^{\beta_1}}\right] \left[\sum_{n=1}^{\infty} \frac{n\theta_2^{\beta_2} \alpha_2^{\beta_2 - 1}}{2^n (n + \alpha_2 \theta_2)^{\beta_2}}\right]$$

$$R(3) = \left[\int_0^{\infty} F_1(y) g_1(y) dy\right] \left[\int_0^{\infty} F_2(y) g_2(y) dy\right] \left[\int_0^{\infty} \bar{F}_3(y) g_3(y) dy\right]$$

$$R(3) = \left[\int_0^{\infty} \left(\frac{2\left(1 - e^{\frac{-y}{\alpha_1}}\right)}{2 - e^{\frac{-y}{\alpha_1}}}\right) \left(\frac{\theta_1^{\beta_1} y^{\beta_1 - 1} e^{-\theta_1 y}}{\Gamma(\beta_1)}\right) dx\right] \left[\int_0^{\infty} \left(\frac{2\left(1 - e^{\frac{-y}{\alpha_2}}\right)}{2 - e^{\frac{-y}{\alpha_2}}}\right) \left(\frac{\theta_2^{\beta_2} y^{\beta_2 - 1} e^{-\theta_2 y}}{\Gamma(\beta_2)}\right) dy\right]$$
$$\times \left[\int_0^{\infty} \left(1 - \frac{2\left(1 - e^{\frac{-y}{\alpha_3}}\right)}{2 - e^{\frac{-y}{\alpha_3}}}\right) \left(\frac{\theta_3^{\beta_3} y^{\beta_3 - 1} e^{-\theta_3 y}}{\Gamma(\beta_3)}\right) dy\right]$$

$$R(3) = \left[1 - \sum_{n=1}^{\infty} \frac{n\theta_1^{\beta_1} \alpha_1^{\beta_1 - 1}}{2^n (n + \alpha_1 \theta_1)^{\beta_1}}\right] \left[1 - \sum_{n=1}^{\infty} \frac{n\theta_2^{\beta_2} \alpha_2^{\beta_2 - 1}}{2^n (n + \alpha_2 \theta_2)^{\beta_2}}\right] \left[\sum_{n=1}^{\infty} \frac{n\theta_3^{\beta_3} \alpha_3^{\beta_3 - 1}}{2^n (n + \alpha_3 \theta_3)^{\beta_3}}\right]$$

$$R(v) = \left[1 - \sum_{n=1}^{\infty} \frac{n\theta_1^{\beta_1} \alpha_1^{\beta_1 - 1}}{2^n (n + \alpha_1 \theta_1)^{\beta_1}}\right] \left[1 - \sum_{n=1}^{\infty} \frac{n\theta_2^{\beta_2} \alpha_2^{\beta_2 - 1}}{2^n (n + \alpha_2 \theta_2)^{\beta_2}}\right] \cdots$$
$$\times \left[1 - \sum_{n=1}^{\infty} \frac{n\theta_{v-1}^{\beta_{v-1}} \alpha_v^{\beta_{v-1} - 1}}{2^n (n + \alpha_{v-1} \theta_{v-1})^{\beta_{v-1}}}\right] \left[\sum_{n=1}^{\infty} \frac{n\theta_v^{\beta_v} \alpha_v^{\beta_v - 1}}{2^n (n + \alpha_v \theta_v)^{\beta_v}}\right]$$

$$R(v) = \left[\sum_{n=1}^{\infty} \frac{n\theta_v^{\beta_v} \alpha_v^{\beta_v - 1}}{2^n (n + \alpha_v \theta_v)^{\beta_v}}\right] \prod_{i=1}^{v} \left[1 - \sum_{n=1}^{\infty} \frac{n\theta_{i-1}^{\beta_{i-1}} \alpha_i^{\beta_{i-1} - 1}}{2^n (n + \alpha_{i-1} \theta_{i-1})^{\beta_{i-1}}}\right]$$

3. NUMERICAL EVALUATION

For different cases of stress strength distributions viz. M-Transformed Exponential, Lindley, Two Parameter Gamma and Exponential distributions the marginal reliabilities $R(1), R(2), R(2)$ and the system reliability R_3 have been evaluated using R for some distinct values of the parameters included in the expressions of $R(1), R(2), R(2)$.

Table1. Exponential strength and M-transformed exponential stress

α_1	α_2	α_3	λ_1	λ_2	λ_3	$R(1)$	$R(2)$	$R(3)$	R_3
1	1	1	1	1	1	0.6137	0.2371	0.0916	0.9424
1	1	1	2	2	2	0.4548	0.2480	0.1352	0.8380
1	1	1	3	3	3	0.3645	0.2316	0.1472	0.7433
2	2	2	1	1	1	0.4548	0.2480	0.1352	0.8380
2	2	2	3	3	3	0.2315	0.1778	0.1367	0.5460
1	1	0.3	1	1	0.3	0.6137	0.2371	0.1406	0.9914
1	1	0.2	2	2	0.2	0.4548	0.2480	0.2893	0.9921
1	1	0.1	3	3	0.1	0.3645	0.2316	0.4012	0.9973
2	2	0.3	1	1	0.3	0.4548	0.2480	0.2800	0.9828
2	2	0.2	2	2	0.2	0.3052	0.2120	0.4698	0.9870
2	2	0.1	3	3	0.1	0.2315	0.1778	0.5865	0.9958

From Table 1 it is clear, R_3 decreases by increasing the value of $\alpha_1, \alpha_2, \lambda_1$ and λ_2. But if the value of α_3 and λ_3 is decreased R_3 increases. For example if $\alpha_1, \alpha_2 = 1$, $\lambda_1, \lambda_2 = 1,2,3$ and for varying values of α_3, λ_3 decreases R_3 increases from 0.7433 to 0.9973. Particularly, when$\alpha_1, \alpha_2 = 1, \lambda_1, \lambda_2 = 3$ and $\alpha_3, \lambda_3 = 0.3$, the system reliability is maximum

Table 2. Lindley strength and M-transformed exponential stress

α_1	α_2	α_3	λ_1	λ_1	λ_3	$R(1)$	$R(2)$	$R(3)$	R_3
1	1	1	1	1	1	0.7246	0.1996	0.0549	0.9791
1	1	1	2	2	2	0.5313	0.2490	0.1167	0.8970
1	1	1	3	3	3	0.4180	0.2432	0.1416	0.8028
2	2	2	1	1	1	0.5695	0.2452	0.1055	0.9202
2	2	2	3	3	3	0.2731	0.1985	0.1443	0.6159
1	1	0.3	1	1	0.3	0.7246	0.1996	0.0746	0.9988
1	1	0.2	2	2	0.2	0.5313	0.2490	0.2185	0.9988
1	1	0.1	3	3	0.1	0.4180	0.2432	0.3385	0.9997
2	2	0.3	1	1	0.3	0.5695	0.2452	0.1823	0.9970
2	2	0.2	2	2	0.2	0.3708	0.2333	0.3938	0.9979
2	2	0.1	3	3	0.1	0.2731	0.1985	0.5280	0.9996

From Table 2 it is clear, R_3 decreases by increasing the value of λ_1, λ_2 and λ_3. But if the value of λ_1, λ_2 and λ_3 is decreased R_3 increases. For example if $\alpha_1, \alpha_2, \alpha_3 = 1$ and $\lambda_1, \lambda_2, \lambda_3 = 1,2,3$ then R_3 decreases from 0.9791 to 0.8028. Also for some proper values of the parameters system reliability can be attained very close to one

Table 3. M-transformed exponential strength and M-transformed exponential stress

α_1	α_2	α_3	β_1	β_2	β_3	$R(1)$	$R(2)$	$R(3)$	R_3
1	1	1	1	1	1	0.5000	0.2500	0.1250	0.8750
1	1	1	2	2	2	0.3333	0.2222	0.1481	0.7036
1	1	1	3	3	3	0.2500	0.1875	0.1406	0.5781
2	2	2	1	1	1	0.3333	0.1111	0.0740	0.5184
2	2	2	3	3	3	0.2000	0.2400	0.1440	0.5840
1	1	0.3	1	1	0.3	0.5000	0.2500	0.1250	0.8750
1	1	0.2	2	2	0.2	0.3333	0.2222	0.2222	0.7777
1	1	0.1	3	3	0.1	0.2500	0.1875	0.2812	0.7187
2	2	0.3	1	1	0.3	0.6666	0.2222	0.0555	0.9443
2	2	0.2	2	2	0.2	0.5000	0.2500	0.1250	0.8750
2	2	0.1	3	3	0.1	0.4000	0.2400	0.1800	0.8200

Clearly from Table 3, the system reliability can be approached closer to one by taking particular values of the parameters. For example $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 2$ and $\alpha_3 = \beta_3 = 0.3$, the system reliability is 0.9443. Asβ_i , $i = 1,2,3$ the system reliability decreases and it increases when $\alpha_3$and β_3 is decreased

Table 4. M-transformed exponential strength and two parameter gamma stress

α_1	α_2	α_3	β_1	β_2	β_3	θ_1	θ_2	θ_3	$R(1)$	$R(2)$	$R(3)$	R_3
1	1	1	2	2	2	1	1	1	0.2218	0.0492	0.0109	0.2819
1	1	1	2	2	2	2	2	2	0.4586	0.2483	0.1344	0.8413
1	1	1	2	2	2	3	3	3	0.6426	0.2296	0.0821	0.9543
1	1	1	2	2	2	6	6	6	0.9981	0.0018	0.0000	0.9999
2	2	2	1	1	1	2	2	2	0.6105	0.2378	0.0926	0.9409
3	3	3	1	1	1	2	2	2	0.4630	0.2486	0.1335	0.8451
4	4	4	1	1	1	2	2	2	0.3739	0.2342	0.1467	0.7548
2	2	2	2	2	2	1	1	1	0.2293	0.1767	0.1362	0.5422
2	2	2	3	3	3	1	1	1	0.1250	0.1094	0.0957	0.3301
2	2	2	4	4	4	1	1	1	0.0722	0.0670	0.0621	0.2013
2	2	2	1	1	1	4	4	4	0.7478	0.1886	0.0475	0.9839

From Table 4 it is clear, R_3 increases by increasing the value of θ_1, θ_2 and θ_3. But if the value of θ_1, θ_2 and θ_3 is decreased R_3 decreases. For example if $\alpha_1, \alpha_2, \alpha_3 = 1$ and $\beta_1 = \beta_2 = \beta_3 = 2$ then R_3 increases from 0.2819 to 0.9999 by increasing the value of θ_1, θ_2 and θ_3. Also for some proper values of the parameters system reliability can be attained very close to one

4. CONCLUSION

In this paper, a system is considered with standby redundancy and have presumed that the distinct components in the system for both stress and strength variables are independent and have different probability distributions viz. M-Transformed Exponential, Exponential, Gamma and Lindley. In the last section, for different cases of stress strength distributions, the marginal reliabilities $R(1), R(2), R(2)$ and the system reliability R_3 have been evaluated for some distinct values of the parameters included in the expressions of $R(1), R(2), R(2)$ and have showed that reliability of system can be monotonically increasing and monotonically decreasing for specific values of parameter. Thus, by proper choice of parameters leads to high reliability.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Birnbaum ZW, McCarty RC. A distribution-free upper confidence bound for Pr{Y<X}, Based on Independent Samples of X and Y, Ann. Math. Statist. 1958;29(2): 558-562.
2. Church JD, Harris B. The estimation of reliability from stress-strength relationships, Technometrics. 1970;12;49–54.
3. Sriwastav GL, Kakati MC. A stress-strength model with redundancy, IAPQR Trans. 1981;6(1):21-27.
4. Kotz S, Lumelskii Y, Pensky M. The stress-strength model and its generalizations, Theory and Applications. Singapore: World Scientific Press; 2003.
5. Raqab MZ, Kundu D. Comparison of different estimators of P[Y < X] for a scaled burr type X distribution. Communications in Statistics-Simulation and Computation. 2005;34:465–483.
6. Kundu D, Gupta RD. Estimation of P[Y < X] for generalized exponential distribution, Metrika. 2005;61:291–308.
7. Kundu D, Gupta RD. Estimation of R = P[Y < X] for Weibull distributions, IEEE Transactions on Reliability. 2006;55:270–280.
8. Gogoi J, Borah M. Estimation of reliability for multi-component systems using exponential gamma and lindley stress-strength distributions. Journal of Reliability and Statistics Studies. 2012;5(1): 33-41.
9. Khan AH, Jan TR. Reliability estimates of generalized poisson distribution and generalized geometric series distribution. Journal of Modern Applied Statistical Methods. 2014;13(2): 379-388.

10. Khan AH, Jan TR. Estimation of multi component systems reliability in stress-strength models. Journal of Modern Applied Statistical Methods. 2014;13(2):389-398.

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sdiarticle4.com/review-history/69236