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A Mathematical Relationship between Hydrogenic Periodic Property and Nuclear Properties in Furtherance of Bohr's Theory

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

Background: Atomic physics and nuclear matter physics are often exclusively studied. However, atomic properties are a direct function of nuclear properties. Establishing a mathematical relationship between nuclear and atomic properties could serve the interest of nuclear and atomic engineers. Nuclear - and atomic-based instrumentation engineering and nuclear medicine (and perhaps atomic medicine) applications could be the benefits.
Objectives: The research is undertaken to 1) link nuclear property, the mass-radius of the nucleon, and ionization energy of hydrogen via the derivation of appropriate equation and 2) determine the mass-radii of the nucleons and some leptons.
Methods: Theoretical and computational methods.
Results and Discussion: As applicable to the previous results in the literature, the larger the mass of the elementary particles, the longer the radii. For the particles investigated, the order of the radius is muon (μ^-)<proton (p^+)< neutron (n)< tauon (τ^-) corresponding to increasing mass, μ^-<p^+< n<τ^-. The values of the mass radii were respectively ≈ 0.1240, 1.1012, 1.1027, and 2.0855 fm.
Conclusion: Nuclear properties such as the radius of any nucleon (Γ_N) can be mathematically linked to atomic properties such as the ionization energy of hydrogen via equation which shows that Γ_N is inversely proportional to the ionization energy of hydrogen and directly proportional to the rest-mass of the particle.

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1. INTRODUCTION

There is no doubt that a lot of great works had been done in the areas of atomic and nuclear physics characterized with very strong mathematical exposition giving the impression that the facts and principles of physics are not amenable to basic mathematical elucidation. For instance, the proton and neutron root mean square radii can be defined as [1]:

$$R_{\mathrm{n(P)}}^{\mathrm{r.m.s}} = \langle r_{\mathrm{n(P)}}^{2} \rangle^{½} = \left(\frac{\int r_{\mathrm{n(P)}}^{2} \rho_{n(\mathrm{P})}(\acute{r})\, d\acute{r}}{\int \rho_{n(\mathrm{P})}(\acute{r})\, d\acute{r}}\right)^{½} \approx \left(\frac{3}{5} R_{\mathrm{on(P)}}^{2} + \frac{7\pi^{2}}{5} a_{\mathrm{n(P)}}^{2}\right)^{½} \qquad (1)$$

Where, $R_{\mathrm{on(P)}}$, $a_{\mathrm{n(P)}}$, and $\rho_{n(\mathrm{P})}(\acute{r})$ are the half-density radius, diffuseness of the neutron (proton) density distributions, and local proton (neutron) density respectively. The issue with Eq. (1) is that the procedural steps from $\langle r_{\mathrm{n(P)}}^{2} \rangle^{½} = \left(\frac{\int r_{\mathrm{n(P)}}^{2} \rho_{n(\mathrm{P})}(\acute{r})\, d\acute{r}}{\int \rho_{n(\mathrm{P})}(\acute{r})\, d\acute{r}}\right)^{½}$ to the final equation are not given; the steps could constitute a major part of the research that produces a separate paper such that students at any level can learn both the "mathematical language" and the principle of physics being discussed or analyzed. Besides, Eq. (1) gives the impression that root mean square (r.m.s.) radius and by extension mass radii of proton and neutron are the same. The issue of mathematical complexity in almost all theoretical and experimental theses on elementary particle properties is very much contrary to Bohr's theory which had met uncomplimentary remarks as discussed in the literature [2]. This has led to what has been known as new physics, wave/quantum mechanics, whose mathematical formalism is rather, to the layman, too complex and sometimes lacks a stepwise approach in the derivational process. An aspect of this new physics, the uncertainty principle and Schrödinger theory has been harshly criticized as being too difficult and nonsensical by a scholar [3] who is, no doubt, endowed with highly advanced postdoctoral mathematics. In this research, classical model characterized by the Newtonian principle is adopted.

Thus, this part of the introductory section can be described as an overview of the literature pieces of information about issues connected with nuclear matter, proton density distribution, and neutron halo or neutron skin thickness [1, 4]. Here it has become very imperative to humbly offer an advice that can foster pedagogical principles that enable comprehensibility to the advantage of undergraduates, interested scholars as laymen like me who are at the borderline between core physics/chemical physics, core physical chemistry and biochemistry or any other biological science. Physical concepts or terms as above including surface nucleons which are hardly explained ought to be exhaustively defined. A layman may give the impression that the nucleus is an enclosed 3-D space with both internal (or intra-) nuclear nucleons and external or surface nucleons.

Atomic physics and nuclear matter physics are often exclusively treated or studied. However, atomic properties are a direct function of nuclear properties. For instance, the nuclear charge density is a function of the total number of protons and the nuclear volume. Thus, the modern definitions of periodic law such as [5] (a) the properties of the elements are a periodic function of their atomic numbers; and (b) the properties of the elements depend upon their total electron configuration are the very evidence-based reasons why there is a need to link nuclear matter properties with atomic matter properties, by establishing a mathematical relationship between hydrogenic periodic property and nuclear properties in furtherance of Bohr's theory. Indeed, continuous reference to Bohr is as a result of his first-principle equation whose modification in several ways and applications has led to the determination of hitherto indeterminate physical variables or periodic properties such as effective nuclear charge, radii *etc* of any element with the same degree of accuracy applicable to hydrogenic atom and ions [2, 6].

The stability of the electronic configuration expressed in terms of ionization energy is a function of the nuclear electronic configuration. Incidentally, there seems to be emerging interest in what is referred to as density distribution of finite nuclei as if to imply that infinite nuclei may exist. Indeed to the core giants in physics, there may be infinite nuclear stated about the solution of some problems involving nuclear density distribution which is seen to be minimal at a certain density of nucleus corresponding to infinite matter [7]. “Perhaps, it seems the universe is implied”. Any investigation of the density distribution of the so-called finite nuclei requires according to Seif and Mansour [1] accurate information about the r.m.s. radii of proton and neutron density distribution, surface diffuseness, and neutron skin thickness. Diffuseness parameter is defined as the fall-off of the nuclear potential in the tail region of the Coulomb barrier [8]. Thus, this research is undertaken to 1) link nuclear property, mass radius of the nucleon to ionization energy of hydrogen via the derivation of appropriate equation and 2) determine the mass-radii of the nucleons and some leptons.

2. REVIEW OF THEORY AND NEW DERIVATIONS

Previous research [9] has shown that the mass-radius of the nucleons and subatomic particles whose mass is greater than the mass of the nucleons can be determined. However, the equation applies to all particles whose individual mass is greater than the mass of any lepton. The greater the mass of the different particles, the longer the lengths of the mass radii become. This cannot be unreasonable given the fact that while the exact density of the moon may not be known, its radius must be much less than the radius of the earth for a reason that is not farfetched. The equation in the literature [9] which is anchored on Newtonian, de Broglie, and Einstein’s mass-energy principles is given as follows.

$$\Gamma_{\mathrm{P}} = \frac{e^6\, m_{\mathrm{P}}}{4\,\pi\,\varepsilon_0^3\, m_{\mathrm{e}}^2\, h^2 c^4} \tag{2}$$

Where, Γ_{P}, e, m_{P}, ε_0, m_{e}, h, and c are the mass-radius of any particle whose mass is > the mass of the electron, the charge of an electron, the mass of the particle, permittivity in free space, the mass of an electron, Planck constant, and the velocity of light in a vacuum respectively. In line with the policy of a simple step-by-step approach (this simply means that phrase such as after some algebra is avoided) the velocity of light in free space raised to the 4^{th} power is given as (from Eq. (2) [9])

$$c^4 = \frac{e^6 m_{\mathrm{P}}}{4\,\pi\,\varepsilon_0^3\, m_{\mathrm{e}}^2\, h^2\, \Gamma_{\mathrm{P}}} \tag{3}$$

The square of the velocity of light is given as:

$$c^2 = \frac{e^3}{m_{\mathrm{e}}\, h\, \varepsilon_0^{3/2}} \times \sqrt[2]{\frac{m_{\mathrm{P}}}{4\pi\Gamma_{\mathrm{P}}}} \tag{4}$$

In the literature [6], is the equation as follows.

$$a_{\mathrm{x}} = \frac{1}{m_{\mathrm{e}}} \sqrt[2]{\frac{\xi_{\mathrm{H}}}{\xi_{\mathrm{x}}}}\ \frac{n_{\mathrm{x}}\, h^2}{\mu_0 e^2 \pi\, c^2} \tag{5}$$

Where, x denotes any element or atom and a_{x}, ξ_{x}, n_{x}, μ_0 and ξ_{H} are the radius of any atom, ionization energy of any atom except hydrogen atom, principal quantum number otherwise referred to as energy level of any atom other than a hydrogen atom, magnetic constant, and the ionization energy of hydrogen atom respectively. If $\xi_{\mathrm{x}} = \xi_{\mathrm{H}}$, the usual Bohr’s symbol, a_0 for the radius of hydrogen otherwise known as Bohr’s radius for hydrogen applies and n_{x} must be = 1.

Substitution of Eq. (4) into Eq. (5) gives:

$$a_{\mathrm{x}} = \frac{1}{m_{\mathrm{e}}} \sqrt[2]{\frac{\xi_{\mathrm{H}}}{\xi_{\mathrm{x}}}}\ \frac{n_{\mathrm{x}} h^2}{\mu_0 e^2 \pi} \sqrt[2]{\frac{4\pi\Gamma_{\mathrm{P}}}{m_{\mathrm{P}}}}\ m_{\mathrm{e}} h\, \frac{\varepsilon_0^{3/2}}{e^3} \tag{6}$$

Simplification of Eq. (6) gives

$$a_{\mathrm{x}} = \sqrt[2]{\frac{\mathbf{4}\,\xi_{\mathrm{H}}}{\xi_{\mathrm{x}}} \frac{\pi\,\Gamma_{\mathrm{P}}}{m_{\mathrm{P}}}} \times \frac{n_{\mathrm{x}}\, h^3\, \varepsilon_0^{3/2}}{\mu_0\, e^5\, \pi} \tag{7}$$

It is known in the literature [2] that for any element, both hydrogenic and non-hydrogenic elements the equation as follows holds.

$$a_{\mathrm{x}} = \frac{n_{\mathrm{x}} h}{\pi (8\, m_{\mathrm{e}})^{½}} \frac{1}{\xi_{\mathrm{x}}^{½}} \tag{8}$$

Where, a_{x} is the Bohr’s radius for any atom; for the purpose of emphasis and clarity, Eq. (8) gives exactly the same result or value of Bohr’s radius for hydrogen if the old Bohr’s equation is used. The reason is that only one electron is involved in hydrogenic atom at the lowest energy level. The subscript x refers to any atom with respect to Eq. (8) that is generalizable to all elements whose ionization energy is known.

It follows based on Eqs (7) and (8) that

$$\frac{n_{\mathrm{x}} h}{\pi (8\, m_{\mathrm{e}})^{½}} \frac{1}{\xi_{\mathrm{x}}^{½}} = \sqrt[2]{\frac{\mathbf{4}\,\xi_{\mathrm{H}}}{\xi_{\mathrm{x}}} \frac{\pi\,\Gamma_{\mathrm{P}}}{m_{\mathrm{P}}}} \times \frac{n_{\mathrm{x}}\, h^3\, \varepsilon_0^{3/2}}{\mu_0 e^5 \pi} \tag{9a}$$

Simplification of Eq. (9a) gives

$$\frac{1}{m_{\mathrm{e}}} = 32\, \xi_{\mathrm{H}} \frac{\pi\,\Gamma_{\mathrm{P}}}{m_{\mathrm{P}}} \frac{h^4\, \varepsilon_0^3}{\mu_0^2\, e^{10}} \tag{9b}$$

Making Γ_{P} subject of the formula in Eq. (9b) gives the equation that reproduces a result equal to what can be obtained from Eq. (1). Thus,

$$\Gamma_{\mathrm{P}} = \frac{m_{\mathrm{P}}\,\mu_0^2\,e^{10}}{32\,h^4\,\varepsilon_0^3\,\pi\,m_{\mathrm{e}}\,\xi_{\mathrm{H}}} \qquad (10)$$

Apart from the hydrogen atom, ξ_H is always the same because for every hydrogenic atom of the form ${}_{Z}^{A}X^{(Z-1)+}$ where, *A* and *Z* are the mass number and atomic number respectively, $n^2\xi_{(Z-1)}/Z^2$ (where *n* =1) as well as $n_X^2\xi_X/Z_{\mathrm{eff}}^2$ is always = ξ_H where Z_{eff} is the effective nuclear charge [2, 6]. With reference to the literature [2, 6] the preceding claim is illustrated as follows. Given the ionization of lithium and sodium, ≈ 520 and 495.8 kJ/mol. respectively (Wikipedia) with corresponding effective nuclear charge, ≈ 1.259 and 1.842 [2], the values expected from the equation are as follows: 131235.778 and 1315133.317 J/mol. The difference in result is as a result of errors in reported data available in the literature. The constant in Eq. (10) is difficult to choose because ξ_H though experimentally and theoretically determinable, it is nevertheless a constant parameter going by the preceding analysis. Thus, if all the parameters in Eq. (10) are constant, then the dependent parameter must be constant as long as the rest-mass of the particle in question is constant.

3. MATERIALS AND METHODS

There was no need for materials and equipment because the research entails purely theoretical and computational or calculational methods.

4. RESULTS AND DISCUSSION

As stated earlier, recent results reported for the length of the radii of nucleons in the literature [9] are different from results reported in older literature [10, 11]. Such pieces of information include the observation that the proton charge radii in the literature are 0.84 ± 0.05 fm [9], 0.856 fm [10], 0.84 ± 0.0004 fm [12-15], 0.84087 fm [16] and finally for the purpose of this research, 0.831 ± 0.012 fm [17]. This latter figure for the proton charge radius refutes the claim that 0.84 fm may be the most accurate if lower figures seem to point to a more accurate values!!! [9]. Yet, again the value of 0.831 ± 0.012 fm [17] may not be the shortest going by the paper by Hare and Papini [18] which showed a value equal to 0.7 fm.

Other reports are those based on an unfamiliar chiral bag model which views the radius of the nucleon to be a bag of radius ≈ 0.8 fm [19] and that of Bochkarev *et al* [4] which is also 0.8 fm being according to them, the r.m.s radius of the nucleon. According to the latter authors [4], proton point densities instead of charge densities can be used to determine the finite size of the proton by the prescription, $r_{\mathrm{P}}^2 = r_C^2 - r_N^2$, where r_{P}, r_{C}, and r_{N} (where r_{N} is = 0.8 fm) are the r.m.s radii of proton, charge, and nucleon density distribution respectively.

The Γ_{P} values for illustration are determined for the nucleons and muon in muonic hydrogen only as shown in Table 1. The muon is a lepton whose mass is ≈ 207 times heavier than the electron, a negatively charged lepton; the muon-proton mass ratio, η according to Pohl *et al* [20] is 0.1126095272.

Based on Eq. (10), the radius of any elementary particle is mainly a function of its mass. Expectedly, therefore, the larger particle with higher mass has a longer radius similar to report elsewhere [8]. Substitution of ionization energy of hydrogen into Eq. (10) however, gave results whose difference from previous results for the proton and neutron are ≈ 0.03406 % of the results in the literature [8].

Table 1. The masses and radii of baryons and leptons

Baryons	**Symbols**	**Mass/exp (–27) kg**	**Radii / fm**
Proton	p^+	1.672621777$^{(C)}$	1.101171175
Neutron	*n*	1.674927351$^{(C)}$	1.102689051
Leptons	**Symbols**	**Mass/exp (–27) kg**	**Radii / fm**
Muon	μ^-	0.188427357	0.123979254
Tauon	τ^-	3.167790098	2.085515799

The superscript (C) means that the values were from CODATA recommended values [21]. Other mass values were calculated using the equation of mass-energy equivalence (GeV/c^2) with data in the literature (www.Sciencedirect.com /topics/chemistry/leptons). Ionization energy of hydrogen atom was calculated using a_0 (=5.2917721092 exp (–11) m), e (=1.602176565 exp (–19) C), and ε_0 (=8.854187817 exp (–12) F/m). The usual Bohr's equation is $\xi_H = e^2/8\,\pi\,\varepsilon_0\,a_0$.

In recent literature, elastic electron-proton scattering (e-p) and the spectroscopy of hydrogen atoms are the two methods traditionally used to determine the proton charge radius, r_p. Another method, using muonic hydrogen atom, in which measurement of Lamp-shift was taken, found a substantial discrepancy compared with previous results [17]. The shorter length of the proton radius led to what has been termed the proton radius puzzle [20]. But a greater puzzle ought to be expected if despite the freely available literature materials the scholars did not notice the values of proton radius shorter than 0.831 fm. The proton radius in question was obtained from the measurement of the Lamb shift in the muonic hydrogen atom.

The term Lamb shift came to reality after the notion of the degeneracy of $2s_{½}$ and $2p_{½}$ states of the hydrogen atom popularized by Dirac's unfamiliar one-particle relativistic theory was replaced with the observation that $2s_{½}$ is higher than $2p_{½}$ (www.sciencedirect.com/topics/chemistry/lambshift). Nevertheless, one may need to know if kinetic energy is referred to at least from layman's perspective otherwise given what s and p stands for, if not mistaken, the potential energy of p should be higher than s. No extra information in this regard is available in the literature for clarification. Lamb shift is the shift of atomic energy level given suitable conditions, e.g. the interaction of the electron with the virtual photon and vacuum electric current (www.sciencedirect.com/topics/chemistry/lambshift), conditions that may not be unexpected in the experimental process involving muonic hydrogen.

It has been explained in the literature [8] why the r.m.s. radius of the proton will continue to shorten on the basis of the implication of the experimental procedure, the unusual electron-proton scattering (e-p) approach involving particles of opposite charge as against the positron-proton (p-p) scattering approach. The current approach in this research has dual characteristics in the sense that, as Eq. (10) shows, Γ_P being the mass-radius of any particle whose mass is equal to the mass of any of the nucleon or larger particles, is inversely proportional to the ionization energy of hydrogen atom, a parameter which can be determined experimentally and theoretically. Besides, current equation Eq. (10) differs from Eq. (2) due largely to the presence of magnetic constant as nominator while Eq. (2) contains the velocity of light.

5. CONCLUSION

Nuclear properties such as the radius of any nucleon (Γ_N) can be mathematically linked to atomic properties such as ionization energy of hydrogen via equation which shows that Γ_N is inversely proportional to ionization energy of hydrogen and directly proportional to the rest mass of the particle. Expectedly, the heavier particles have longer mass radii than the lighter particles. The chemistry of any atom is a function of its nuclear property. Thus, a link between hydrogenic and nuclear properties in furtherance of Bohr's theory is not out of place and it is not intended to be restricted to hydrogenic atoms as long as ionization energy is known. Feature research may focus on the mathematical relationship between nuclear property and other periodic properties of any element.

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COMPETING INTERESTS

The author have declared that there is no known competing financial interests or non-financial interests or personal relationships that could have appeared to influence the work reported in this paper.

REFERENCES

1. Seif WM, Mansour H, Systematics of nucleon density distributions and neutron skin of nuclei. Int. J. Mod. Phys. E. 2015;24(11): 1-14.
2. Udema II. Renaissance of Bohr's model via derived alternative equation. Am. J. Mod. Phys. 2017;6(2):23-31.
3. Mills RL. The fallacy of Feynman's and related arguments on the stability of the hydrogen atom according to quantum mechanics. Annales de la Fondation Louis de Broglie. 2005;30(2):129-149.

4. Bochkarev OV, Chulkov LV. Egelhof P, Geissel H, Golovkov MS, Irnich H, *et al*. Evidence for a neutron skin in ^{20}N. Eur. Phys. J.1998;1:15-17.
5. Kneen WR, Rogers MJ Wand Simpson P. Chemistry. Facts, patterns, and principles. 1st ed. London: The English Language Book Society and Addison-Wesley Publishers Limited; 1972.
6. Udema II. Revisiting Bohr's theory via a relationship between magnetic constant and Bohr radius of any element. Asian J. Phys. Chem. Sci. 2018;6(1):1-11.
7. Strutinsky VM, Magner AG, Denisov VYu. Density distribution in nuclei. Z. Phys. A-Atoms & Nuclei. 1985;322: 49-156.
8. Gharaei R, Hadikhani A, Zanganeh V. An explanation for the anomaly problem of diffuseness parameter of the nucleus-nucleus potential in heavy-ion fusion reactions: A possible thermal relation. Nucl. Physics A. 2019;990:47-63.
9. Udema II. Theoretical determination of the mass radii of the nucleons and heavier subatomic particles. Asian J. Res. Rev. Phys. 2020;3(4):1-10.
10. Lyuboritskij VE, Gutsche Th, Faessler A. Electromagnetic structure of the nucleon in the perturbative chiral quark model. Phys. Rev. C - Nuclear Phys. 2001; 64 (6): 652031-652316.
11. Gentile TR, Crawford CB. Neutron charge radius and the neutron form factor. Phys. Rev. C. 2011;83:1-6.
12. Carson CE. The proton radius puzzle. Prog. Part. Nucl. Phys. 2015;82:59-77.
13. Pohl R, Antognini A, Nez F, Amaro FD, Biraben F, Cardoso JMR, *et al*. The size of the proton. Nature. 2010;466:213-216.
14. Pohl R, Nez F, Fernandes LMP, Amaro FD, Biraben F, Cardoso JMR, Covita DS, Dax A. Dhawan S, Diepold M, *et al*. Laser spectroscopy of muonic deuterium. Science. 2016;353:669-673.
15. Peset C, Pineda A. The Lamb shift in muonic hydrogen and the proton radius from effective field theories. Eur. Phys. J. A. 2015;51(156):arXiv.
16. Kelkar NG, Mart T, Nowakowski M. Extraction of the proton charge radius from experiments. Makara J. Sci. 2016;20(3):1-10.
17. Xiong W, Gasparian A, Gao H, Dutta D, Khandaker M, Liyanage N, et al. A small proton charge radius from electron – proton scattering experiment. Nature. 2019;575(7781):147-170.
18. Hare HG, Papini G. Mass radius of the nucleon. Canadian J. Phys. 1972;50:1163-1168.
19. Byrne J. The mean square charge radius of the neutron. Neutron News. 1994; 5(4): 15-17.
20. Pohl R, Gilman R, Miller GA. Pachnucki K. Muonic hydrogen and the proton radius puzzle. Annu. Rev. Nucl. Part. Sci. 2013;63:175-205.
21. Mohr PJ, Taylor BN, Newel DB. CODATA recommended values of the fundamental physical constants-2010. J. Phys. Chem. Ref. Data. 2012;41(4):1–84.

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