Research Article

Influence of Lorentz Invariation Violation on Arbitrary Spin Fermion Tunneling Radiation in the Vaidya-Bonner Space-Time

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In the space-time of the nonstationary spherical symmetry Vaidya-Bonner black hole, an accurate modification of Hawking tunneling radiation for fermions with arbitrary spin is researched. Considering a light dispersion relationship derived from string theory, quantum gravitational theory, and the Rarita-Schwinger equation in the nonstationary spherical symmetry space-time, we derive an accurately modified dynamic equation for fermions with arbitrary spin. By solving the equation, the modified tunneling rate of fermions with arbitrary spin, Hawking temperature, and entropy at the event horizon of the Vaidya-Bonner black hole are presented. We find that the Hawking temperature will increase, but the entropy will decrease compared with the case without the Lorentz Invariation Violation modification.

1. Introduction

The theory of Hawking thermal radiation reveals the relationship between gravitational theory, quantum theory, and statistical thermal dynamic mechanics [1]. After the research of Hawking thermal radiation to all kinds of black holes [2], Kraus and Wilczek did some modifications to the Hawking thermal radiation adopting self-gravitational interaction [3]. Hereafter, researchers studied the Hawking tunneling radiation for many types of black holes [4–9]. In 2007, Kerner and Mann proposed a semiclassic method to investigate the tunneling radiation of fermions with spin 1/2 [10, 11]. In the later research, this semiclassic method is widely used to calculate the tunneling radiation of the other type of particles [12–14]. Yang and Lin developed Kerner and Mann's theory and proposed that the Hamilton-Jacobi method is efficient for the tunneling of fermions [15, 16]. According to references [15, 16], after choosing a suitable Gamma matrix and considering the commutation relation of the Pauli matrix in the Dirac equation, which describes the dynamic of the fermion quite well, the Hamilton-Jacobi equation in the curved

space-time can be derived. This result means that the Hamilton-Jacobi equation is also a very important equation in the research of the tunneling theory of fermions. In recent years, the Lorentz light dispersion relationship is generally regarded as a basic relation in modern physics. It seems that both general relativity and quantum mechanics are built on this relationship. However, the research of quantum gravitational theory indicates that the Lorentz relationship should be modified in the high-energy case. Although scientists have not built a successful light dispersion relationship in the high-energy case, current researches are helpful to the development of this theory. People usually estimate that the magnitude of this modification should be in the Plank scale. It is confirmed that both the Dirac equation and the Hamilton-Jacobi equation must be modified if the Lorentz Invariation Violation is considered. In such a case, only an accurate modification can efficiently research fermion tunneling radiation from a black hole, such as the Vaidya-Bonner black hole. In this paper, the most important progress is that we use a new method which is suitable for fermions with an arbitrary spin. We will research the exact modification of tunneling

radiation for fermions with an arbitrary spin, considering the Lorentz Invariation Violation.

2. Exact Modification of Arbitrary Spin Fermion Rarita-Schwinger Equation and Hamilton-Jacobi Equation

In the research of string theory, the authors proposed a relation [17–21]:

$$P_0^2 = p^2 + m^2 - (LP_0)^\alpha p^2. \tag{1}$$

In the natural unit, P_0 and p are the energy and momentum of the particle with the static mass m, respectively. L is a constant in the magnitude of the Plank scale, which comes from the Lorentz Invariation Violation theory. In Equation (1), $\alpha = 1$ is adopted in the Liouville-string model. Kruglov obtained a modified Dirac equation considering $\alpha = 2$ [22]. Therefore, we substitute $\alpha = 2$ into Equation (1) and get a general Rarita-Schwinger equation in the flat space:

$$\left[\bar{\gamma}^\mu \partial_\mu + \frac{m}{\hbar} - \lambda \hbar \bar{\gamma}^t \partial_t \bar{\gamma}^j \partial_j\right] \Psi_{\alpha_1 \cdots \alpha_k} = 0, \tag{2}$$

where $\hbar$ is the reduced Plank constant, which equals 1 in the natural units. λ is a very small constant. $\Psi_{\alpha_1 \cdots \alpha_k}$ is a wave function, where the value of α_k corresponds to a different spin. The larger the α_k, the higher the spin is. The wave function satisfies following supplementary condition:

$$\bar{\gamma}^\mu \Psi^\mu_{\alpha_2 \cdots \alpha_k} = \partial_\mu \Psi^\mu_{\alpha_2 \cdots \alpha_k} = \Psi^\mu_{\mu_{\alpha_3 \cdots \alpha_k}} = 0. \tag{3}$$

When $k = 0$ and $\Psi_{\alpha_1 \cdots \alpha_k} = \Psi$, Equation (2) changes to the Dirac equation for spin $1/2$ and condition (3) disappears automatically. When $k = 1$, Equation (2) describes the dynamic of fermions with spin $3/2$ and the condition (3) also disappears automatically. Note that the commutation relation

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} I. \tag{4}$$

In the curved space-time, the Rarita-Schwinger equation can be rewritten as

$$\left[\gamma^\mu D_\mu + \frac{m}{\hbar} - \lambda \hbar \gamma^t D_t \gamma^j D_j\right] \Psi \alpha_1 \cdots \alpha_k = 0, \tag{5}$$

where $D_\mu = \partial_\mu + (i/\hbar) e A_\mu$, $\lambda \ll 1$, and $\lambda \hbar \gamma^t D_t \gamma^j D_j$ is a very small term. For fermions with an arbitrary spin, the wave function is

$$\Psi_{\alpha_1 \cdots \alpha_k} = \xi_{\alpha_1 \cdots \alpha_k} e^{(i/\hbar)S}, \tag{6}$$

where $\xi_{\alpha_1 \cdots \alpha_k}$ and S are matrices and the action of the fermion, respectively. The line element of the nonstationary Vaidya-Bonner black hole represented in an advanced Eddington coordinate [23] is given by

$$dS^2 = -F(r, v)dv^2 + 2drdv + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{7}$$

$$F(r, v) = 1 - \frac{2M(v)}{r} + \frac{Q^2(v)}{r^2}, \tag{8}$$

where v is the Eddington time and $M(v)$ and $Q(v)$ represent the mass and charge of the black hole changes with time, respectively. When $Q(v) = 0$, the nonstationary Vaidya-Bonner black hole is reduced to the Vaidya black hole. The electromagnetic four-potential of the Vaidya-Bonner black hole is

$$A_\mu = \left(\frac{Q}{r}, 0, 0, 0\right) = (A_0, 0, 0, 0). \tag{9}$$

Corresponding to the line element (7), the inverse metric tensor is

$$g^{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & \dfrac{\Delta}{r^2} & 0 & 0 \\ 0 & 0 & r^{-2} & 0 \\ 0 & 0 & 0 & r^{-2}\sin^{-2}\theta \end{pmatrix}, \tag{10}$$

where

$$\Delta = r^2 - 2Mr + Q^2. \tag{11}$$

Because the component of the inverse metric tensor $g^{00} = 0$ in the curved space-time of line element (7), so Equations (5) and (6) become

$$\begin{aligned}[i\gamma^0(\partial_v S + eA_0) + i\gamma^j \partial_j S + m \\ + \lambda(\partial_v S + eA_0)\gamma^0\gamma^i\partial_i S]\xi_{\alpha_1 \cdots \alpha_k} = 0.\end{aligned} \tag{12}$$

In this paper, the range for i and j in the superscript and subscript satisfies $i, j = 1, 2, 3$. μ and ν in the superscript and subscript are defined as $\mu, \nu = 0, 1, 2, 3$. Setting

$$\Gamma^\mu = i\gamma^\mu + \lambda(\partial_v S + eA_0)\gamma^0\gamma^\mu, \tag{13}$$

Equation (12) becomes

$$[m + \Gamma^\mu(\partial_\mu S + eA_\mu)]\xi_{\alpha_1 \cdots \alpha_k} = 0. \tag{14}$$

Multiplying $\Gamma^\nu(\partial_\nu S + eA_\nu)$ in both sides of Equation (14), then

$$\Gamma^\nu(\partial_\nu S + eA_\nu)\Gamma^\mu(\partial_\mu S + eA_\mu)\xi_{\alpha_1 \cdots \alpha_k} - m^2\xi_{\alpha_1 \cdots \alpha_k} = 0. \tag{15}$$

Exchanging ν and μ in Equation (15), we get

$$\Gamma^\mu(\partial_\mu S + eA_\mu)\Gamma^\nu(\partial_\nu S + eA_\nu)\xi_{\alpha_1 \cdots \alpha_k} - m^2\xi_{\alpha_1 \cdots \alpha_k} = 0. \tag{16}$$

Equations (15) and (16) are equivalent. Considering $\gamma^0\gamma^0 = g^{00} = 0$, firstly adding the left side and the right side

of Equations (15) and (16), respectively, and then dividing the new equation by 2, finally combining with Equation (13), we obtain

$$\left\{ 2g^{0j}(\partial_v S + eA_0)\partial_j S + g^{ij}\partial_i S \partial_j S \right.$$
$$- i2\lambda(\partial_v S + eA_0)g^{0i}\partial_i S \gamma^\mu(\partial_\mu S + eA_\mu) \qquad (17)$$
$$\left. - \lambda^2\left[(\partial_v S + eA_0)g^{0j}\partial_j S\right]^2 + m^2 \right\}\xi_{\alpha_1\cdots\alpha_k} = 0.$$

Defining

$$m_l = \frac{-2g^{0j}(\partial_v S + eA_0)\partial_j S - g^{ij}\partial_i S \partial_j S}{2(\partial_v S + eA_0)g^{0i}\partial_i S}$$
$$+ \frac{-m^2 + \lambda^2\left[(\partial_v S + eA_0)g^{0j}\partial_j S\right]^2}{2(\partial_v S + eA_0)g^{0i}\partial_i S}, \qquad (18)$$

Equation (17) changes to

$$i\lambda\gamma^\mu(\partial_\mu S + eA_\mu)\xi_{\alpha_1\cdots\alpha_k} + m_l\xi_{\alpha_1\cdots\alpha_k} = 0. \qquad (19)$$

Multiplying $i\lambda\gamma^\nu(\partial_v S + eA_v)$ at both sides, we get

$$\lambda^2\gamma^\mu\gamma^\nu(\partial_\mu S + eA_\mu)(\partial_v S + eA_v)\xi_{\alpha_1\cdots\alpha_k} + m_l^2\,\xi_{\alpha_1\cdots\alpha_k} = 0. \quad (20)$$

By exchanging μ and ν for Equation (20), then

$$\lambda^2\gamma^\nu\gamma^\mu(\partial_v S + eA_v)(\partial_\mu S + eA_\mu)\xi_{\alpha_1\cdots\alpha_k} + m_l^2\,\xi_{\alpha_1\cdots\alpha_k} = 0. \quad (21)$$

Combining Equations (20), (21), and

$$\gamma^\nu\gamma^\mu + \gamma^\mu\gamma^\nu = 2g^{\mu\nu}I, \qquad (22)$$

it is easy to get

$$\left[\lambda^2 g^{\mu\nu}(\partial_\mu S + eA_\mu)(\partial_v S + eA_v) + m_l^2\right]\xi_{\alpha_1\cdots\alpha_k} = 0. \qquad (23)$$

Equation (23) is a matrix equation. In fact, it is an eigenvalue matrix equation. The condition for the nonsingular solution of this eigenvalue matrix equation requires that the corresponding value of the determinant is zero. Combining Equations (18), (23), and (7), we get

$$\frac{2g^{0j}(\partial_v S + eA_0)\partial_j S + g^{ij}\partial_i S \partial_j S + m^2}{2(\partial_v S + eA_0)g^{0i}\partial_i S}$$
$$+ \frac{-\lambda^2\left[(\partial_v S + eA_0)g^{0j}\partial_j S\right]^2}{2(\partial_v S + eA_0)g^{0i}\partial_i S} - \lambda m = 0. \qquad (24)$$

Therefore,

$$2g^{0j}(\partial_v S + eA_0)\partial_j S + g^{ij}\partial_i S \partial_j S + m^2$$
$$- 2\lambda m(\partial_v S + eA_0)g^{0i}\partial_i S \qquad (25)$$
$$- \lambda^2\left[(\partial_v S + eA_0)g^{0j}\partial_j S\right]^2 = 0.$$

As $\lambda \ll 1$, $o(\lambda^2)$ is a high-order term. For accuracy of modification, the term $o(\lambda^2)$ is kept in Equations (17), (23), and (24). If $o(\lambda^2)$ is ignored in Equation (17), one cannot obtain a correct result. For the Vaidya-Bonner black hole, only this derivation can get a correct result. In fact, Equation (25) is the dynamic equation describing an arbitrary spin fermion in the Vaidya-Bonner space-time. Moreover, this equation is derived from the Rarita-Schwinger equation in the curved space-time with the Lorentz Invariance Violation. So this equation is a deformation of the Hamilton-Jacobi equation or can be called exactly as the Rarita-Schwinger-Hamilton-Jacobi equation. The first two terms of this equation can be expressed as $g^{\mu\nu}(\partial_\mu S + eA_\mu)(\partial_v S + eA_v)$. Considering Equation (10), one can obtain the first two terms in Equation (25), so Equation (25) can be rewritten as

$$g^{\mu\nu}(\partial_\mu S + eA_\mu)(\partial_v S + eA_v) + m^2$$
$$- 2\lambda m(\partial_v S + eA_0)g^{0i}\partial_i S \qquad (26)$$
$$- \lambda^2\left[(\partial_v S + eA_0)g^{0j}\partial_j S\right]^2 = 0.$$

From this equation, we can get the action of the fermion and then study the modified tunneling radiation of fermions. Equation (26) is a highly accurate dynamic equation because the term $o(\lambda^2)$ is not ignored during the derivation and the Lorentz Invariance Violation is included. If it is not done so, an accurate modification cannot be obtained. Note that the modification of boson Hamilton-Jacobi equation is different from this method [24, 25]. This indicates the significance of accurate modification of tunneling for particles with an arbitrary spin. In the following, we will derive the thermal dynamic characteristics at the horizon of the Vaidya-Bonner black hole.

3. Tunneling Modification for Fermions with Arbitrary Spin in Vaidya-Bonner Black Hole

The Vaidya-Bonner black hole is a charged nonstationary spherical black hole; the line element is shown in Equation (7). The event horizon of the Vaidya-Bonner black hole is determined by the zero supercurved equation

$$g^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial f}{\partial x^\nu} = 0. \qquad (27)$$

From Equations (10) and (27), we find that the event horizon r_H satisfies

$$r_H^2 - 2Mr_H + Q^2 - 2\dot{r}_H r_H^2 = 0, \qquad (28)$$

where $\dot{r}_H = dr_H/dv$ is the change rate of r_H with time. Solving Equation (28), we get

$$r_H = \frac{M \pm \left[M^2 - Q^2(1 - 2\dot{r}_H)\right]^{1/2}}{1 - 2\dot{r}_H}, \qquad (29)$$

where "+" denotes the event horizon of the Vaidya-Bonner black hole. From Equations (10) and (25), the accurate dynamic equation of fermions with an arbitrary spin is

$$\frac{\Delta}{r^2}\left(\frac{\partial S}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2 + \frac{1}{r^2 \sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2$$
$$+ 2\left(\frac{\partial S}{\partial v} + eA_0\right)\left(\frac{\partial S}{\partial r}\right) + m^2 - 2\lambda m\left(\frac{\partial S}{\partial v} + eA_0\right) \quad (30)$$
$$\cdot\left(\frac{\partial S}{\partial r}\right) - \lambda^2\left(\frac{\partial S}{\partial v} + eA_0\right)^2\left(\frac{\partial S}{\partial r}\right)^2 = 0.$$

The key to research the tunneling is to get the action S of fermions. For this black hole, the key is the solution of the action in the direction of the radius, so a tortoise coordinate transformation is necessary,

$$r_* = r + \frac{1}{2\kappa}\ln\frac{r - r_H(v)}{r_H(v_0)}, \quad (31)$$

$$v_* = v - v0, \quad (32)$$

where κ is the surface gravity, r_H is the event horizon of the black hole, and v_0 is a special moment when the fermion escapes from the event horizon. Both v_0 and κ are constants. From Equations (31) and (32), we have

$$\frac{\partial}{\partial r} = \left[1 + \frac{1}{2\kappa(r - r_H)}\right]\frac{\partial}{\partial r_*}, \quad (33)$$

$$\frac{\partial}{\partial v} = \left[\frac{\partial}{\partial v_*} + \frac{\dot{r}_H}{2\kappa(r - r_H)}\right]\frac{\partial}{\partial r_*}. \quad (34)$$

We make a separation of the variable to action S as

$$S(v_*, r_H, \theta, \varphi) = R(v_*, r_H) + Y(\theta, \varphi), \quad (35)$$

and set

$$\frac{\partial S}{\partial v_*} = \frac{\partial R}{\partial v_*} = -\omega. \quad (36)$$

Substituting Equations (31), (32), (33), (34), (35), (36) into Equation (30), then Equation (30) becomes

$$\left(\frac{\partial R}{\partial r_*}\right)^2\left\{\frac{\Delta}{r^2}\left[\frac{1 + 2\kappa(r - r_H)}{2\kappa(r - r_H)}\right]^2 + \frac{2\dot{r}_H[1 + 2\kappa(r - r_H)]}{[2\kappa(r - r_H)]^2}\right.$$
$$- 2\lambda m\dot{r}_H\frac{[1 + 2\kappa(r - r_H)]}{[2\kappa(r - r_H)]^2} - \lambda^2\frac{1}{[2\kappa(r - r_H)]^2}$$
$$\left.\cdot\left(\frac{\partial R}{\partial v_*} + eA_0\right)^2[1 + 2\kappa(r - r_H)]\right\} \quad (37)$$
$$+ 2\left(\frac{\partial R}{\partial v_*} + eA_0\right)(1 - \lambda m)\frac{[1 + 2\kappa(r - r_H)]}{\kappa(r - r_H)}\frac{\partial R}{\partial r_*}$$
$$+ m^2 + \frac{\lambda}{r^2} + o\left(\lambda'^2\right) = 0,$$

where $\lambda' = \lambda\dot{r}_H$. For simplification, it is suitable to keep the first-order term of λ in the final results. Multiplying $2\kappa(r - r_H)$ to both sides of Equation (37), and taking the limit for the condition $r \longrightarrow r_H$, we get

$$\left(\frac{\partial R}{\partial r_*}\right)^2 - 2(1 - \lambda m)(\omega - \omega_0)\left(\frac{\partial R}{\partial r_*}\right) = 0, \quad (38)$$

where $\omega_0 = eQ/r_H$. The limit of the coefficient of $(\partial R/\partial r_*)^2$ is

$$\lim_{r \to r_H}\frac{1}{2\kappa(r - r_H)r^2}\left[(r^2 - 2Mr + Q^2)[1 + 2\kappa(r - r_H)]\right.$$
$$\left. + 2\dot{r}_H r^2 - 2\lambda m\dot{r}_H r^2 - \lambda^2(\omega - \omega_0)^2 r^2\right] = 1. \quad (39)$$

From Equations (33) and (38)

$$\frac{\partial R}{\partial r} = \frac{1 + 2\kappa(r - r_H)}{2\kappa(r - r_H)}\frac{\partial R}{\partial r_*} = 2\frac{1 + 2\kappa(r - r_H)}{2\kappa(r - r_H)}(1 - \lambda m)(\omega - \omega_0). \quad (40)$$

Using the residue theorem to solve R in Equation (40), we get

$$R_\pm = \int 2\frac{1 + 2\kappa(r - r_H)}{2\kappa(r - r_H)}(1 - \lambda m)(\omega - \omega_0)dr$$
$$= \frac{i\pi}{\kappa}(1 - \lambda m)[(\omega - \omega_0) \pm (\omega - \omega_0)]. \quad (41)$$

From Equation (39), κ in Equation (41) can be obtained as

$$\kappa = \frac{r_H - M - 2\dot{r}_H r_H - 2\lambda m\dot{r}_H r_H - \lambda^2(\omega - \omega_0)^2 r_H}{2Mr_H - Q^2}. \quad (42)$$

Due to $\lambda \ll 1$, the final result can only retain the λ term. In Equation (41), "+" and "-" represent outgoing and ingoing waves, respectively, from the horizon of the back hole.

So according to the tunneling theory, we get the tunneling rate for fermions with an arbitrary spin in the Vaidya-Bonner space-time.

$$\Gamma = \exp(-2 \text{ Im } S) = \exp(-2 \text{ Im } R_\pm)$$
$$= \exp\left[-\frac{2\pi}{\kappa'}(\omega - \omega_0)\right] = \exp\left(-\frac{\omega - \omega_0}{T'_H}\right). \quad (43)$$

The $\kappa' = \kappa/(1 - \lambda m)$ in Equations (42) and (43) is the modified surface gravitational force at the event horizon of the black hole. T_H in Equation (43) is

$$T'_H = \frac{\kappa'}{2\pi} = \frac{r_H - M - 2\dot{r}_H r_H - 2\lambda m\dot{r}_H r_H - \lambda^2(\omega - \omega_0)^2 r_H}{2\pi(1 - \lambda m)(2Mr_H - Q^2)}. \quad (44)$$

This is a new form of the Hawking temperature after modification in the Vaidya-Bonner black hole. Obviously, the tunneling rate and temperature of the black hole have been significantly modified. Adopting the Taylor expansion for $1/(1 - \lambda m)$ and neglecting the high-order items of λ, Equation (44) becomes

$$
\begin{aligned}
T'_H &= \frac{r_H - M - 2\dot{r}_H r_H - 2\lambda m \dot{r}_H r_H - \lambda^2 (\omega - \omega_0)^2 r_H}{2\pi (2Mr_H - Q^2)} \\
&\quad \cdot \left[1 + \lambda m + (\lambda m)^2 + \cdots \right] \\
&\approx \frac{r_H - M - 2\dot{r}_H r_H + \lambda m (r_H - M - 4\dot{r}_H r_H)}{2\pi (2Mr_H - Q^2)} \\
&= T_H + \frac{\lambda m (r_H - M - 4\dot{r}_H r_H)}{2\pi (2Mr_H - Q^2)},
\end{aligned}
\tag{45}
$$

where T_H is the Hawking temperature without the Lorentz Invariation Violation modification. Usually, $\dot{r}_H \ll r_H$, so the Hawking temperature becomes higher than that without the Lorentz Invariation Violation modification.

4. Conclusions and Discussions

In this paper, based on the modified Dirac equation proposed by Kruglov, we extend his work to the Rarita-Schwinger equation which can describe fermions with an arbitrary spin and accurately modify the semiclassic Hamilton-Jacobi equation. The characteristics of tunneling radiation from a nonstationary spherical Vaidya-Bonner black hole are derived. The results show that the tunneling rate, surface gravitational force, and Hawking temperature, all of them should be modified by a term related to parameter λ. Although it is a minor modification term, it is still valuable for further research. The tunneling rate and Hawking temperature indicate all these characteristics are still spherically symmetrical. Another important parameter in thermal kinetics is entropy. The modification of the Hawking temperature must induce the change of entropy. According to the first law of thermal kinetics of a black hole,

$$
dM = TdS + VdJ + UdQ,
\tag{46}
$$

where V and U are the rotation potential and electromagnetic potential of the black hole, respectively. For the Vaidya-Bonner black hole, the modified entropy at $r = r_H$ is

$$
dS = \frac{dM - UdQ}{T'_H}.
\tag{47}
$$

Equations (43), (44), (45), (46), (47) show a few new results. We find that the modification of the Hawking temperature and entropy is not only related to λ but also related to $\dot{r}_H$ for the Vaidya-Bonner black hole. For the general case of $\dot{r}_H \ll r_H$, the Hawking temperature will increase, but the entropy will decrease compared with the fiducial results without the Lorentz Invariation Violation modification. As $\dot{r}_H = 0$, our results can reduce to the results of the Reissner-Nordström black hole; as $\dot{r}_H = Q = 0$, our results can reduce to the results of the Schwarzschild black hole.

For the curved space-time with a component of inverse metric tensor $g^{00} = 0$, the modified results will include λ and λ^2. In the results, keeping the main λ term is enough since λ is very small. In our work, we have chosen the condition $\alpha = 2$ for the modified light dispersion relation. In the general case, this condition should be cut off. Therefore, a more general modification method to the Rarita-Schwinger equation and the tunneling it describes should be further discussed.

Moreover, for the tunneling radiation of bosons, $g^{\mu\nu}$ in the normal Hamilton-Jacobi equation of bosons should be substituted by $g^{\mu\nu} + \lambda u^\mu u^\nu$, where u is an etheric-like vector. Applying the inverse metric tensors of the Vaidya-Bonner black hole and choosing a suitable u, one can obtain the dynamical equation for bosons in the Vaidya-Bonner space-time. The solution process of this new Hamilton-Jacobi equation is similar to the method in this paper, such as tortoise coordinate transformation and separation of a variable. We will do a series of researches to these problems in the future work.

Data Availability

The data used to support the findings of this study are included within the article. The researchers can also contact the corresponding author for more information.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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