

## Research Article

# Physical Attributes of Bardeen Stellar Structures in $f(R)$ Gravity

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The main aim of our study is to explore some relativistic configurations of compact object solution in the background of  $f(R)$  gravity, by adopting the Krori-Barua spacetime. In this regard, we establish the field equations for spherically symmetric spacetime along with charged anisotropic matter source by assuming the specific form of the metric potentials, i.e.,  $\nu(r) = Br^2 + C$  and  $\lambda(r) = Ar^2$ . Further, to calculate the constant values, we consider the Bardeen model as an exterior spacetime at the surface boundary. To ensure the viability of the  $f(R)$  gravity model, the physical characteristics including energy density, pressure components, energy bonds, equilibrium condition, Herrera cracking concept, mass-radius relation, and adiabatic index are analyzed in detail. It is observed that all the outcomes by graphical exploration and tabular figures show that the Bardeen black hole model describes the physically realistic stellar structures.

## 1. Introduction

In modern cosmology, the discussion of accelerated evolution of the universe is considered as the most significant topic. One can believe that the formation of the universe depends upon ordinary matter, dark matter, and dark energy [1–4]. Various astronomical observations suggested that the key intent of this mysterious expansion of our universe is dark energy, as it preserves huge negative pressure as compared to the pressure of dark matter. Although general relativity (GR) provides many realistic results in exploring the nature of the universe, in the case of dark energy, GR fails to provide satisfactory outcomes. Thus, to achieve the fruitful results, numerous modified theories have been proposed and gained much popularity in the preceding decades. These gravitational theories are formulated by modifying the Einstein-Hilbert action and provide the natural replacement of GR. The extensive range of these modified theories of gravity is  $f(R)$ ,  $f(R, T)$ ,  $f(G)$ ,  $f(R, G)$  [5–12]. However,  $f(R)$  theory of gravity is one of the most famous and realistic alternatives to GR and gains the trust of researchers by providing the most interesting outcomes in the field of dark energy problems. Several forms of  $f(R)$  theory have already been presented by many researchers [13–15]. This theory

has shown the significant outcomes in dealing with different cosmological constraints, such as galactic scale, the cosmic microwave background test, early time inflation, the late-time cosmic evolution, phantom fields, and most importantly in the exploration of the mystery of expansion of the universe [16–23]. It is worthwhile to mention here that several physical aspects like Newtonian limit [24–26], the solar system test [27–29], formulation of singularities [30], and gravitational stability [31–33] were comprehensively explored through  $f(R)$  gravity. The  $f(R)$  theory of gravity was first introduced by Buchdahl [34]. Later on, this modification was used to solve some cosmic acceleration and early inflation problems [35, 36]. Moreover, Nojiri and Odintsov [37] generalized the Einstein-Hilbert action by using the Ricci scalar function and observed some interesting results. Erickcek et al. [38] obtained the unique solution for the exterior spacetime of the stellar structure by using the matching constraints in the background of matter distribution. In this regard, Kainulainen et al. [39] discussed the interior spacetime of compact objects in the Palatini formulation of the  $f(R)$  theory of gravity. Some auxiliary features in the study of the massive compact objects have been added by the  $f(R)$  gravity modeling [40]. Moreover, some viable  $f(R)$  theory of gravity rainbow models was demonstrated by Hendi et al.

[41]. A class of nonlinear electrodynamics ranging from three-dimensional spacetime is also explored by the same authors [42]. Starobinsky [43] presented a new family of  $f(R)$  theory models that represented the interesting outcomes for the solar system laboratory testing.

In astrophysics, the gravitational collapse of stellar objects and the formulation of new compact stars is always considered as the most interesting topic. The gravitational collapse appears at the point where internal pressure of the stellar objects fails to keep up the pressure against the outer gravitational force. The degeneracy pressure produces stability against the collapsing of the star, and the outcomes of collapsing of these compact stars lead to the birth of white dwarfs, neutron stars, or black holes. These stars emerged with apex densities because these stars were considered to be massive objects but volumetrically smaller. However, the exact features of these compact stars are yet to be explored, one can believe that they are actually the massive ones with a very small radius. All these types of objects other than black holes are commonly categorized as degenerate stars. To investigate the vital existence of such stellar systems, the solutions of Einstein field equations (EFE) are mandatory. The very first solution of these EFE was determined by Schwarzschild [44]. In this regard, the realistic and complex nontraversable models of the celestial objects have been investigated by Tolman [45] and Oppenheimer and Volkoff [46] in the frame of observational data. Later, Oppenheimer and Snyder [47] demonstrated the effects of gravitational collapse with a homogeneity-based dust sphere. The realistic features of the compact objects show the connection between the interior pressure and the force of gravitation that ultimately led to an equilibrium state. This phenomenon has considerable importance in the study of interior structure of compact objects. Further, Baade and Zwicky [48] examined the compact stellar system and proposed that the supernova might convert into a smaller stellar structure once observing the strongly magnetized spinning neutrons. Folomeev and Singleton [49] analyzed the spherically symmetric polytropic matter distribution for nonminimal coupling and showed that the stellar system had regular, static, and asymptotically flat behavior. Dzhunushaliev et al. [50] explored the stability features of symmetric matter fluid by using scalar fields. Moreover, the idea that nuclear density of the compact object illustrates anisotropic nature at the center of the star was first presented by Ruderman [51].

For modeling of static objects, the spherical symmetric geometry is considered as a very natural and effective source while there are several possibilities in the choice of matter distribution. In the beginning, it has been assumed that the formulation of a star's core consists of perfect fluid which leads to the isotropic state. However, isotropy might be a suitable attribute, but it does not show a usual characteristic of a stellar compact, whereas, in case of pressure anisotropic and viscosity fluid, the anisotropy disturbs the stability of the structure relative to local isotropic cases. Anisotropy in fluid normally arises due to the existence of a combination of various kinds of fluids, rotation, magnetic field, viscosity, etc. Many models of anisotropic compact stars have been introduced in the literature [52–58]. The modeling of the mass-

radius relation of the neutron star was studied by Egeland [59], and he claimed that density of the vacuum space is the main cause of cosmological constant. Mak and Harko [60] discussed the physical features of the strange stars and obtained an exact solution by using spherical symmetric matter distribution. Yet, the presence of electric charged anisotropy enhances the equilibrium state and stability of the celestial system [61–63]. Hossein et al. [64] described a couple of useful characteristics of anisotropic fluid with cosmological constants. Rahaman et al. [65] discussed the Chaplygin gas equation of state (EoS) and presented the extended version of the Krori-Barua [66] model. Further, Herrera and his collaborators [67–69] investigated the interesting physical aspects of anisotropic stars in different contexts. Lobo [70] studied the compact stars along with a barotropic EoS and provided the extension of Mazur-Mottola Gravatar models by employing the matching conditions between static source and Schwarzschild spacetime. Later, Sunzu et al. [71] investigated the theory of MIT bag EoS for quark stars and they concluded that an increase in anisotropic parameters will gradually decrease the energy density, due to which the EoS becomes stiffer. Later, Illyas [72] also discussed the charged anisotropic fluid for compact stars by considering some viable  $f(G)$  gravity models. In particular, either to demonstrate the core of the astrophysical elements or to explore different aspects of the astrophysical elements, charged anisotropic fluid is always considered as the most favourable condition.

The concept of the black hole is considered as a crucial topic in literature. However, one can believe that there are two different classifications of black holes, one consisting of a horizon having a singularity inside it and the other without any singularity. In general, regular black holes are those black holes which are singularity free. The Bardeen black hole [73] was the very first example of the regular black hole, and this idea came to light by Bardeen in 1968. Later, the stability features of the Bardeen model were discussed by Moreno and Sarbach [74]. Later, Zhou et al. [75] analyzed the geodesic configuration of test objects in the context of the Bardeen spacetime. Further, the Bardeen model in the frame of GR has been studied by Shamir et al. [76] and they provided some interesting observations of a viable celestial system in the light of charged perfect fluid. The theory about the Bardeen model states that this model can be defined as a gravitational collapsing magnetic monopole produced by the certain structure of nonlinear electrodynamics and was introduced by Ayon-Beato and Garcia [77]. Some literature is available in the context of the Bardeen model [78–81]. In addition, the charged stellar structure along with the Bardeen sphere in the background of conformal motion was explored [82]. In a recent paper [83], compact star solutions have been discussed for the Bardeen sphere by using the famous Karmarkar approach. Being motivated from the above literature, here, we explore the studies of Mustafa et al. [83] in the context of modified  $f(R)$  theory by assuming the charged compact structures along with the Bardeen model.

The aim of this study is to explore some realistic stellar charged anisotropic model in the context of  $f(R)$  gravity,

by utilizing the Krori-Barua [66] spacetime. The illustration of metric potential confirms that the metric tensors exhibit continuous, nonsingular, and well-behaved nature. In particular, we study the physical attributes of the compact objects against various values of  $M$  for different ranges of  $R$ , i.e.,  $R = 10.10$  km with  $M = 1.2, 1.3, 1.4, 1.5(M_e)$ ,  $R = 9.10$  km with  $M = 1.2, 1.4(M_e)$ , and  $R = 8.10$  km with  $M = 1.2(M_e)$ . For this purpose, we take specific electric fields  $E^2 = kQr$  [83] and establish physically appropriate models of compact stars.

The arrangement of our current study is as follows: in the following portion, the field equations for the  $f(R)$  gravity have been presented for anisotropic configuration. A realistic  $f(R)$  theory of gravity model is also presented in the same section. Section 3 is dedicated to the Bardeen charged model and junction conditions. In segment IV, we explore the physical features and graphical responses of the compact stars. In the last segment, we summarized our work.

## 2. Some Basic Modified Field Equations

Firstly, we evolve the field equations of the  $f(R)$  gravity. In this regard, we assume the action of  $f(R)$  theory of gravity [10] shown as

$$S = \int \left[ \frac{f(R)}{2\kappa} + \mathcal{L}_m \right] \sqrt{-g} d^4x. \quad (1)$$

Here,  $f$  is a function of Ricci scalar and  $\mathcal{L}_m$  is the matter of the Lagrangian field. By varying the action (1) respecting the metric potential  $g_{\eta\zeta}$ , we develop the following  $f(R)$  field equation:

$$FR_{\eta\zeta} - \frac{1}{2}f(R)g_{\eta\zeta} - \nabla_\eta \nabla_\zeta F + g_{\eta\zeta} \square F = -\kappa T_{\eta\zeta}, \quad (2)$$

where  $\nabla_\eta$  and  $\square$  indicate covariant derivative and D'Alembertian symbol, i.e.,  $\square \equiv \nabla_\eta \nabla^\eta$  and  $F = df(R)/dR$ . Furthermore, for the investigation of the stellar objects, we assume the static spherically symmetric spacetime as

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2). \quad (3)$$

Here,  $\nu(r)$  and  $\lambda(r)$  are functions of  $r$  only. We divide the energy-momentum tensor into two parts, i.e.,

$$T_{\eta\zeta} = T_{\eta\zeta}^{(af)} + T_{\eta\zeta}^{(ef)}. \quad (4)$$

Anisotropic fluid is considered as the basic generalization of isotropic perfect fluid, in which the transverse pressure is not equal to radial pressure. In comparison with isotropic fluid, anisotropic spheres are assumed to describe more compact stellar objects like neutrons and gravastars [84, 85]. In case of necessity, anisotropic fluid narrates the inner structure of spherical symmetric spheres and the solid crust of the compact stars [86]. In the field of stellar objects, local pressure anisotropy is considered to be the most realistic assumption for demonstrating the nature of matter distri-

bution [87–99]. The stress-energy tensor corresponding to an anisotropic matter distribution, in an orthonormal basis, is presented by the density and pressure (radial and transverse) of the compact star which is correlated with the metric potential functions  $\nu(r)$  and  $\lambda(r)$  given in Equation (3).

$$T_{\eta\zeta} = (\rho + p_r)u_\eta u_\zeta + p_t g_{\eta\zeta} + (p_r - p_t)\mathcal{X}_\eta \mathcal{X}_\zeta, \quad (5)$$

where  $\rho$ ,  $p_r$ , and  $p_t$  designate the energy density, radial, and transverse pressure component, respectively. Further,  $u^\eta$  and  $\mathcal{X}^\eta$  are its timelike four velocity and a spacelike unit vectors orthogonal to  $u^\eta$ , respectively, which fulfill the following correlations:

$$\begin{aligned} u^\eta &= e^{-\nu(r)/2} \delta_0^\eta, \\ \mathcal{X}^\eta &= e^{-\lambda(r)/2} \delta_1^\eta, \\ u^\eta u_\eta &= -\mathcal{X}^\eta \mathcal{X}_\eta = 1. \end{aligned} \quad (6)$$

The energy-momentum tensor for electromagnetic field is given as

$$T_{\eta\zeta}^{(ef)} = \frac{1}{4\pi} \left( g_{\eta\eta} \mathbb{F}^{\eta\mu} \mathbb{F}_{\zeta\mu} - \frac{1}{4} g_{\eta\eta} \delta_\zeta^\eta \mathbb{F}_{\mu\nu} \mathbb{F}^{\mu\nu} \right). \quad (7)$$

Moreover,  $\mathbb{F}^{\eta\mu}$  is the Maxwell stress tensor and its equation can be represented as

$$\begin{aligned} \mathbb{F}^{\eta\mu} &= \beta_{\mu,\eta} - \beta_{\eta,\mu}, \\ \mathbb{F}^{\eta\mu}{}_{;\mu} &= -4\pi J^\eta. \end{aligned} \quad (8)$$

Here,  $\beta$  is the four potential and  $J^\eta$  is the four current which is presented as  $J^\eta = \sigma v^\eta$ , where  $\sigma$  indicates as charge density. Now, for spherically symmetric static line element, the nonzero components of four potential are  $J^0$  and for Maxwell tensor is  $\mathbb{F}^{01}$ , described as

$$\mathbb{F}^{01} = -\mathbb{F}^{10} = \frac{q}{r^2} e^{(\nu+\lambda)/2}, \quad (9)$$

where

$$q = 16\pi \int_0^r \sigma \rho^2 e^{\lambda/2} d\rho. \quad (10)$$

The term  $q$  denotes charge within the core of compact objects corresponding to  $r$ . Furthermore, the expression for electric field can be considered as  $E^2 = kQr$  [83] and  $E^2 = \mathbb{F}^{10} \mathbb{F}_{01}$ , which is expressed as  $E^2 = q^2/r^4$ . The  $f(R)$  field equations for (3) are given as

$$\begin{aligned} 8\pi\rho + E^2 &= e^{-\lambda} F \left( \frac{\nu''}{2} + \frac{\nu'}{r} + \frac{\nu'(\nu' - \lambda')}{4} \right) \\ &+ \frac{1}{2} f(R) - e^{-\lambda} F'' - e^{-\lambda} F' \left( \frac{\lambda'}{2} + \frac{2}{r} \right), \end{aligned} \quad (11)$$

$$8\pi p_r - E^2 = e^{-\lambda} F \left( -\frac{v''}{2} + \frac{\lambda'}{r} - \frac{v'(v' - \lambda')}{4} \right) - \frac{1}{2} f(R) + e^{-\lambda} F' \left( \frac{v' + 2\lambda'}{2} + \frac{2}{r} \right), \quad (12)$$

$$8\pi p_t + E^2 = F \left( \frac{1 - e^{-\lambda}}{r^2} + e^{-\lambda} \frac{(\lambda' - v')}{2r} \right) - \frac{1}{2} f(R) + e^{-\lambda} F'' + e^{-\lambda} F' \left( \frac{v' + \lambda'}{2} + \frac{1}{r} \right), \quad (13)$$

$$\sigma = \frac{e^{-\lambda/2}}{4\pi r^2} (r^2 E)', \quad (14)$$

here, “prime” symbolizes the  $r$  derivative. Further, the anisotropic factor  $\Delta$  is presented as

$$\Delta = 8\pi p_t - 8\pi p_r. \quad (15)$$

Next, by utilizing Equations (12) and (13), the following equation is generated:

$$\Delta = F \left[ \frac{1 - e^{-\lambda}}{r^2} + e^{-\lambda} \left( \frac{v''}{2} - \frac{(v' + \lambda')}{2r} + \frac{v'(v' - \lambda')}{4} \right) \right] + e^{-\lambda} F'' - e^{-\lambda} F' \left( \frac{\lambda'}{2} + \frac{1}{r} \right). \quad (16)$$

It is worthwhile to mention here that to simplify these Equations (11)–(14), we have considered Krori-Barua [66] spacetime, i.e.,  $v(r) = Br^2 + C$  and  $\lambda(r) = Ar^2$ , where  $A$ ,  $B$ , and  $C$  are any arbitrary constants. The formulation of singularities inside the compact star is believed as the key feature in the investigation of stellar objects. The geometry of any spacetime is actually presented by its metric potentials. For the realistic model of the  $f(R)$  theory of gravity, the metric potentials  $g_{rr}$  and  $g_{tt}$  should exhibit positive and regular behavior. Thus, in our case, Figure 1 shows that these metric potentials narrate the monotonically increasing, singularity-free graphical illustration and attain maximum value at the surface boundary which confirm that the model under consideration reveals physically realistic behavior. Further, to investigate the nature of the compact stars, here, we consider a realistic model of  $f(R)$  theory of gravity [100].

$$f(R) = R - (1 - \gamma)\xi^2 \left( \frac{R}{\xi^2} \right)^\gamma, \quad (17)$$

here,  $\xi$  and  $\gamma$  are any constant. This model (17) actually relates  $f(R)$  gravity with scalar-tensor gravity and is employed to explore the gravitational impacts of cosmic acceleration [100]. Faulkner et al. [100] argued that confor-

mal coupling is the significant aspect of this model (17). Moreover, this model also showed consistent outcomes for the Chameleon mechanism and solar system constraints [101]. To probe the stellar object existence for (17) model, the constant parameters selected in such a way that  $\rho$ ,  $p_r$ , and  $p_t$  reveal the positive and finite nature.

### 3. Matching Conditions

In this section, we assume the Bardeen black hole model as an exterior spacetime of the compact objects given by [73].

$$ds^2 = -h(r)dt^2 + h(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (18)$$

where  $h(r) = 1 - 2Mr^2/(q^2 + r^2)^{3/2}$ . The Bardeen model can be derived as exact solutions of suitable nonlinear electrodynamics coupled to gravity. The nonzero Einstein tensor in the Bardeen black hole can be related to the energy-momentum tensor of a nonlinear electromagnetic Lagrangian [74]. Moreover, the existence of Bardeen model solutions does not deny the singularity theorems [102]. It is notable that the sphere asymptotically responses as

$$h(r) = 1 - \frac{2M}{r} + \frac{3Mq^2}{r^3} + O\left(\frac{1}{r^5}\right). \quad (19)$$

One can observe from (19) that the expression  $1/r$  relates with the mass  $M$  of the compact star configuration, although the term relating  $1/r^3$  increased the importance in this discussion and became different from the usual Reissner-Nordstrom solution [103]. Here, in our present work, we take  $h(r) = 1 - (2M/r) + (3Mq^2/r^3)$ . The continuity condition for (3) and (18) at  $r = R$  takes the form

$$\begin{aligned} g_{rr}^+ &= g_{rr}^-, \\ g_{tt}^+ &= g_{tt}^-, \\ \frac{\partial g_{tt}^+}{\partial r} &= \frac{\partial g_{tt}^-}{\partial r}, \end{aligned} \quad (20)$$

here, “+” indicates the exterior metric and “-” identifies the interior. Next, we utilize Equations (3), (18), and (20) and obtain the values of  $A$ ,  $B$ , and  $C$ , defined as

$$\begin{aligned} A &= \frac{1}{R^2} \ln \left[ \frac{R}{R - 2M + 3MR^2 E^2} \right], \\ B &= \frac{2M + 3MR^2 E^2}{2R^2 (R - 2M + 3MR^2 E^2)}, \\ C &= \ln \left[ \frac{R - 2M + 3MR^2 E^2}{R e^{BR^2}} \right]. \end{aligned} \quad (21)$$

Further, we determine the mathematical values of the parameters by assigning specific values to the free parameters  $R$ ,  $M$ , and  $\xi$  and fixing some parameters, such that  $\gamma = 2$ ,  $k = 0.000001$ , and  $Q = 0.00001$ , as represented in Table 1.

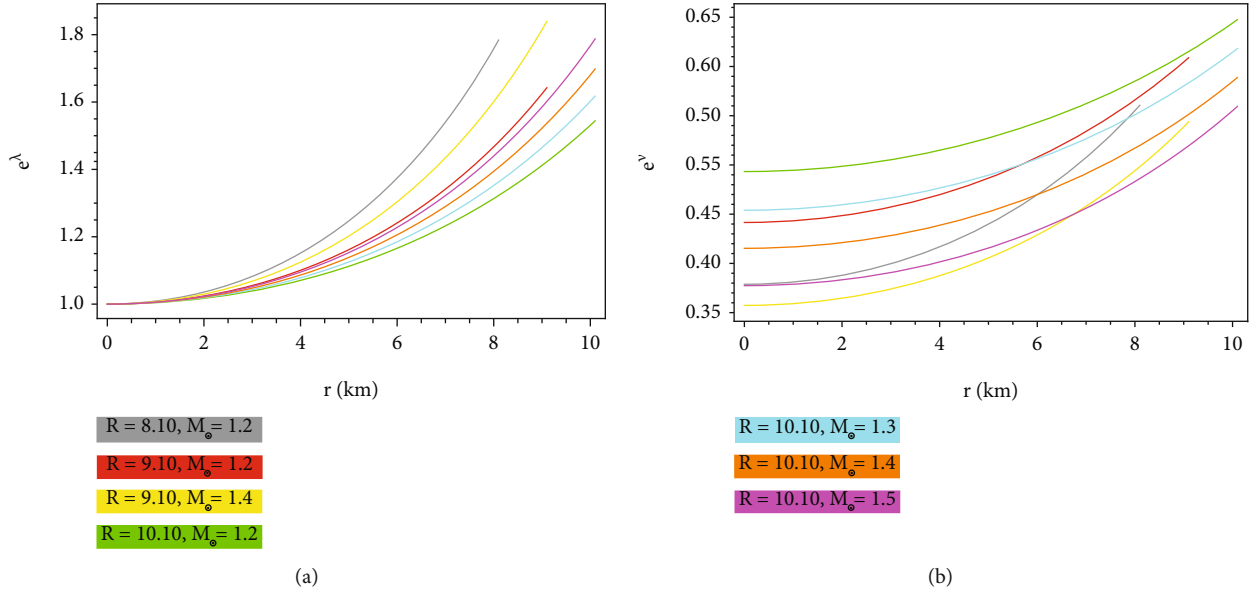

 FIGURE 1: Variation of  $g_{rr}$  (a) and  $g_{tt}$  (b).

 TABLE 1: Constant values of  $\xi$ ,  $A$ ,  $B$ , and  $C$  for considered compact stars.

(a)				
$M(M_{\odot})$	$\xi$	$R = 8.10 \text{ km [82]}$		
		$A$	$B$	$C$
1.2	0.05	0.008822	0.0059742	-0.970785

(b)				
$M(M_{\odot})$	$\xi$	$R = 9.10 \text{ km [82]}$		
		$A$	$B$	$C$
1.2	0.043	0.005992	0.003879	-0.817404
1.4	0.044	0.007359	0.005068	-1.02913

(c)				
$M(M_{\odot})$	$\xi$	$R = 10.10 \text{ km [82]}$		
		$A$	$B$	$C$
1.2	0.035	0.0042596	0.002667	-0.706625
1.3	0.036	0.004715	0.003027	-0.789717
1.4	0.037	0.005192	0.003422	-0.878726
1.5	0.038	0.005693	0.003859	-0.974462

Further, here, we mention the mandatory requirements, which have to be fulfilled for the well-behaved nature of the stellar system.

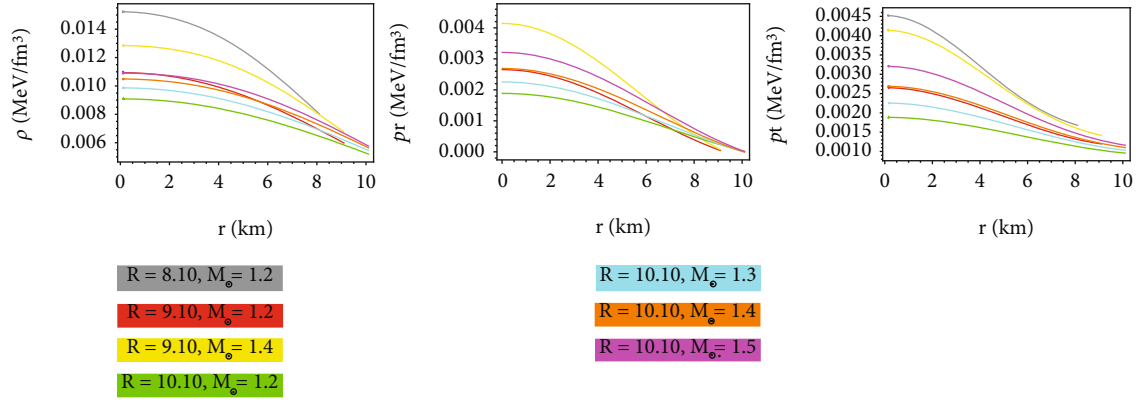
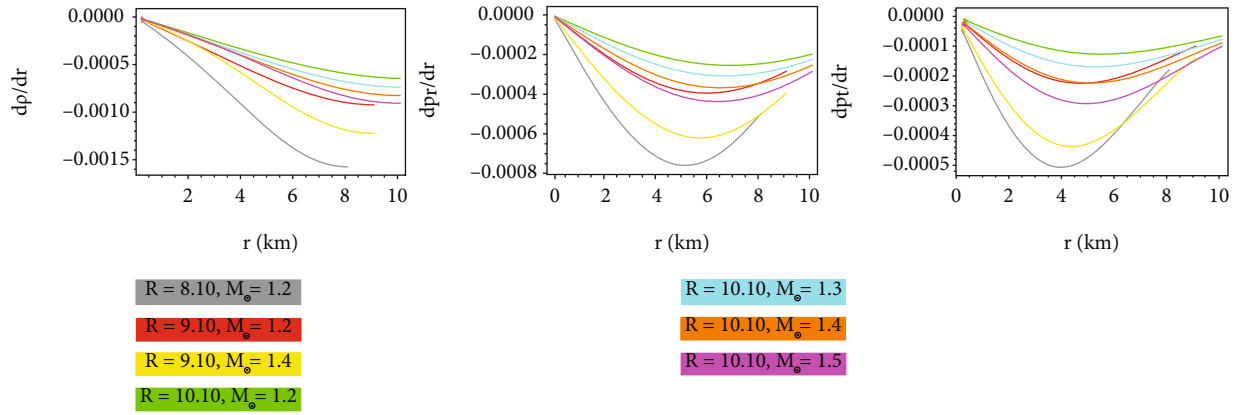
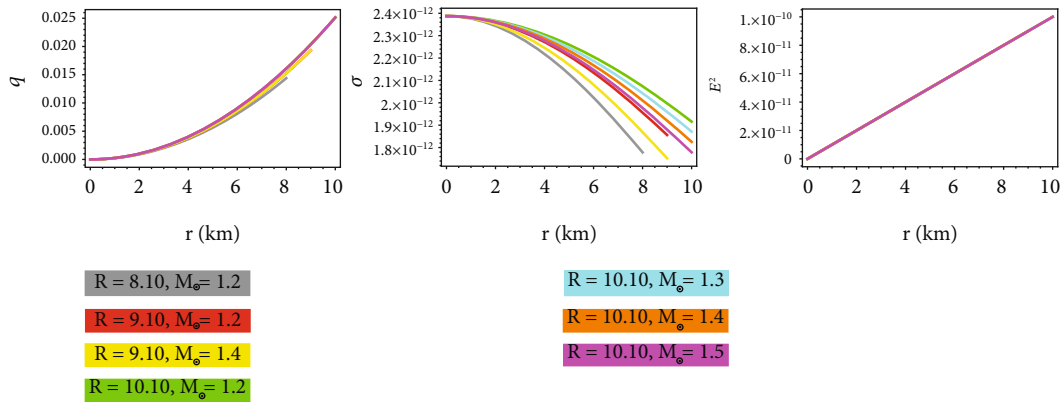
- (i) The  $g_{rr}$  and  $g_{tt}$  should be singularity free. Also, the anisotropic matter configuration must fulfill  $e^{\lambda(r=0)} = 1$  and  $e^{\nu(r=0)} = \text{constant}$
- (ii)  $\rho, p_r$ , and  $p_t$  must be positive and maximum at the center of the star

- (iii) The gradient of  $\rho, p_r$ , and  $p_t$  should be negative inside the limit  $0 \leq r \leq R$
- (iv) The obtained solution must fulfill the energy bonds NEC, WEC, SEC, and DEC
- (v) The condition of EoS must satisfy, i.e.,  $0 < w_r, w_t < 1$
- (vi) The velocity sounds must lie within  $[0, 1]$ , i.e.,  $0 < v_r^2, v_t^2 < 1$
- (vii) Redshift function must show monotonic decreasing behavior while mass and compactness should narrate monotonic increasing illustration
- (viii) Adiabatic index for anisotropic matter configuration must be greater than  $4/3$

#### 4. Physical Attributes of the Stellar Structure

Here, in this section, we will examine the graphical responses of the considered compact objects in the framework of the  $f(R)$  gravity model. For this, we will perform different physical tests such as energy density, pressure components, energy conditions, equilibrium constraints, causality condition, mass function, and redshift. All these attributes play a valuable role in illustrating the nature of compact stars.

**4.1. Energy Density and Pressure Progression.** Here, we explore the graphical responses of the density, radial pressure, and transverse pressure in the light of the  $f(R)$  gravity model. It is clearly shown from Figure 2 that all these plots illustrate positive and singularity-free nature. Also, these plots attain the highest values at the center and touch the surface at the boundary which shows that these plots exhibit satisfactory behavior. We also discuss the graphical illustration of gradients of  $\rho, p_r$ , and  $p_t$ , and their plots are shown in Figure 3.

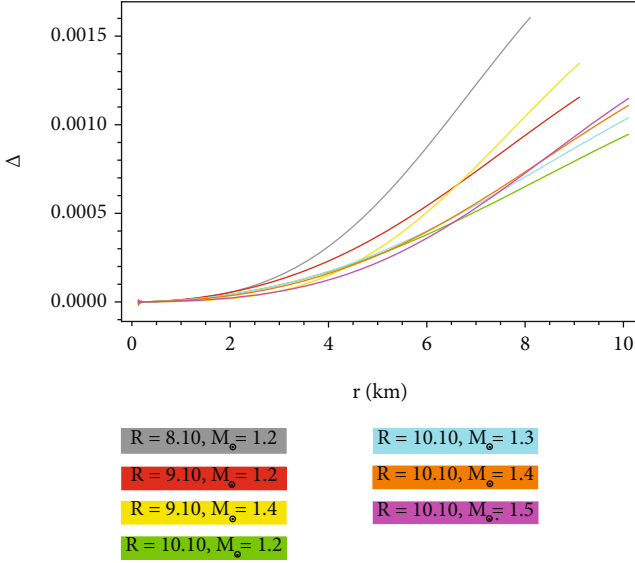
FIGURE 2: Evolution of  $\rho$ ,  $p_r$ , and  $p_t$ .FIGURE 3: Variation of  $d\rho/dr$ ,  $dp_r/dr$ , and  $dp_t/dr$ .FIGURE 4: Variation of  $q$ ,  $\sigma$ , and  $E^2$ .

The gradient of  $\rho$ ,  $p_r$ , and  $p_t$  presented negative behavior, which can be seen from Figure 3. These aspects indicate the apex compactness behavior of the compact stars.

**4.2. Charge Density and Electric Field.** Further, we examined the graphical response impact of  $q$ ,  $\sigma$ , and  $E^2$ . In Figure 4, the plot of electric charge shows increasing responses, whereas the nature of charge density is decreasing. One can easily observe that  $\sigma$  attains its highest value at center.

Moreover, the electric field illustrates that its graph vanishes when  $r = 0$  and then increases in a positive direction.

**4.3. Anisotropy Evolution.** In this segment, we investigate the graphical responses of anisotropy parameter  $\Delta$ , given in Equation (15) [104], since the repulsive nature of  $\Delta$  validates the existence of compact objects. It can be notable from Figure 5 that for a considered  $f(R)$  gravity model, anisotropy parameter exhibits repulsive nature as  $\Delta > 0$ . This implies that our system is well-behaved and stable.

FIGURE 5: Behavior of  $\Delta$ .

**4.4. Energy Conditions.** To verify the existence of the compact objects, energy constraints' role is very crucial. The famous energy constraints [105] are categories as follows: null energy (NEC), weak energy (WEC), strong energy (SEC), and dominant energy conditions (DEC), and defined as

- (i) NEC:  $\rho + E^2 \geq 0$
- (ii) WEC:  $\rho + p_r \geq 0, \rho + p_t + E^2 \geq 0$
- (iii) SEC:  $\rho + p_r + 2p_t + E^2 \geq 0$
- (iv) DEC:  $\rho - p_r + E^2 \geq 0, \rho - p_t \geq 0$

It has been observed from Figure 6 that all energy bonds showed well-fitted nature for the considered  $f(R)$  gravity model.

**4.5. Equilibrium Constraint.** Further, we examine the equilibrium state of the  $f(R)$  gravity model for the succeeding forces: gravitational force, hydrostatics force, anisotropic force, and electric force. For this purpose, we explore the Tolman-Oppenheimer-Volkoff equation [45, 46] along electric charge, given as

$$\frac{M_G(r)(\rho + p_r)}{r} e^{(\lambda-\nu)/2} + \frac{dp_r}{dr} - \frac{2}{r}(p_t - p_r) - \sigma(r)E(r)e^{\lambda(r)/2} = 0. \quad (22)$$

The effective gravitational mass  $M_G(r)$  represented by

$$M_G(r) = \frac{1}{2} v' r e^{(\nu-\lambda)/2}. \quad (23)$$

By replacing  $M_G(r)$  in Equation (22), we get

$$\frac{v'}{2}(\rho + p_r) + \frac{dp_r}{dr} - \frac{2}{r}(p_t - p_r) - \sigma(r)E(r)e^{\lambda(r)/2} = 0, \quad (24)$$

where  $\mathcal{F}_g = -v'/2(\rho + p_r)$ ,  $\mathcal{F}_h = -dp_r/dr$ ,  $\mathcal{F}_a =$ , and  $\mathcal{F}_e = \sigma(r)E(r)e^{\lambda(r)/2}$ . Here,  $\mathcal{F}_g$  symbolizes as gravitational force,  $\mathcal{F}_h$  indicates hydrostatic force,  $\mathcal{F}_a$  defines as anisotropic force, and  $\mathcal{F}_e$  identifies as electric force. For a realistic model of  $f(R)$  gravity, the joint sum of the above-mentioned four forces should be exactly equal to zero. This implies that for an equilibrium system, all these forces cancel out the effect of each other and maintain the balancing state, i.e.,

$$\mathcal{F}_g + \mathcal{F}_h + \mathcal{F}_a + \mathcal{F}_e = 0. \quad (25)$$

From Figure 7, it can be clearly noticed that all forces are satisfied by the necessary constraints of equilibrium.

**4.6. Equation of State.** Next, we determine the EoS for the  $r$  component and  $t$  component. The two ratios of EoS are mentioned as

$$w_r = \frac{p_r}{\rho}, \quad (26)$$

$$w_t = \frac{p_t}{\rho}.$$

In the graphical illustration shown in Figure 8, it can be noticed that  $0 < w_r$ , and  $w_t < 1$ . This implies that our system exhibits well-behaved nature.

**4.7. Stability Condition.** The velocity sound for  $r$  component and  $t$  component is denoted by  $v_r^2$  and  $v_t^2$  and is defined as

$$v_r^2 = \frac{dp_r}{d\rho}, \quad (27)$$

$$v_t^2 = \frac{dp_t}{d\rho}.$$

To prove the physical stability of the considered  $f(R)$  model, we employ the Herrera technique [87] presented as  $0 \leq v_r^2$  and  $v_t^2 \leq 1$ . In this regard, we discussed one more worthy approach presented by Abreu et al. [106] to investigate the potentially unstable (stable) configuration of the stellar objects. It is important to mention that potentially unstable (stable) realms within the compact structure are managed by the modification of sound propagation. The anisotropic realm is considered as potentially stable where  $-1 \leq v_t^2 - v_r^2 \leq 0$ . This means that the horizon in which the transverse speed component is less than the radial speed sound component is recognized as potentially stable otherwise not. It is worthy to highlight here that the considered  $f(R)$  model is entirely stable as shown in Figure 9.

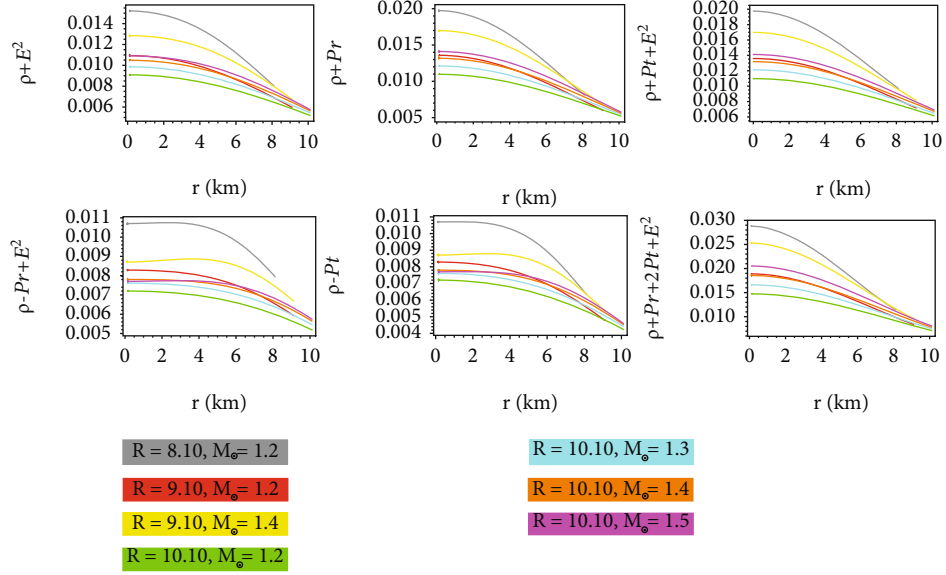
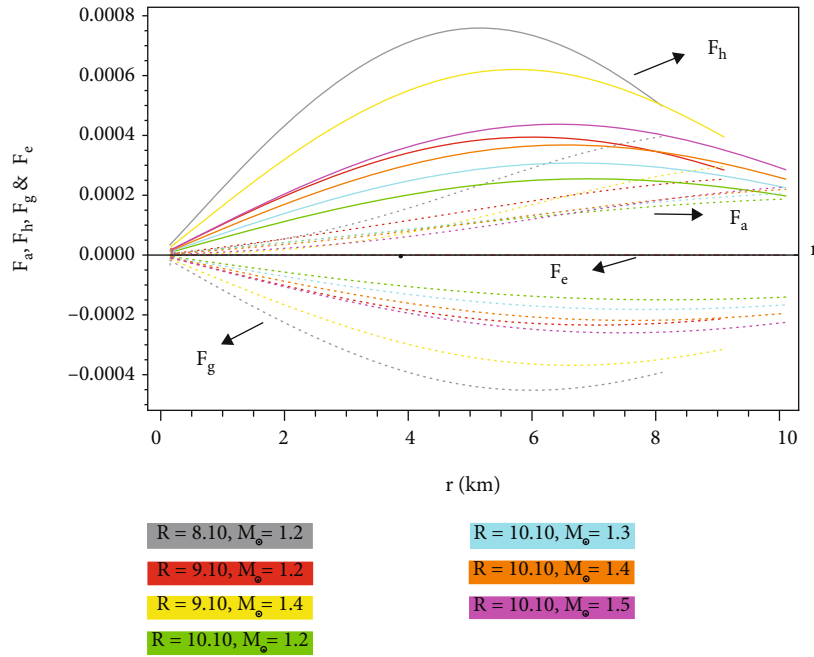


FIGURE 6: Evolution of energy conditions for our proposed model.

FIGURE 7: Behavior of  $\mathcal{F}_h$ ,  $\mathcal{F}_g$ ,  $\mathcal{F}_a$ , and  $\mathcal{F}_e$  for our proposed model.

**4.8. Mass, Compactness, and Redshift Evolution.** For the existence of a realistic and viable model, its mass function denoted by  $\mathcal{M}(r)$ , compactness factor symbolizes by  $\mathcal{U}(r)$ , and the redshift function identifies as  $Z(r)$  are considered as the necessary constraints in the analysis of a stellar system. Generally, one can believe that objects having anisotropic fluid should not be arbitrarily massive for a viable modified theory model. In the frame of isotropic fluid, the concept of maximum permissible mass-radius ratio, i.e.,  $2M/R < 8/9$ , was studied by Buchdahl [107]. Later, Mak and

Harko [108] generalized the concept of Buchdahl and argued that this mass-radius ratio holds for isotropic as well as anisotropic fluid. Hence, the mass function for anisotropic fluid is given by

$$\mathcal{M}(r) = 4\pi \int_0^R r^2 \rho dr = \frac{R}{2} (1 - e^{-\lambda}). \quad (28)$$

This response of  $\mathcal{M}(r)$  implies that  $\mathcal{M}(r)$  is regular and



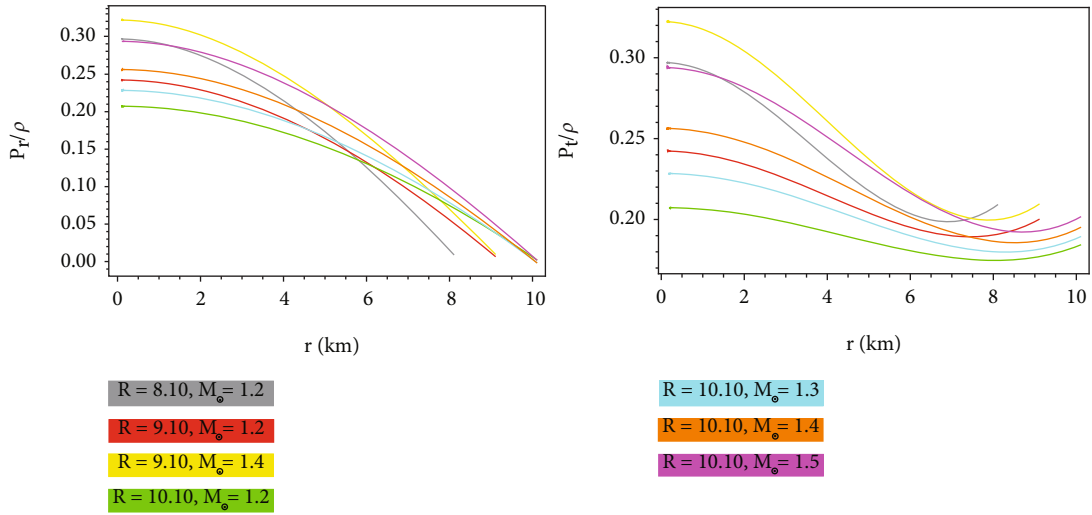


FIGURE 8: Evolution of  $w_r$  and  $w_t$  for our proposed model.

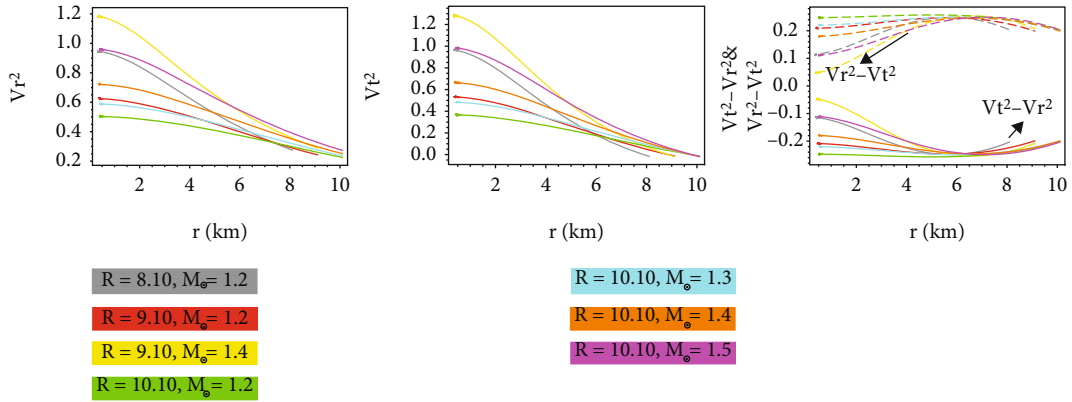


FIGURE 9: Behavior of  $v_r^2$ ,  $v_t^2$ , and  $v_t^2 - v_r^2$  for  $f(R)$  gravity model.

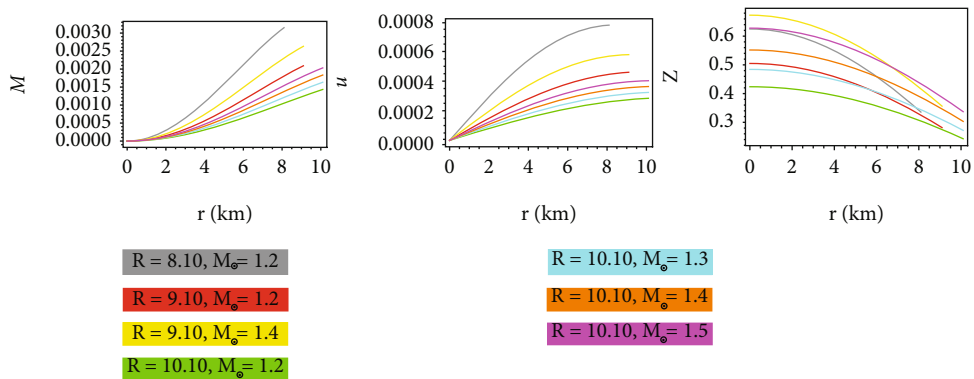
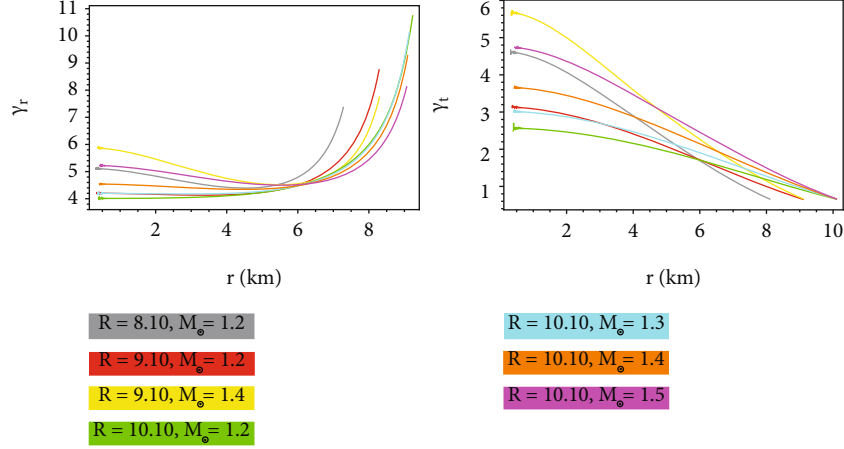


FIGURE 10: Behavior of  $\mathcal{M}(r)$ ,  $\mathcal{U}(r)$ , and  $Z(r)$ .

FIGURE 11: Variation of  $\gamma_r$  and  $\gamma_t$ .

attains maximum values at the boundary. Further, we explore the compactness factor which is presented as a mass-radius ratio [108] and is defined as

$$\mathcal{U}(r) = \frac{2\mathcal{M}(r)}{r} = \frac{2e^{-2Ar^2}(-1 + e^{Ar^2})(-1 + e^{Ar^2} - 3Br^2 - B^2r^4 + Ar^2(2 + Br^2))}{r^3}. \quad (29)$$

Next, the redshift  $Z(r)$  is given as [104]

$$Z(r) = e^{-\nu/2} - 1 = -1 + e^{-(C/2) - (Br^2/2)}. \quad (30)$$

From Figure 10, it is notable that the graphs of  $\mathcal{M}(r)$  and  $\mathcal{U}(r)$  illustrate the monotonically increasing nature, whereas the behavior of  $Z(r)$  is decreasing.

**4.9. Adiabatic Index.** Adiabatic index plays one of the most important roles in the stability of the stars. In general, it expresses the stiffness of EoS for energy density and categorizes the relativistic and nonrelativistic celestial objects according to their stability condition. The concept of the dynamical stability against infinitesimal radial adiabatic perturbation of the celestial structures was proposed by Chandrasekhar [109]. Later, his idea became much popular among cosmologists and can be tested for both isotropic and anisotropic fluid [110–115]. For a physically acceptable stellar model, the value of adiabatic index should be greater than  $4/3$ . Moreover, the formula of the adiabatic index for  $r$  component and  $t$  component is shown as

$$\gamma_r = \frac{\rho + p_r}{p_r} \left( \frac{dp_r}{d\rho} \right) = \frac{\rho + p_r}{p_r} v_r^2, \quad (31)$$

$$\gamma_t = \frac{\rho + p_t}{p_t} \left( \frac{dp_t}{d\rho} \right) = \frac{\rho + p_t}{p_t} v_t^2.$$

Hence, from Figure 11, it is clear that this condition is fulfilled at each point of our stellar system.

## 5. Conclusion

Here, we investigate the existence of stellar structure by considering the charged static spherical symmetric spacetime along with the Bardeen sphere in the background of a viable model for the  $f(R)$  theory of gravity. For this purpose, we assume the metric potentials by adopting the Krori-Barua [66] model, i.e.,  $\nu(r) = Br^2 + C$  and  $\lambda(r) = Ar^2$ , where  $A$ ,  $B$ , and  $C$  are any constant parameters. Moreover, in order to determine the values of these constant parameters, we relate the interior geometry with the exterior Bardeen sphere [73]. To validate the existence of our system, we evaluate different mandatory features related to the celestial structure. To do so, we discuss the graphical behavior of various figures of  $M$  for different values of  $R$ , i.e.,  $R = 10.10$  km with  $M = 1.2, 1.3, 1.4, 1.5(M_\odot)$ ,  $R = 9.10$  km with  $M = 1.2, 1.4(M_\odot)$ , and  $R = 8.10$  km with  $M = 1.2(M_\odot)$ .

The main goal of our investigation is to generate a new realistic family of  $f(R)$  theory of gravity solutions in the presence of charge anisotropic fluid. The significant outcomes are enlisted beneath.

- (i) Metric potentials are generally used to narrate the nature of spacetime. It can be easily seen from Figure 1 that the graphs of the metric potentials  $g_{rr} = e^\lambda$  and  $g_{tt} = e^\nu$  are positive, singularity free, and fulfill the necessity, i.e.,  $e^{\lambda(r=0)} = 1$  and  $e^{\nu(r=0)} = e^C$ . It is noticed that  $g_{rr}$  and  $g_{tt}$  attain the maximum values and show monotonically increasing behavior, which indicates that the chosen system exhibits a justifiable behavior
- (ii) The variation of density and pressure components corresponding to radial coordinate  $r$  for the considered model is positive and regular inside the sphere. From Figure 2, it is observed that the graphs of  $\rho, p_r$ , and  $p_t$  attain maximum

value at the center and decreasing downward, which confirms that the considered  $f(R)$  gravity model is well-behaved in nature. Also, the graphical representation of gradients of  $\rho$ ,  $p_r$ , and  $p_t$ , shown in Figure 3, yields satisfactory outcomes

- (iii) Next, discussion is based on the graphs of  $q$ ,  $\sigma$ , and  $E^2$ . One can clearly see in Figure 4 that the graphical response of  $E^2$  is maximum at  $r = R$ , while on the other hand,  $\sigma$  demonstrates the decreasing behavior. Moreover, the plot of charge  $q$  shows positive behavior and remained consistent for the entire system. Further, Figure 5 clarifies that  $\Delta$  reveals consistently positive nature for the considered  $f(R)$  gravity model. Thus, the repulsive nature of anisotropy confirms the physically acceptable behavior of the chosen stars
- (iv) Figure 6 shows that all the energy constraints are fulfilled
- (v) Figure 7 represents the balancing behavior of  $(\mathcal{F}_g)$ ,  $(\mathcal{F}_h)$ ,  $(\mathcal{F}_a)$ , and  $(\mathcal{F}_e)$  forces for our proposed viable  $f(R)$  gravity model
- (vi) For the well-behaved nature of the compact objects, the graphical response of EoS must fulfill the condition  $0 < w_r$  and  $w_t < 1$ . The corresponding plots given in Figure 8 show the consistent behavior of our proposed model of the  $f(R)$  gravity
- (vii) For stellar objects, the values of velocity of sounds  $v_r^2$  for  $r$  and  $v_t^2$  for  $t$  should be furnished within limits of  $[0, 1]$ . It is clear from Figure 11 that the causality constraints exhibit satisfactory nature for considered stellar structure. Furthermore, the Abreu approach outcomes are also consistent for the considered  $f(R)$  gravity model
- (viii) It has been noted from Figure 9 that the plots of  $\mathcal{M}(r)$  and  $\mathcal{U}(r)$  exhibit monotonically increasing nature while the plot of  $Z(r)$  is monotonically decreasing; this implies that we establish a stable system
- (ix) The radial and transverse adiabatic index values denoted by  $\gamma_r$  and  $\gamma_t$  for the chosen model are greater than  $4/3$ , as shown in Figure 10, which confirms the stability of our system

Thus, our presented work satisfied all those results proposed by Mustafa et al. [83] under the frame of GR. This confirms that our considered  $f(R)$  gravity model is entirely stable and physically acceptable.

## Data Availability

There is no data involved in the study. It is a theoretical work.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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