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A Study of Multi-Node and Dual-Hop Collaborative Communication Performance Based on Harmonic Mean Method

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Abstract: Closed form expressions for the PDF and MGF of the harmonic mean of two independent exponential variates are cited and derived, and then applied to study the performance of cellular multi-node and dual-hop cooperative communication systems with non-regenerative relays over flat Rayleigh fading channels. We derive the probability density function (PDF) and asymptotic symbol error rate (SER) expression with MRC scheme. Then we use Matlab to simulate the performance.

Keywords: harmonic mean, cooperative communication, multi-node, dual-hop, MGF

1. Introduction

MIMO technique has been regarded as the essential technique for beyond 3G mobile cellular networks. The Benefits of MIMO system have been extensively studied by researchers in both academic and industry. It successfully meets the rapidly growing demand for high rate, voice and especially for multimedia services [1].

In wireless cellular networks, base station can be equipped with multiple antennas and keeps them spatially separated. Unfortunately, it is hardly to fix multiple antennas in portable mobile terminals (also called as mobile terminals or users) due to insufficient antenna space, energy, and price, etc. So, the bottleneck of capacity is limited and the diversity technique of mobile terminals could not be realized from the traditional end-to-end transmission systems. In order to break this embarrassing situation, a novel concept, namely cooperative communication (or user cooperative diversity) was introduced by Sendonaris *et al*. [2].

Mazen O Hasna *et al*. in [3,4] firstly applied the harmonic mean to the cellular multi-node and dual-hop cooperative communication systems. They respectively derived the probability density function (PDF), the cumulative distribution function (CDF) and the moment generating function (MGF) of the expression. It's a new train of thought of cooperative diversity scheme. This paper applied the method of the harmonic mean to derive PDF and MGF of two independent exponential variates are cited and derived to study the performance of cellular multi-node and dual-hop cooperative communication systems with non-regenerative relays over flat Rayleigh fading channels.

2. Harmonic Mean of Exponential Variates

2.1 Definitions

1) Harmonic Mean
Given two numbers X1 and X2, the harmonic mean of X1 and X2, $\mu_H(X_1, X_2)$ is defined as the reciprocal of the arithmetic mean of the reciprocals of X1 and X2 [3] that is:

$$\mu_H(X_1, X_2) = \frac{2}{\frac{1}{X_1}+\frac{1}{X_2}} = \frac{2X_1X_2}{X_1+X_2} \quad \textbf{(1)}$$

2) Exponential RV
X follows an exponential distribution with parameterβ > 0 if the PDF of X is given by $p_X(x) = \beta e^{-\beta x}U(x)$, where $U(\cdot)$ is the unit step function.

2.2 Derive the Moment Generating Function

MGF of the Harmonic Mean of Two Exponential RVs Given a RV X ~ E(β), the PDF of Y = 1/X can be evaluated with the help of [9].

Then, $p_Y(y) = \frac{\beta}{y^2}e^{-\frac{\beta}{y}}U(y)$, $M_Y(s) = E_Y\left(e^{-sy}\right)$, K_v (X)= K_{-v} (X)

According to [5], we can get:

$$\int_0^\infty x^{v-1}e^{-\frac{\beta}{x}-\gamma x}dx = 2\left(\frac{\beta}{\gamma}\right)^{\frac{v}{2}} K_v\left(2\sqrt{\beta\gamma}\right)[R_e\beta > 0, R_e\gamma > 0]$$

$$M_\gamma(s) = E_Y\left(e^{-sy}\right) = \int_0^\infty p_Y(y)e^{sy}dy = \int_0^\infty \frac{\beta}{y^2}e^{-\frac{\beta}{y}}e^{sy}dy$$

$$= M_y(s) = 2\sqrt{\beta s}K_1(2\sqrt{\beta s})$$

When X= $\mu_H(X_1, X_2)$, we can get:

$$M_X(s) = E_X\left(e^{-sx}\right) = \int_{-\infty}^{+\infty} e^{-sx}\frac{1}{2}\beta_1\beta_2 e^{-\frac{x}{2}(\beta_1+\beta_2)}$$
$$\left[\left(\frac{\beta_1+\beta_2}{\sqrt{\beta_1\beta_2}}\right)K_1\left(x\sqrt{\beta_1\beta_2}\right)+2K_0\left(x\sqrt{\beta_1\beta_2}\right)\cdot U(x)dx\right]$$
$$=\int_0^\infty e^{-sx}\frac{1}{2}\beta_1\beta_2 e^{-\frac{x}{2}(\beta_1+\beta_2)}\left[\left(\frac{\beta_1+\beta_2}{\sqrt{\beta_1\beta_2}}\right)K_1\left(x\sqrt{\beta_1\beta_2}\right)+2K_0\left(x\sqrt{\beta_1\beta_2}\right)dx\right]$$
$$=\int_0^\infty e^{-sx}\frac{1}{2}\sqrt{\beta_1\beta_2}\,xe^{-\frac{x}{2}(\beta_1+\beta_2)}\cdot(\beta_1+\beta_2)K_1\left(x\sqrt{\beta_1\beta_2}\right)dx+$$
$$\int_0^\infty e^{-sx}\beta_1\beta_2 xe^{-\frac{x}{2}(\beta_1+\beta_2)}K_0\left(x\sqrt{\beta_1\beta_2}\right) \quad \textbf{(2)}$$

where K0(•) is the zero-order modified Bessel function of the second kind defined in [11].

Where $K_1(x)$ is the first order modified Bessel function of the second kind defined in [11].

Here, we find the Formula (2) can be constituted with ① + ②, according to [5].

$$\int_0^\infty x^{v-1}e^{\alpha x}K_v(\beta x)dx =$$
$$\frac{\sqrt{\pi}(2\beta)^v}{(\alpha+\beta)^{\mu+v}}\frac{\Gamma(\mu+v)\Gamma(\mu-v)}{\Gamma\left(\mu+\frac{1}{2}\right)}F\left(\mu+v,v+\frac{1}{2},\mu+\frac{1}{2},\frac{\alpha-\beta}{\alpha+\beta}\right)$$

Then, we can get:

① $\mu=2$, $\alpha=s+\frac{1}{2}(\beta_1+\beta_2)$; $v=1$, $\beta=\sqrt{\beta_1\beta_2}$

② $\mu=2$, $\alpha=s+\frac{1}{2}(\beta_1+\beta_2)$, $v=0$, $\beta=\sqrt{\beta_1\beta_2}$

$$M_X(s)=\frac{16\beta_1\beta_2}{3\left(\beta_1+\beta_2+2\sqrt{\beta_1\beta_2}+s\right)^2}$$
$$\left[\frac{4(\beta_1+\beta_2)}{\left(\beta_1+\beta_2+2\sqrt{\beta_1\beta_2}+s\right)}*{}_2F_1\left(3,\frac{3}{2};\frac{5}{2};\frac{\beta_1+\beta_2-2\sqrt{\beta_1\beta_2}+s}{\beta_1+\beta_2+2\sqrt{\beta_1\beta_2}+s}\right)\right.$$
$$\left.+{}_2F\left(2,\frac{1}{2};\frac{5}{2};\frac{\beta_1+\beta_2-2\sqrt{\beta_1\beta_2}+s}{\beta_1+\beta_2+2\sqrt{\beta_1\beta_2}+s}\right)\right]$$

3. Scheme of Cellular Multi-Node and Dual-Hop Cooperative Communication Systems with Non-Regenerative Relays over Flat Rayleigh Fading Channels

3.1 System Model

Consider an uplink wireless cooperative communication system with only one source, one destination and arbitrary N relays (from mobile terminals to base station). The source terminal transmits Mary phase-shift keying (M-PSK) modulation signals to destination terminal through a direct path along with N dual-hop paths. The channel is assumed to be quasi-static with flat fading. Furthermore, perfect channel state information is assumed available at the receivers, but the channels are unknown at the transmitters. There are two transmission phases. Firstly, the source broadcasts signals to destination and relays. Secondly, the relays which can successfully decode the signals will retransmit them to destination. Otherwise, they remain idle and do not participate in the cooperation.

Figure 1 is the system model of multi-node and dual-hop cooperative communication system. S is the moving terminal, D is the base station. This paper research the MPSK modulation of non-regenerative relay over flat Rayleigh fading channels, and the noise is the additive white Gaussian noise (AWGN). We suppose there are m relays on the uplink, and the gains are Gi. Assume that terminal S is transmitting a signal S(t) which has an average power normalized to 1.

3.2 Performance Analysis

Assume that in the EPA mode, where Ps=Pr=1. In the situation of non-regenerative relay and the MRC receiving mode, the receiving signal are respectively:

$$S_{R_1}(t)=h_1 s(t)+n_1(t)$$

$$S_{d_1}(t)=g_1 G_1\left(h_1 s(t)+n_1(t)\right)+n_2(t)$$

When $G_i=\sqrt{1/h_i}$ can according with the harmonic mean format, as

$$\gamma=\frac{2\frac{h_1^2}{N_0}\cdot\frac{g_1^2}{N_0}}{\frac{h_1^2}{N_0}+\frac{g_1^2}{N_0}}$$

which can use the method of harmonic mean to calculate.

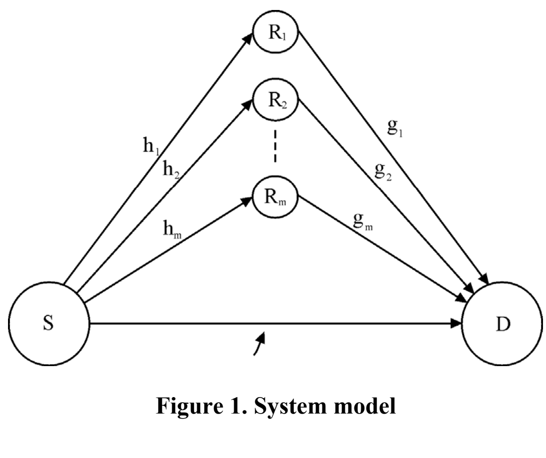

Figure 1. System model

We define $\overline{\gamma_i}$ and $\overline{\gamma_{g_i}}$ are respectively instantaneous SNR of the ith channels. The total SNR is:

$$r = r_f + \sum_{i=1}^{m} r_i$$

According to [4], we can achieve the cumulative distribution function (CDF) of the instantaneous SNR $\overline{\gamma_i}$ of the Multi-node and dual-hop Collaborative Communication channels:

$$F_{r_i}(r) = 1 - \prod_{i=1}^{m} p(r_{h_i} > r)p(r_{g_i} > r)$$

$$F_{r_i}(r) = 1 - \prod_{i=1}^{m} r(\bar{r}_{h_i}\bar{r}_{g_i})^{-1/2} e^{-r/2[(\bar{r}_{h_i})^{-1}+(\bar{r}_{g_i})^{-1}]} K_1[r(\bar{r}_{h_i}\bar{r}_{g_i})^{-1/2}]$$

The probabilitydensity function (PDF) of the instantaneous SNR $\overline{\gamma_i}$ of the Multi-node and dual-hop Collaborative Communication channels:

$$\mathrm{P}_\Gamma(\gamma) = \sum_{i=1}^{m} \frac{2\gamma e^{-\gamma\left(\frac{1}{\overline{r_{h_i}}}+\frac{1}{\overline{r_{g_i}}}\right)}}{\overline{r_{h_i}}\cdot\overline{r_{g_i}}} \left[\begin{array}{l} \dfrac{\overline{r_{h_i}}+\overline{r_{g_i}}}{\sqrt{\overline{r_{h_i}}\cdot\overline{r_{g_i}}}} K_1\left(\dfrac{2\gamma}{\sqrt{\overline{r_{h_i}}\cdot\overline{r_{g_i}}}}\right) \\ +2K_0\left(\dfrac{2\gamma}{\sqrt{\overline{r_{h_i}}\cdot\overline{r_{g_i}}}}\right) \end{array} \right] U(\gamma)$$

The Outage Probability

$$Pout = P[\gamma \le \gamma_{th}] = \int_0^{\gamma_{th}} P_\Gamma(\gamma)d\gamma$$

On the assumption that the average SNR $\overline{\gamma_i}$ when $\overline{\gamma_{h_i}} = \overline{\gamma_{g_i}} = \overline{\gamma_i}$, can obtain the moment generating function (MGF) of the instantaneous SNR $\overline{\gamma_i}$ of the Multi-node and dual-hop Collaborative Communication channels[4][8]:

$$M_\Gamma(s) = \left(1 - s\overline{\gamma}_f\right)^{-1} \prod_{i=1}^{M} {}_2F_1\left(1,2;\frac{3}{2};-\frac{\overline{\gamma}_i}{4}s\right)$$

$$= \frac{1}{\pi}\sum_{K=0}^{N} \frac{(1+K)!\left(\frac{1}{2}\right)!}{1\cdot3\cdot5\cdot7\cdots(2K-1)\left(-\frac{1}{2}\right)!}\left(\frac{\overline{\gamma}}{2}\right)^K$$

$$\sin^{2k}\left(\frac{\pi}{M}\right)\int_0^{(M-1)\pi/M} \sin^{-2K}\theta d\theta$$

This formula can't get it's closed-form, but we can get the asymptotic expression expressed as:

$$M_\Gamma(s) \approx \sum_{K=0}^{N} \frac{(1+K)!\left(\frac{1}{2}\right)!\left(\frac{\overline{\gamma}_i}{2}\right)^K}{\left(-\frac{1}{2}\right)!(2K-1)!} \times \frac{\sin^{2K}\left(\pi/M\right)}{1+\sin^2\left(\pi/M\right)}$$

The upper limit of average SER of the Multi-node and dual-hop channels is:

$$\mathrm{P}_s(E) \le \frac{\mathrm{M}-1}{\mathrm{M}}\left(1 - s\overline{\gamma}_f\right)^{-1}\prod_{i=1}^{m}\mathrm{M}_\gamma\left(-g_{psk}\right)$$

$$\mathrm{P}_S(E) \le \frac{\mathrm{M}-1}{\mathrm{M}}\left(\frac{\sin^2\theta}{\sin^2\theta + \overline{\gamma}_f\sin^2\left(\frac{\pi}{\mathrm{M}}\right)}\right)$$

$$\prod_{i=1}^{m}\left(\frac{(1)!(2)!}{\left(\frac{3}{2}\right)!(1)!}\left(\frac{\overline{\gamma}_i}{4}\right)^1\sin^2\left(\frac{\pi}{\mathrm{M}}\right) + \frac{(1)!(2)!}{\left(\frac{3}{2}\right)!(2)!}\left(\frac{\overline{\gamma}_i}{4}\right)^2\sin^4\left(\frac{\pi}{\mathrm{M}}\right)+\cdots\right)$$

$$\le \frac{\mathrm{M}-1}{\mathrm{M}}\left(\frac{\sin^2\theta}{\sin^2\theta + \overline{\gamma}_f\sin^2\left(\frac{\pi}{\mathrm{M}}\right)}\right)\sin^{2m}\left(\frac{\pi}{\mathrm{M}}\right)$$

$$\prod_{i=1}^{m}\frac{\overline{\gamma}_i}{4}\left(\frac{(1)!(2)!}{\left(\frac{3}{2}\right)!(1)!} + \frac{(1)!(2)!}{\left(\frac{3}{2}\right)!(2)!}+\cdots\right)$$

When $\frac{\overline{\gamma}_i}{4} \le 1$,as $i \in (1,m)$

$$P_S(E) \le \frac{\mathrm{M}-1}{\mathrm{M}}\left(\frac{\sin^2\theta}{\sin^2\theta + \overline{\gamma}_f\sin^2\left(\frac{\pi}{\mathrm{M}}\right)}\right)$$

$$\sin^{2m}\left(\frac{\pi}{\mathrm{M}}\right)\left(\prod_{i=1}^{m}\frac{\overline{\gamma}_i}{4}\right)\cdot\sum_{k=0}^{N}\frac{(1)_K(2)_K}{\left(\frac{3}{2}\right)_K K!}$$

4. Numerical Results and Performance Analysis

In this paper, we will respectively simulate and analysis the PDF of instantaneous SNR $\overline{\gamma_i}$ of about different situation the Multi-node and dual-hop Collaborative Communication channels. We research the M-ary phase-shift keying (M-PSK) modulation, M=4,8; the number of relays m=4,10. We respectively simulate the situation of high SNR and low SNR with 20 sampling nodes.

Figure 2 is the compare of PDF about different sampling ranks and different relay numbers. From this figure,

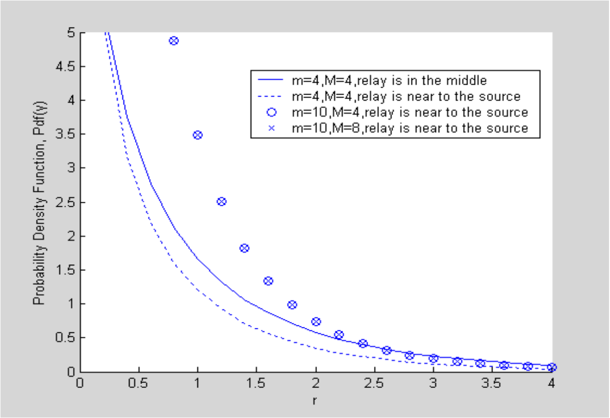

Figure 2. The compare of PDF about different sampling ranks and different relay numbers

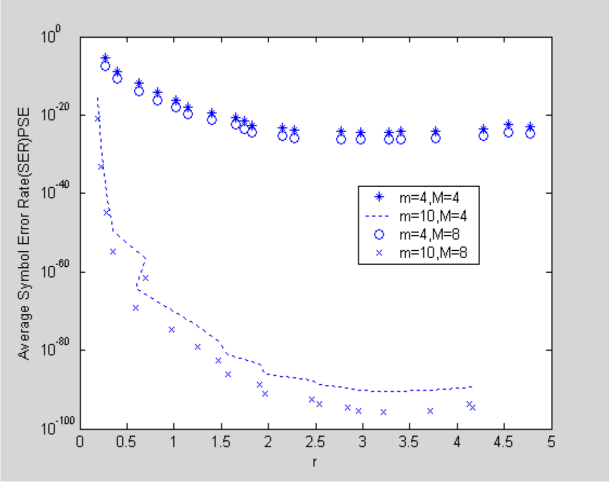

Figure 3. The compare of error symbol rate (SER) about the different sampling ranks and different relay numbers

we can find on the same SNR, the performance is better when the relay number is more. On the same relay nodes, the performance is almost the same when the high sampling ranks and low sampling ranks.

Figure 3 is the compare of error symbol rate (SER) about the different sampling ranks and different relay numbers. We respectively simulate the situation of m=4,10, M=4,8, with 20 sampling nodes. Because of the little SER, we adopt the logarithmic scale coordinate method. We can find on the same SNR, the performance is better when the relay number is more. When the same sampling nodes, the performance is better which is adopt with the high rank modulation. It's also approved that the performance of Multi-node and dual-hop Collaborative Communication can be improved when adopt the high rank modulation while the user's number is definite.

5. Conclusions

This paper applies the method of the harmonic mean to derive PDF and MGF of two independent exponential variates which are cited and derived to study the performance of cellular multi-node and dual-hop cooperative communication systems with non-regenerative relays over flat Rayleigh fading channels.

We derive the probability density function (PDF) and asymptotic symbol error rate (SER) expression. These numerical results indicate that the number of relay is more, the performance is better. It also indicates the ascendance performance of cooperative diversity.

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