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An Extended Six-parameter Generalised Linear Mixed Effects Model for Achievement Gaps among Public and Private Basic Schools of Ghana

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

The mixed generalized linear model extension has distinct advantages over generalised linear models and hierarchical linear models by reducing estimation and precision errors, and increasing power. This paper aimed at building a six parameter Generalised Mixed linear Model with five structural parameters that explain students, teachers, and administrative-logistic characteristics and interaction effects as well as random effect parameter. Our model, which has Bernoulli response, and Logit link function outperformed existing models and was robust across varied sample sizes on the basis of AIC and BIC. Parameters of the model were

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estimated by Maximum Likelihood Estimation Method. The nonparametric contour-3D stratification was used to compare performance of public and private school pupils directly with respect to their BECE performance under each element of a given factor at a time. Our advanced model simultaneously takes into account the variance as a result of the relatively small samples taken from each category of private school as well as any bias that may have been introduced by the different participation rates between the private and public-school categories. Our model identified class competition, concern and parental support for their children's academic output, timely provision of books and learning materials for students, daily quality supervision of the head teacher and teacher by superiors, and conducive teaching and learning environments as the leading causes of performance gaps in the BECE examination in Ghana. It is recommended for the inferences made in this paper to be used by stake-holders of the basic schools to help optimise teaching and learning processes which will in the long run enhance performance of the pupils in the BECE examination.

Keywords: Private school; public-school; performance gaps; generalised mixed effects; BECE; basic education.

1 Introduction

Recent discussions have brought attention to the importance of school sector (whether private or public) and its impact on students' academic success. According to conventional opinion and prior studies, private schools perform better academically [1-4]. Disparities in academic achievements among basic school learners learning under same environmental and learning conditions have been studied and found to differ. Teaching Methods and teachers' effectiveness in using content knowledge, professional values and skills have also been found to control basic school learners' attitude and believes about the subjects they learn at school [1-4].

Many recent works have reported the long-standing issue of private schools outperforming Public-schools in the Basic Education Certificate Examination (BECE) [5-11], in addition to the abundance of research works to understand the causes of learners' performance [12-21].

In order to close the achievement gap between public elementary school students and those in private schools, there is a difficulty with maximizing the learning outcomes of these students. The causes of the differences have been well investigated in the scholarly literature, but the mechanics and dynamics of these influences on BECE performance are still unknown or cannot be quantified appropriately. Stakeholders can better influence particular variables to improve learning outcomes when these dynamics have been well investigated and understood [5-21].

By a wide margin, hierarchical linear and hierarchical generalised linear models have been the main tools used to examine national student performance disparities. Organizations that conduct research on student performance, including well-known authorities like the National Assessment of Educational Progress (NAEP), which is well-known in the USA, and the International Finance Corporation (IFCo), a subsidiary of the World Bank that is also well-known for its educational reports in many countries, have used hierarchical linear and hierarchical generalised linear models to produce their findings. These studies as well as others [22-27] have reported that the widely used hierarchical linear and hierarchical generalised linear models do not have the ability to handle heterogeneous and unequal variance situations, unequal sample size, strong correlation situations, unequal numbers of repeats, or the ability to capture random factors. The mixed generalized linear model extension has distinct advantages over these models and is able to address these issues while maintaining the benefits of the hierarchical linear and hierarchical generalised linear models. It can reduce estimation errors, increase power and precision, and decrease precision errors [28-38].

Since each group has a unique role in shaping how children are educated, [39]'s research identified parents, students, and supervisors (heads, proprietors) as major stakeholders that should be the primary sources of information to optimize their contributions. As a result, the current study seeks to build a six parameter Generalised Linear Mixed Model with five structural parameters that explains students, teachers, and administrative-logistic characteristics and two interaction effects as well as random effect parameter.

2 Methodology

The probit GLMM and the Clog-log models will be compared to our developed model since they all have binary response distribution. Our developed model has Logistc Link specified as follows:

Logistic (logit) link: $\log\left(\frac{\pi_i}{1-\pi_i}\right) = x_i^t\beta$ (1)

symmetric, linear between 0.2 and 0.8, underlying tolerance dbn is logistic dbn, commonly used in medical field; fitted values are the logarithm of the "odds"

The probit GLMM and the Clog-log have the following link functions.

Probit link: $\pi_i = \phi\left(\frac{t_i-\mu}{\sigma}\right) \Rightarrow \phi^{-1}(\pi_i) = x_i^t\beta*$ (2)

symmetric, similar to logit, tolerance *dbn* is Normal (μ, σ^2); Complementary log-log link:

$$\log(-\log(1-\pi_i)) = x_i^t\beta \quad (3)$$

asymmetric, tolerance *dbn* is Gumbel (extreme value) *dbn* .

For this study, a random sample of basic schools from both the public and private sectors in Ghana's Ada East and West District was taken. To compute the required sample size with confidence interval 95% (Z = 1.960), proportion of 50% (for unknown population) and 6.965% margin of error, the required sample of size n, was calculated using Cochran's sample size estimator as;

$$n = \frac{z^2 \times \widehat{p}(1-\widehat{p})}{\varepsilon^2} = \frac{1.96 \times \frac{50}{100} \times \left(1 - \frac{50}{100}\right)}{0.06965^2} \approx 198. \quad (4)$$

Two alternative criteria are instrumental in comparing survival models both in discrete and criterion (AIC) and the Bayesian Information Criterion (BIC). They are designed to handle complex models such as nonlinear models, in spite of the possibility of over fitting. In such cases, there is an alternative to conduct split-analysis by estimating the model with 75% of the data, holding back 25% to check the validity of the model; continuous data even in the event of misspecification. These include the Akaike's information.

The Akaike information criterion (AIC) is given by:

$$AIC = -2\log L + 2K \quad (5)$$

and the Bayesian information criteria (BIC) by:

$$BIC = -2\log L + K\log(N) \quad (6)$$

Where p is the number of model parameters; L is the maximum likelihood.

The Maximum Likelihood Estimation will be the basis for parameter estimation. The Akaike Information Criterion (AIC) was used to compare models for the Bayesian Information Criterion (BIC) study [40,41].

The Generalised Linear model as shown in (a) would be modified to Generalised Linear Mixed Effect Model

$$E\left(Y|\boldsymbol{\beta}\right) = g^{-1}\left(\boldsymbol{\beta}X\right) \quad (7)$$

The exponential family form as explicated in Equation (b) will be used to extract the canonical parameters of the Bernoulli response distribution [42-46].

$$f(y,\theta,\varphi) = \exp\left[\frac{y\theta - b(\theta)}{a(\varphi)} + c(y,\varphi)\right] \quad (8)$$

The canonical parameter is θ_i, the dispersion parameter is φ ; $b(\theta)$ is the cumulant function

For Bernoulli response, Equation (8) equivalence of the exponential family yields Equation (9) as follows;

$$\left\langle \begin{array}{c} a(\varphi)=1 \\ c(y,\varphi)=0 \\ b(\theta)=\log\left(1+\exp\{\theta\}\right) \\ \theta=\log\left(\dfrac{\pi}{1-\pi}\right) \end{array} \right. \tag{9}$$

3 Generalised Linear Mixed Effect Model Formulation

Consider the response Y_{ij} with $i=1,2,...,m$ and $j=1,2,...,m_i$ which are based on the assumption of conditionally independent given the random effects under the random $Z_1, Z_2, ...Z_n$.

Generalized linear models incorporate other types of distributions from the exponential family, and include a link function g(.) relating the mean μ, or stated differently, the estimated fitted values E(y), to the linear predictor Xβ, often denoted η. The general form is thus

$$\left(\begin{array}{c} g(\mu)=\beta X \\ g(\mu)=\eta \end{array} \right. \tag{10}$$

$$E(y)=\mu=g^{-1}(\eta) \tag{11}$$

Consider the Generalised Linear Mixed Model (GLMM) that contains a linear mixed model inside the inverse link function. This model component is referred to as the linear predictor,

$$E(Y|\rho,\varpi)=g^{-1}(\rho X_0+\varpi Z) \tag{12}$$

Where Y represents the (n × 1) vector of observed data and Z is a (r × 1) vector of random effects. X_0 represents observed structural component.

Equation (13) represents the structural component broken into three components that measure teacher factors, Pupils factors and Administrative-Logistic factors

$$E(Y|\varpi)=g^{-1}\left(\rho\left(X_{0T}+X_{0P}+X_{0A}\right)+\varpi Z\right) \tag{13}$$

Equation (14) introduces interaction component due to a possible interaction between Pupil and Teacher factors.

$$E(Y|\varpi)=g^{-1}\left(\rho_{0T}X_{0T}+\rho_{0P}X_{0P}+\rho_{0A}X_{0A}+\rho_{PT}X_{0T}\cdot X_{0P}+\varpi Z\right) \tag{14}$$

Equation (15) is a further extended form of Equation (14) with added interaction between Teacher and Administrative-Logistic factors.

$$E(Y|\varpi)=g^{-1}\left(\rho_{0T}X_{0T}+\rho_{0P}X_{0P}+\rho_{0A}X_{0A}+\rho_{PT}X_{0T}\cdot X_{0P}+\rho_{AT}X_{0T}.X_{0A}+\varpi Z\right) \tag{15}$$

where $g(\cdot)$ is a differentiable monotonic link function and $g^{-1}(\cdot)$ is its inverse.

The matrix X is a (n×p) matrix of rank k, and Z is a (n×r) design matrix for the random.

Due to the underlying stakeholder relationship that tends to complement the overall purpose of education, the two interaction terms were included.

Our response variable, Y, is assumed to follow the Bernoulli distribution for the purposes of this study, and hence the probability mass function (PMF) has the following form:

$$f(y,\theta,\varphi) = \exp\left[\frac{y\theta - b(\theta)}{a(\varphi)} + c(y,\varphi)\right] \tag{16}$$

Let Equation (16) be a PMF on $(0,\infty)$ of a positive random variable ξ, which characterizes a two-parameter exponential family of distributions for positive random variables. The Bernoulli response variable has the form as explicated in Equation (17),

$$f(y) = P(Y=y) = \mu^{y}(1-\mu)^{1-\mu} = \begin{cases} \mu \quad if & y=1 \\ 1-\mu \quad if & y=0 \end{cases} \tag{17}$$

The range of y is $\{0,1\}$.

$$f(y|\mu) = \exp\left(y\log\mu + (1-y)\log(1-\mu)\right) \tag{18}$$

The logarithm of the probability mass function is

$$\log\left(p\left(y|\mu\right)\right) = \log\left(1-\mu\right) + y\log\left(\frac{\mu}{1-\mu}\right), 0<p<1, y=0,1. \tag{19}$$

Thus, the canonical link function (also known as the natural parameter) is the logit link

$$\theta = \eta = g\left(\mu\right) = \log\left(\frac{\mu}{1-\mu}\right) \tag{20}$$

When $\mu = P\left[y=1\right]$, thus the students belong to a private school, the quantity $\dfrac{\mu}{1-\mu}$ is the odds ratio (in the range (0, ∞)) and g is the logarithm of the odds ratio, sometimes called “log odds”. The inverse link is

$$\mu\left(\eta\right) = g^{-1}\left(\eta\right) = \frac{e^{\eta}}{1+e^{\eta}} = \frac{1}{1+e^{-\eta}} \tag{21}$$

For the canonical link function, the derivative of its inverse is the variance of the response.
To that end,

$$\frac{d\mu}{d\eta} = \frac{e^{\eta}}{\left(1+e^{\eta}\right)^{2}} = \frac{1}{1+e^{-\eta}}\cdot\frac{e^{-\eta}}{1+e^{-\eta}} = \mu\left(1-\mu\right) = Var\left(y\right) \tag{22}$$

The log-linear and logistic link functions are used to connect $\boldsymbol{\mu}_i$ and p_i to covariates for the purposes of interpretation. The logit of 'success' is also known as the log-odds.

The binary response model for the clustered and longitudinal data is given in Equation (23).

$$\ln(\mu_i) = \rho_{_{0T}} X_{0T} + \rho_{_{0P}} X_{0P} + \rho_{_{0A}} X_{0A} + \rho_{_{PT}} X_{0T} \cdot X_{0P} + \rho_{_{AT}} X_{0T} . X_{0A} + \varpi Z \tag{23}$$

Therefore, following from Equation (7), our five-parameter modified Generalised mixed effect model assumed to follow Bernoulli Exponential Family with four structural component parameters and one random effect parameter assumed to be conditionally independent is given by Equation (24).

$$\text{logit}(p_i) = \log\left(\frac{p_i}{1-p_i}\right) = \rho_{_{0T}} X_{0T} + \rho_{_{0P}} X_{0P} + \rho_{_{0A}} X_{0A} + \rho_{_{PT}} X_{0T} \cdot X_{0P} + \rho_{_{AT}} X_{0T} . X_{0A} + \varpi Z \tag{24}$$

4 Results and Discussion

4.1 Checking for model assumptions

According to Fig. 1, since the Filliben Correlation coefficient of 0.9468 at 5% significance level is less than 0.9927, we reject the null hypothesis that the dataset came from a population with a normal distribution. Hence, we apply the Generalised Linear Mixed Model for non-normal outcome of this nature. This will transform the nonlinear component with a link function. Alternatively, the percentiles in the sample and the theoretical percentiles do not relate in a linear way. The Filliben Correlation coefficient indicates that the criteria that the error terms be not normally distributed has been met, hence is a necessary to employ generalized linear modeling. The quantile residual plot against the index reveals no significant problems with outliers. There are no solitary residuals that seem to have departed from the typical random distribution of residuals, thus there is no outlier issue.

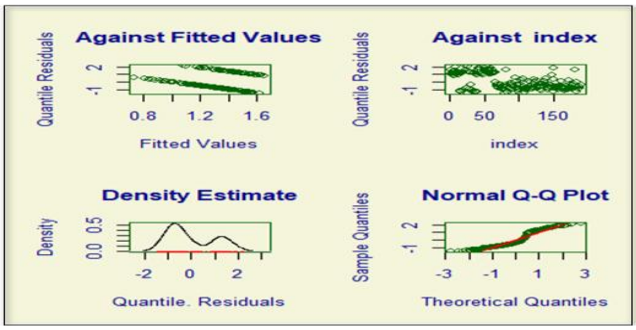

Fig. 1. Graphical test for nonlinearity, outliers and homoscedasticity

4.2 Assessing strength of correlation

We convert the factor level type to numeric in order to produce a heat map with the Spearman method-calculated coefficient of correlation as shown in Fig. 2. It is obvious that there is a serious issue with collinearity in our dataset because the entire heat map appears dark. This is a sign that MIXED models will handle the correlated data and unequal variances better than GLM; the stronger the correlation between any two being compared, the larger the number and darker the hue. In cases when survey respondents or test subjects are repeatedly measured, strongly correlated data such as those in our scenario are frequently observed. GLM's repeated measurements models are extended by MIXED to support an uneven number of repetitions. Additionally, when experimental units are nested in a hierarchy, it can handle more complicated circumstances.

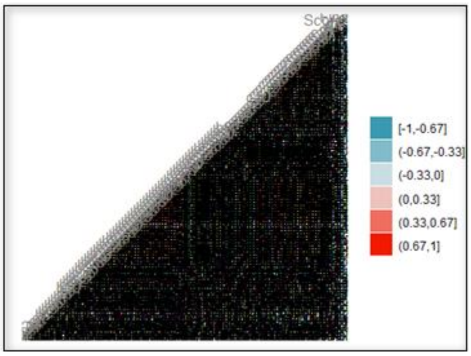

Fig. 2. Graph of heat map correlation check

4.3 Two-way ANOVA test

A significant P-value ($p<0.05$) for Factor1 will mean the Gender of pupils significantly affect performance at private and public-school. A significant P-value ($p<0.05$) for Factor2 will mean the Religion of pupils significantly affect performance at private and public-school.

Table 1. Two-way ANOVA test: dependent variable: type of school

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	11.448[a]	28	0.409	2.245	0.001
Intercept	14.663	1	14.663	80.526	0
PupilGd*TeacherGd	0.931	1	0.931	5.115	0.025
TeacherGd* Subjtot	2.022	2	1.011	5.551	0.005
TeacherGd*Rel.Deno	3.005	3	1.002	5.5	0.001

The normality assumptions of residuals homogeneity of variance (Levene's test) were satisfied for two-way ANOVA.

The results of the two-way ANOVA demonstrate that a number of variables, including interaction between student-gender and instructor-gender, have a significant impact ($p<0.05$) on BECE performance levels in both public and private schools. This indicates that some students think they will perform better if they take certain subject classes from teachers of a particular gender. This is supported by another finding that suggest that there is a significant interaction effect between the teacher's gender and the subject(s) he/she teaches on the BECE performance of students from both private and public-school s ($p<0.05$). Due to the strong connection between teacher gender and school characteristics (such as the sort of religious group that works with government), this have been found to significantly affect BECE performance.

4.4 Model selection criteria

Table 2 shows the model selection criteria and determination of robustness. The output shows that our modified GLMM was robust across sample sizes where it remained the best model with the lowest values of AIC and BIC. The selected model is significantly robust since the AIC difference between the selected model and each of the other two candidate models is generally more than 2 units irrespective of sample size variation.

Table 2. Model selection criteria by AIC, BIC

Sample Variation	Model	BIC	AIC	Delta (AIC)
100%	Modified GMM	**323.7**	**213.6**	0
	Probit GMM	339.4	219.6	6
	Clog-log GMM	344.8	221.8	8.2
	Modified GMM	**258.32**	**248.61**	0

	Probit GMM	263.46	250.51	1.9
70%	Clog-log GMM	260.51	250.8	2.19
	Modified GMM	**267.43**	**248.02**	0
	Probit GMM	268.69	252.5	4.48
50%	Clog-log GMM	271.04	251.63	3.61
	Modified GMM	**269.71**	**250.3**	0
	Probit GMM	270.95	254.76	4.46
30%	Clog-log GMM	274.94	252.29	1.99

The essence of the Delta (AIC) is to compare the difference of tentative models to the Modified model. Automatically, the Modified model's comparison to itself will be zero. But the main interest is the Delta (AIC) for tentative models. A Delta (AIC) of at least 2 signifies statistical significance.

Table 3. Reduced model estimations

Factors	Parameter Estimates (E)	P-value	Standard Error	Exp(E)
Teacher Factors				
Identifies and remediates learners' difficulties or misconceptions	0.87447	0.00902	0.1989	2.4
meaningfully communicates progress clearly to parents and learners	0.9036	0.00221	0.33489	2.5
Enhanced parent-teacher relationship	0.6094	0.01442	0.2491	1.8
Highly supervised to achieve set targets	0.63720	0.042448	0.31403	1.9
Reflects to modify outputs	0.8141	0.00571	0.2946	2.3
Pupil Factors				
Influence of Private Home Tutor	1.09589	0.01798	0.46317	3
Engages in Holiday Classes	0.4303	0.0305	0.1989	1.5
Class competition	1.06365	0.048147	0.53827	2.9
Well organised, disciplined with time management due to school culture that translates into academic life	0.70675	0.044826	0.35227	2
Prep Time Culture	0.6593187	0.03842	0.3184649	1.9
Pupil well managed and supervised at home by parents to focus on academics as a continuity from school	0.6775728	0.04168	0.3326766	2
Adm. Log. Factors				
Daily Quality Supervision of Head teacher and Teacher by superiors	1.42558	0.000875	0.42836	4.2
Timely Provision of Books and learning materials by Parents/Stakeholders for Pupils	1.4006656	0.00136	0.4374075	4.0
Conducive Teaching /Learning environment	1.2121599	0.03617	0.5785873	3.4
Interaction Factors				
Concern and parents' support parents towards their Pupils' Academic Output (PTA)	1.1848393	0.04422	0.5888820	3.3

Random Effects: Variance = 0.933

Table 3 shows the parameter estimates of the reduced model which captures fifteen (15) significant factors (out of 70 factors examined) that explains the disparities in BECE performance among private and public basic school student. The random Effect Variance was 9.017. Table 3's findings indicate that private school teachers are twice as likely to detect and correct their students' misconceptions or learning difficulties (B21 factor), which results in better BECE performance than their public-school counterparts (B =0.87447, p<0.05, se = 0.1989). As a result, private school students are twice as likely to receive positive feedback regarding their misconceptions or learning difficulties.

4.4.1 Factor B21: Identifies and remediates learners' difficulties or misconceptions

Fig. 3 displays a contour graph of the area projected to be affected by this factor in terms of BECE performance, with the red area representing performance in public-school s and the yellow region representing performance in private schools. The 3D figure provides a sharper picture, showing that private candidates perform 34% better than public candidates in the BECE with regard to the "Identifies and remediates learners' difficulties or misconceptions" factor.

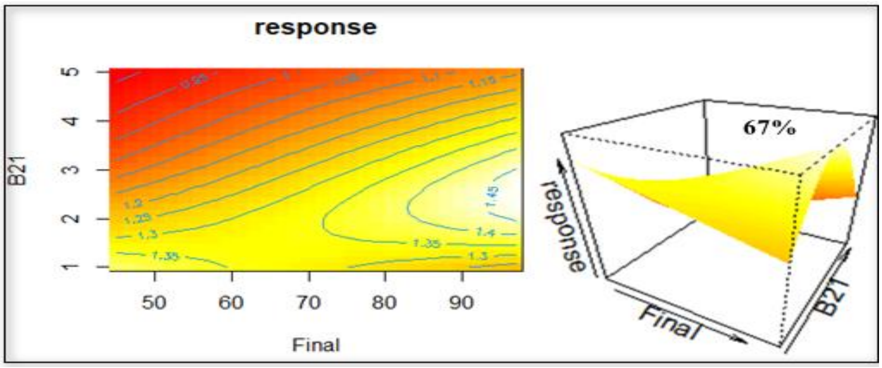

Fig. 3. Graph predicting performance with respect to factor "identifies and remediates learners' difficulties or misconceptions"

4.4.2 Meaningfully communicates progress clearly to parents and learners

Teachers in private schools tend to be three times more effective than those in public-schools at significantly and plainly communicating progress to students and parents according to Table 3 (B = 0.9036, p<0.05, se = 0.33489). Fig. 4 shows a contour graph of the area expected to be impacted by this factor in terms of BECE performance. A clearer picture is given by the 3D figure, which demonstrates that private candidates outperform public candidates in the BECE with regard to the B24 factor by 8 %.

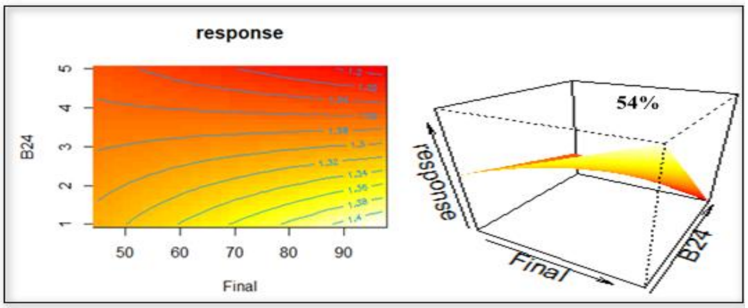

Fig. 4. Graph predicting performance with respect to factor "meaningfully communicates progress clearly to parents and learners"

4.4.3 Enhanced parent-teacher relationship

The positive effects of enhanced teacher-parent relationship (B18) on performance in BECE examination is 2-fold greater in private school than in the public-school (B = 0.6094, p<0.05, se = 0.2491). According to the contour graph in Fig. 5, private school students perform 56% better on the BECE as a result of improved parent-teacher ties, which enable parents to give their children timely interventions.

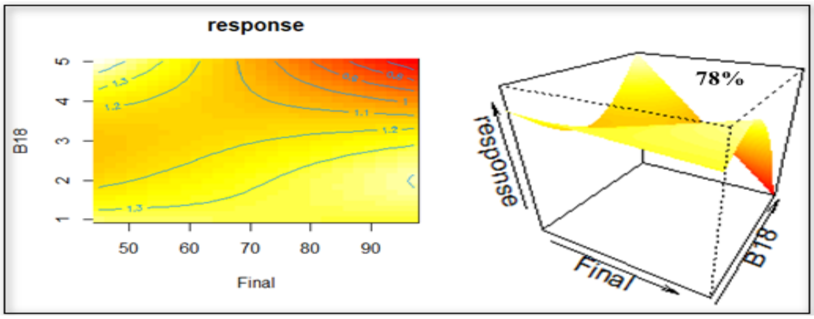

Fig. 5. Graph Predicting Performance with respect to Factor "Enhanced parent-teacher relationship"

4.4.4 Highly supervised to achieve set targets

Due to the frequent monitoring, they receive from their superiors and the eventual instillation of the culture of alertness and vigilance to improve students' output, private teachers are twice as likely to exhibit numerous high-quality composure and constant alertness to achieve short- to long-term academic goals (B = 0.63720, $p<0.05$, se = 0.31403). Fig. 6 demonstrates how the BECE scores of private school students were 18% higher than those of public-school students as a result of private instructors' improved ability to respond to Factor "Highly supervised to achieve set targets".

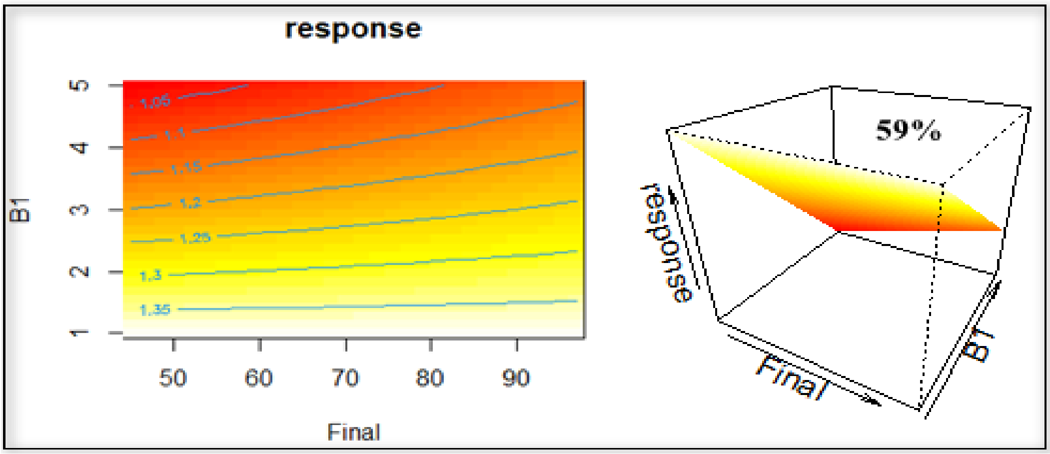

Fig. 6. Graph predicting performance with respect to factor "highly supervised to achieve set targets"

4.4.5 Reflects to modify outputs

Private school teachers are two-fold better able to evaluate their work and see the need to change it to produce effective results (B = 0.63720, $p<0.05$, se = 0.31403). According to Fig. 7's result, Factor B3's influence is responsible for private school pupils performing 10% better on the BECE than their public-school counterparts.

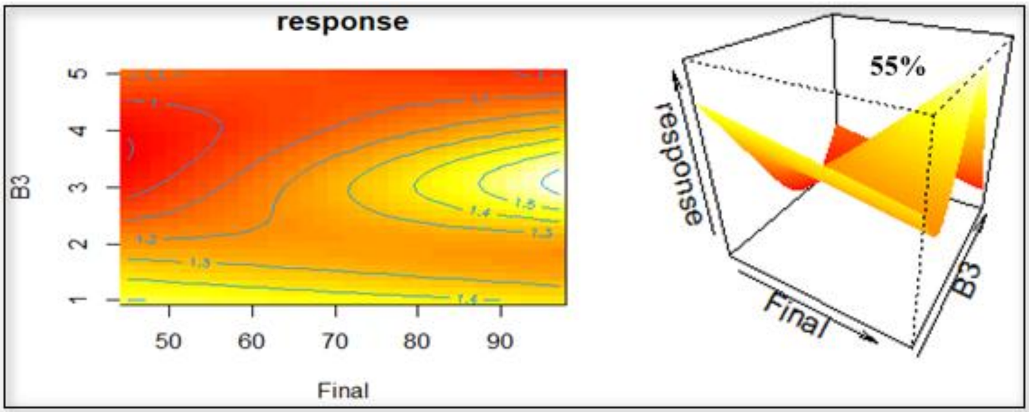

Fig. 7. Graph predicting performance with respect to factor "reflects to modify outputs"

4.4.6 Influence of private home tutor

Private school pupils are three times as good in academics due to the Influence of Private Home Tutor compared to the performance of public-school students who have no influence of a home teacher (B = 1.09589, p<0.05, se = 0.46317). Fig. 8 suggests that Factor "Influence of Private Home Tutor "accounts for a 28% improvement in BECE performance for students in private schools compared to those in public-school s.

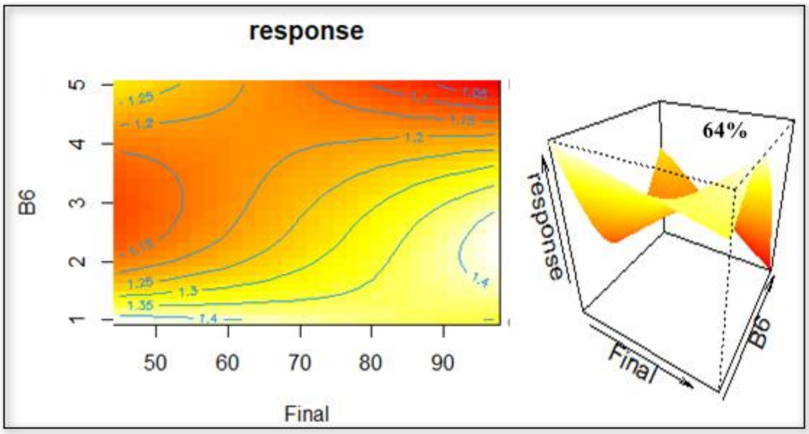

Fig. 8. Graph predicting performance with respect to factor "influence of private home tutor "

4.4.7 Engages in holiday classes

Private students tend to attend Extra or Holiday classes which makes them outperform students in public-schools in terms of academic achievement by a factor of two (B = 0.4303, p<0.05, se = 0.1989). From Fig. 9, it can be concluded that Factor "Engages in Holiday Classes" is responsible for the 12% improvement in BECE scores for students in private schools as compared to those in public-schools.

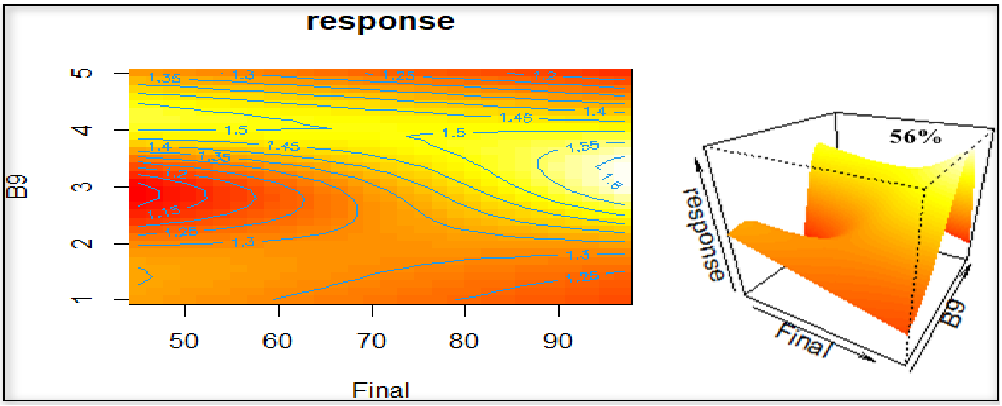

Fig. 9. Graph predicting performance with respect to factor "engages in holiday classes"

4.4.8 Class competition

There is a three-fold higher tendency for students in private schools to outperform those in public-schools in BECE, which can be attributed to the highly competitive atmosphere among students, where they strive for academic brilliance to increase their class performance rank (B = 1.06365, p<0.05, se = 0.53827). Fig. 10 suggests that Factor" Class competition" is responsible for the 60% improvement in BECE scores between pupils in private schools and those in public-school s.

4.4.9 Well organised, disciplined with time management due to school culture that translates into academic life

Private school students typically exhibit greater levels of organization, discipline, and time management, which are traits and qualities developed from proper school culture and translate into their academic life, giving them a two-fold advantage over their public-school counterparts in BECE success (B = 0.70675, p<0.05, se = 0.35227). Fig. 11 suggests that Factor "Well organised, disciplined with time management due to school culture that translates into academic life" is responsible for the 50% improvement in BECE performance in private school students as compared to public-school students.

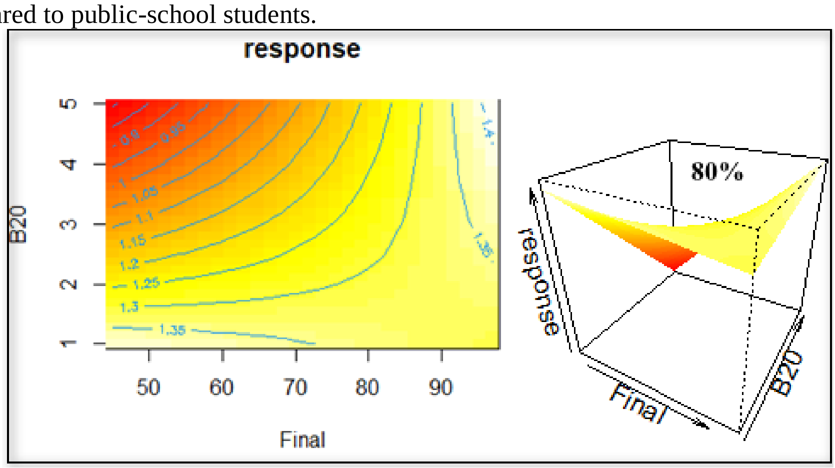

Fig. 10. Graph predicting performance with respect to factor" class competition"

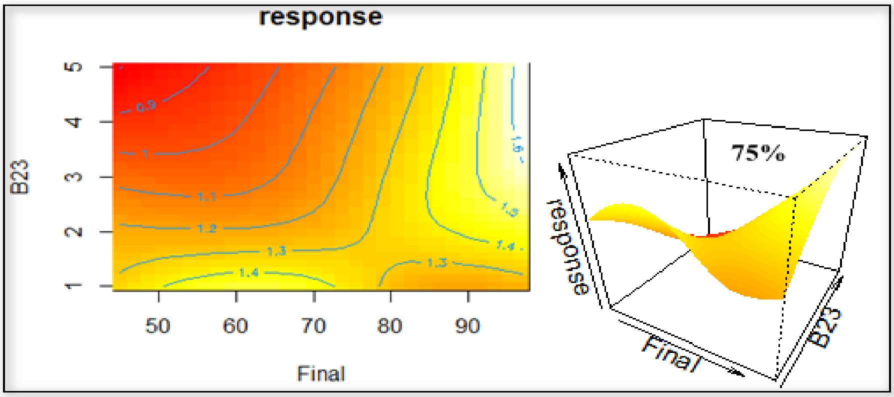

Fig. 11. Graph predicting performance with respect to factor "well organised, disciplined with time management due to school culture that translates into academic life"

4.4.10 Prep time culture

Students in private schools have a stronger propensity to adhere to the Prep Time Culture, which translates into a 2-fold greater propensity to pass the BECE than candidates from public-school s ($B = 0.6593187$, $p<0.05$, $se = 0.3184649$). Fig. 12 suggests that Factor "Prep Time Culture" is responsible for the 32% improvement in BECE performance among students in private schools compared to those in public-school s.

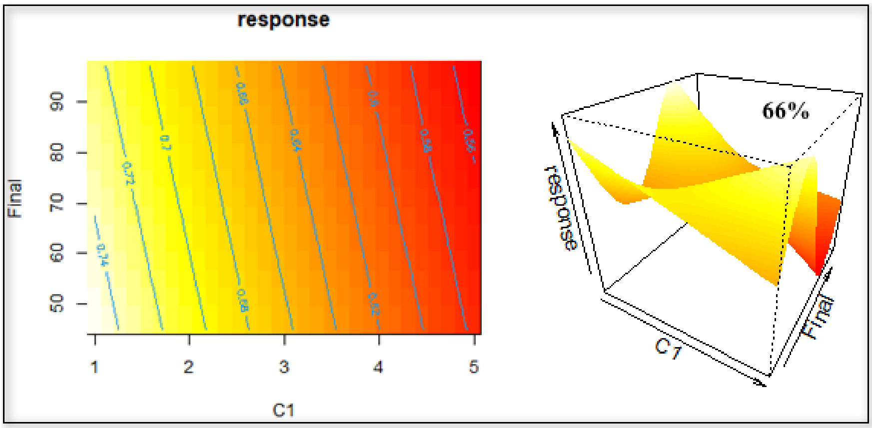

Fig. 12. Graph predicting performance with respect to factor "prep time culture"

4.4.11 Pupil well managed and supervised at home by parents to focus on academics as a continuity from school

Since private school students are typically well-managed and closely watched at home by their parents, they are more likely than pupils from public-school s to succeed in the BECE exam by a factor of two ($B = 0.6775728$, $p<0.05$, $se = 0.3326766$). Fig. 13 suggests that Factor "Pupil well managed and supervised at home by parents to focus on academics as a continuity from school "is responsible for the 50% improvement in BECE scores between pupils in private schools and those in public-schools.

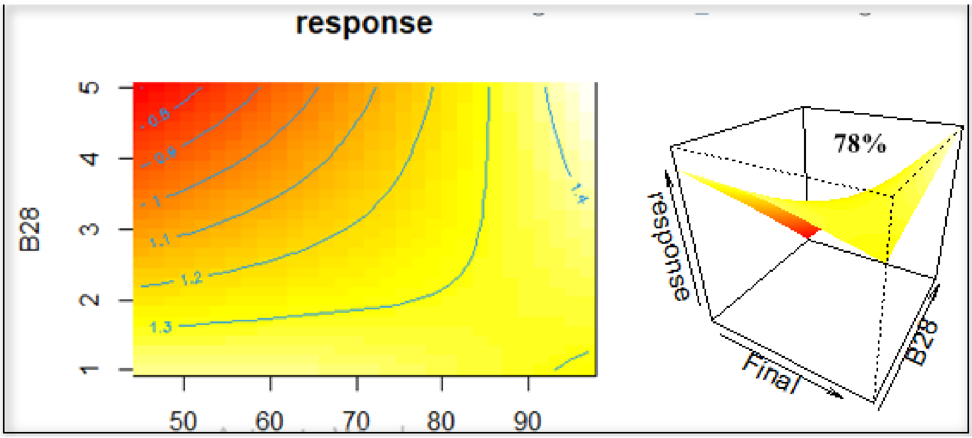

Fig. 13. Graph predicting performance with respect to factor "pupil well managed and supervised at home by parents to focus on academics as a continuity from school "

4.4.12 Daily quality supervision of head teacher and teacher by superiors

Daily high-quality teacher monitoring by school owners and teacher supervision by school heads results in excellent teaching and learning, which contributes for a 4-times greater likelihood for private school students to succeed in BECE than for students in public-school s (B = 1.42558, $p<0.05$, se = 0.42836). Fig. 14 suggests that Factor "Daily Quality Supervision of Head teacher and Teacher by superiors" is responsible for the 74% improvement in BECE performance among students in private schools compared to those in public-schools.

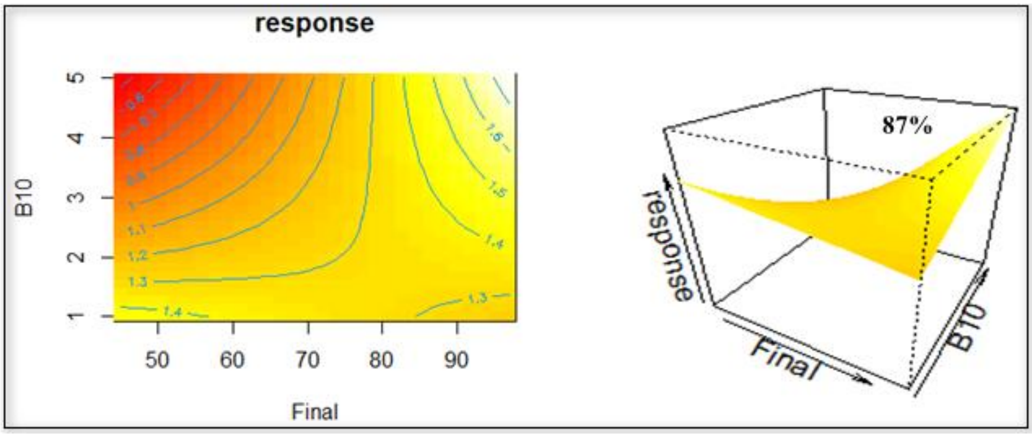

Fig. 14. Graph predicting performance with respect to factor "daily quality supervision of head teacher and teacher by superiors"

4.4.13 Timely provision of books and learning materials by parents/stakeholders for pupils

Private school students are four times more likely than public-school students to succeed in the BECE as their parents tend to provide the required books and materials on time as instructed by the school (B = 1.4006656, $p<0.05$, se = 0.4374075). Fig. 15 suggests that Factor "Timely Provision of Books and learning materials by Parents/Stakeholders for Pupils" is responsible for the 58% increase in BECE performance among students in private schools as compared to those in public-schools.

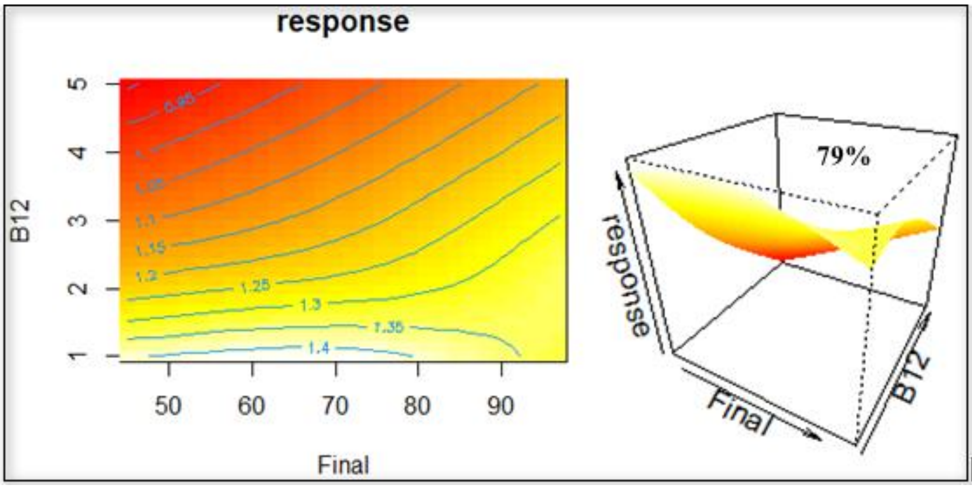

Fig. 15. Graph predicting performance with respect to factor "timely provision of books and learning materials by parents/stakeholders for pupils"

4.4.14 Conducive teaching /learning environment

Due to the teaching and learning environment, BECE results are typically better for students from private schools. In the BECE, pupils from private schools have a three times greater tendency to score better than those from public-school s as a result of this factor (B = 1.2121599, p<0.05, se = 1.2121599). Fig. 16 suggests that Factor "Conducive Teaching /Learning environment "is responsible for the 64% improvement in BECE performance among students in private schools compared to those in public-school s.

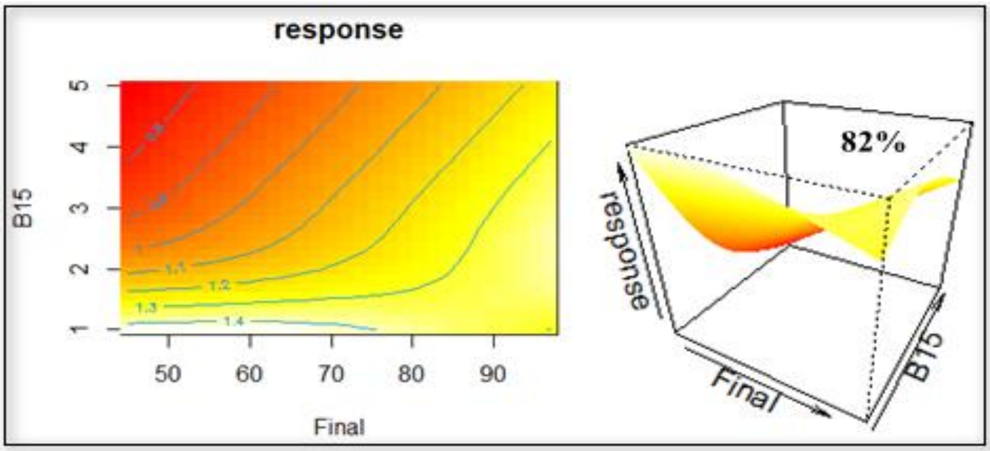

Fig. 16. Graph predicting performance with respect to factor "conducive teaching /learning environment "

4.4.15 Concern and parents' support parents towards their Pupils' Academic Output (PTA)

The performance of students attending private schools tends to be three times better than that of students attending public-school s in the BECE due to the high level of parental participation and desire to support the aims of the private schools attended by their wards (B = 1.1848393, p<0.05, se = 0.5888820). Fig. 17 indicates that Factor "Concern and parents' support parents towards their Pupils' Academic Output (PTA)" is responsible for the 50% improvement in BECE performance among students in private schools as compared to those in public-school s.

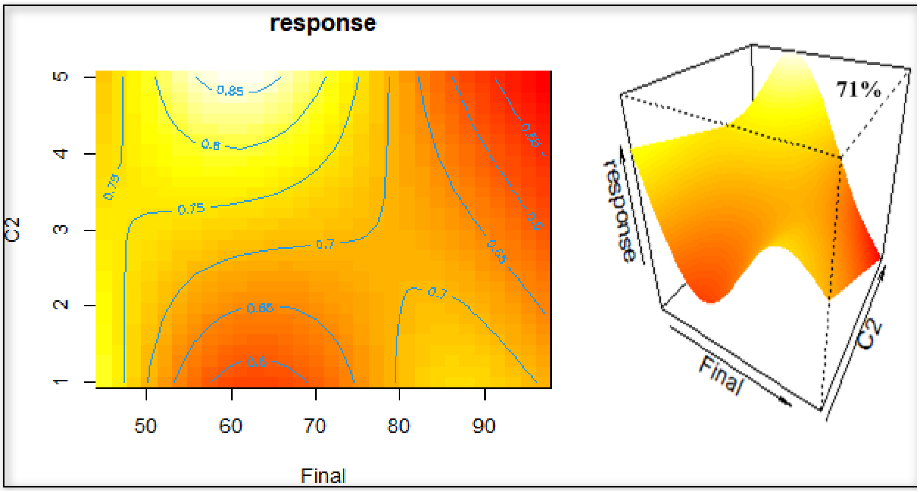

Fig. 17. Graph predicting performance with respect to factor "concern and parents' support parents towards their pupils' academic output (PTA)"

5 Conclusion

The study sought to build a six-parameter Generalised Mixed linear Model with five structural parameters that explain students, teachers, and administrative-logistic characteristics and two interaction effects as well as random effect parameter. The study identified fifteen (15) significant factors out of 70 factors examined to be responsible for disparities in BECE performance among private and public basic school student. The probabilities associated with the dynamics of each of the significant factors have been estimated by the parameters of the model. Our model has been found to be efficient across varied sample sizes as opposed to existing models. The two-way ANOVA was used to assess.

Mean difference of significant factors including interaction effects. The top five factors that influence BECE performance gaps are, in order, Daily Quality Supervision of Head Teacher and Teacher by Superiors, Timely Provision of Books and Learning Materials by Parents/Stakeholders for Pupils, Conducive Teaching/Learning Environment, Concern and Parental Support for Pupils' Academic Output, and Class Competition. Researchers [47–58] have agreed that basic school students making academic progress under the same circumstances encountered differences. Our modelling approach deals with heterogeneous and unequal variance situations, unequal sample size, strong correlation situations, unequal numbers of repeats, and random factors, in contrast to earlier works by renowned authorities like the National Assessment of Educational Progress (NAEP) and the International Finance Corporation (IFCo), which used hierarchical linear models to produce their findings. Again, our analysis fills a gap left by [59], who urged for additional research to explore the precise variability related to interaction components and unobserved random effects, such as a generalized linear mixed effect model.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Dangaraa US, Chinyere GM. Quality assurance in Nigeria's education system: prospect and challenges, EduLine. J Educ Learn Innov ISSN. 2019;1(1) (2021):2775-6173 (online).
DOI: 10.35877/454RI.eduline422

[2] Hussain N, Rigoni U, Orij RP. Corporate governance and sustainability performance: Analysis of triple bottom line performance. J Bus Ethics. 2018;149(2):411-32.
DOI: 10.1007/s10551-016-3099-5

[3] Adamu N. Impact of learning environment on the performance of students in social studies in junior secondary schools in Taraba State, Nigeria [masters degree thesis]. Zaria, Nigeria: School of Postgraduate Studies, Ahmadu Bello University; 2015.

[4] Adediwura A, Tayo B. Perception of teachers' knowledge attitude and teaching skills as predictor of academic performance in Nigerian secondary schools. Educ Res Rev. 2007;2:165-71.

[5] Mawusi Nugba R, Quansah F, Ankomah F, Ewoenam Tsey E, Rosemary Ankoma-Sey V. A trend analysis of junior high school pupils' performance in the Basic Education Certificate Examination (BECE) in Ghana. IJEEDU. 2021;10(3):79.
DOI: 10.11648/j.ijeedu.20211003.15

[6] Bonsu DH. A comparative analysis of academic performance of public and private junior high schools in the basic education certificate in Sekondi/Takoradi, European journal of basic and applied sciences, European. J Basic Appl Sci ISSN 2059-305. 20163(1).

[7] Tooley J, Dixon P, Amuah I. Private and public-school ing in Ghana: A census and comparative survey. Int Rev Educ. 2007;53(4):389-415.
DOI: 10.1007/s11159-007-9042-3

[8] Tooley J, Dixon P, Gomathi SV. Private schools and the millennium development goal of universal primary education: A census and comparative survey in Hyderabad, India. Oxf Rev Educ. 2007;33(5):539-60.
DOI: 10.1080/03054980701425664

[9] Tooley J. Is private education good for the Poor? [working paper]. Centre for Market Solutions in Education. Newcastle: University of Newcastle; 2005.

[10] Tooley J, Dixon P, Olaniyan O. Private and public-school ing in low-income areas of Lagos State, Nigeria: A Census and Comparative Survey. Int Rev Educ. 2005;43(3):125-46.

[11] Tooley J, Dixon P. ”De facto” Privatisation of Education and the Poor: Implications of a Study from Sub- Saharan Africa and India. Compare. 2006;36(4):443-62.
DOI: 10.1080/03057920601024891

[12] Senyefia B. Effect of physical and virtual manipulative on the mathematical achievement of junior high school students in the topic of transformation in Ghana. Asian Res J Math. 2017;4(4):1-14:Article no. ARJOM.34238, ISSN: 2456-477X.
DOI: 10.9734/ARJOM/2017/34238

[13] Bosson-Amedenu S. Archbishop Porter Girls' senior high school students' perception of difficult concepts in the senior high school further mathematics curriculum in Ghana. Asian Res J Math. 2017;4(3):1-14:Article no. ARJOM.32331, ISSN: 2456-477X.
DOI: 10.9734/ARJOM/2017/32331

[14] Bosson-Amedenu S. Pre-SHS students' perception of difficult concepts in junior high school mathematics curriculum in Ghana. Asian Res J Math. 2017;3(2):1-11:Article no.ARJOM.32329 ISSN: 2456-477X.
DOI: 10.9734/ARJOM/2017/32329

[15] Bosson-Amedenu S. Remedial students' perception of difficult concepts in senior high school core mathematics curriculum in Ghana. Asian J Math. 2017;3(3):1-13:Article no. ARJOM.32330, ISSN: 2456-477X.
DOI: 10.9734/ARJOM/2017/32330

[16] Bosson-Amedenu S. Effect of use WAEC syllabus on the mathematical achievement of WASSCE candidates in Ghana. ARJASS. 2018;6(4):1-8:Article no. ARJASS.34695, ISSN: 2456-4761.
DOI: 10.9734/ARJASS/2018/34695

[17] Bosson-Amedenu S. Predictive validity of mathematics mock examination results of senior and junior high school students' performance in WASSCE and BECE in Ghana. Asian Res J Math. 2017;3(4):1-8:Article no. ARJOM ISSN: 2456-477X.
DOI: 10.9734/ARJOM/2017/32328

[18] Bosson-Amedenu S, Osei-Asibey E, Wiah EN. Evaluation of assessment as learning teaching strategy among basic school teachers in Ghana. J Sci Res Rep. 2020;26(3):109-18:Article no.JSRR.56382, ISSN: 2320-0227.

[19] Bosson-Amedenu S, Osei-Asibey E, Wiah EN. Use of assessment of learning teaching strategy among basic school teachers in Ghana. Asian J Educ Soc Stud. 2020;7(4):1-11:Article no. AJESS.56005 ISSN: 2581-6268.

[20] Bosson-Amedenu S, Osei-Asibey E, Otoo H. Factor analysis model for diagnostic assessment and instructional approaches in Ghana's new curriculum: the case of differentiation, scaffolding, archives of current research. International. 2020;20(4):1-16:Article no. ACRI.57232, ISSN: 2454-7077.

[21] Bosson-Amedenu S, Osei-Asibey E, Acquah J. Multivariate study of the use of assessment strategies and Cross Cutting issues by basic school teachers: the case of Ghana's new curriculum, advances in research. Vol. 21(5). ID: NLM: 101666096; 2020; Article no. AIR.56593 ISSN: 2348-0394, 14-27.

[22] Riley RW. What teally tnatters in American education. United States Department of Education [online]; 1997. Available from: http://ed.gov/Speeches/09-1997/index.html.

[23] Akaguri L. Quality Low-Fee Private Schools for the Rural Poor: perception or reality? Evidence from Southern Ghana. CREATE (Consortium for Research on Educational Access, Transitions and Equity) Pathways to Access. Res Monogr. University of Sussex: CREATE. 2011;69.

[24] McCulloch AW, Marshall PL. K-2 teachers' attempts to connect out-of-school experiences to in-school mathematics learning. J Urban Math Educ. 2011;4(2):44-66.

[25] Bryk AS, Lee VE, Holland PB. Catholic schools and the common good. Cambridge, MA: Harvard University Press; 1993.

[26] Raudenbush SW, Brennan RT, Barnett RC. A multivariate hierarchical model for studying psychological change within married couples. J Fam Psychol. 1995;9(2):161-74.
DOI: 10.1037/0893-3200.9.2.161

[27] Graham JW, Donaldson SI, Piccinin AM, Hansen WB. Resistance-skills training and onset of alcohol use: evidence for beneficial and potentially harmful effects in public-school s and in private Catholic schools; 1997.

[28] Hastie TJ, Tibshirani RJ. Generalized additive models. Statist Sci of Monographs on Statistics and Applied Probability. 1990;1(3).
DOI: 10.1214/ss/1177013604.

[29] Bryk AS, Lee VE, Holland PB. Catholic schools and the common good. Cambridge, MA: Harvard University Press; 1993.

[30] US Department of Education, National Center for Education Statistics. Washington, D.C. Condition Educ. 1997 (NCES 97-388);1997, 150, based on NCES, Digest of Education Statistics:1996 (based on Common Core of Data).

[31] US Department of Education, National Center for Education Statistics. How different, how similar? Comparing key organizational qualities of American public and private secondary schools, by D. Keil (NCES 96-322), and C.T. 1996;35.

[32] Hardin J, Hilbe J. Generalized linear models and extensions. College Station, TX: Stata Press; 2001.

[33] McCullagh P, Nelder JA. Generalized linear models. 2nd ed. London: Chapman & Hall; 1989.

[34] Lubienski C. Instrumentalist perspectives on the 'public' in public education: incentives and purposes. Educ Policy. 2003c;17(4):478-502.
DOI: 10.1177/0895904803254964

[35] Lubienski C. Public-school s in marketized environments: shifting incentives and unintended consequences of competition-based educational reforms. Am J Educ. 2005;111(4):464-86.
DOI: 10.1086/431180

[36] Lubienski C. Incentives for school diversification: competition and promotional patterns in local education markets. J Sch Choice. 2006;1(2):1-31.
DOI: 10.1300/J467v01n02_01

[37] Lubienski ST. Celebrating diversity and denying disparities: A critical assessment. Educ Res. 2003;32(8):30-8.
DOI: 10.3102/0013189X032008030

[38] Lubienski ST, Lubienski C. A new look at public and private schools: student background and mathematics achievement. Phi Delta Kappan. 2005;86(9):696-9.
DOI: 10.1177/003172170508600914

[39] Mishra S, Giordano D, Maiorana F, Csizmadia AP, Marsden S, Riedesel C, et al. New horizons in the assessment of computer science at school and beyond: leveraging on the viva platform. Proceedings of the. I TiCSE Working Group Rep. 2015;2015:117-47.

[40] Senyefia B, Joseph A, Christiana Cynthia N, Ouerfelli N. A comparison between accelerated failure time models in analyzing the survival of breast cancer patients. J Cancer Tumor Int. 2022;12(1):16-28.
DOI: 10.9734/jcti/2022/v12i130166

[41] Bosson-Amedenu S, Acquah J, Nyarko CC, Ouerfelli N. A novel three parameter modified cox frailty model for noncompeting risk patients with breast cancer Malignality, mathematical theory and modeling. 2022;12(1):1-15.

[42] Liang KY, Zeger SL. Longitudinal data analysis using generalized linear models. Biometrika. 1986;73(1):13-22.
DOI: 10.1093/biomet/73.1.13

[43] Lindsey JK. Applying generalized linear models. New York: Springer; 1997.

[44] McCullagh P, Nelder JA. Generalized linear models of Monographs on Statistics and Applied Probability. 2nd ed. London: Chapman & Hall. 1989;37.

[45] Müller M. Estimation and testing in generalized partial linear models—a comparative study. Stat Comput. 2001;11(4):299-309.
DOI: 10.1023/A:1011981314532

[46] Nelder JA, Wedderburn RWM. Generalized linear models. J R Stat Soc A. 1972;135(3):370-84.
DOI: 10.2307/2344614

[47] Dangara UY, Madudili CG. Eval eff learn treat stud's Acad perform Niger; 2019. Eunice et al. AJPAS. 2022;20(3):132-49:Article no.AJPAS.93512147.

[48] Hussain RM, Rizvi A. A Effect of classroom learning treatment on students' academic achievement in mathematics at secondary level. Bull Educ Res. 2018;40(2):207-18.

[49] Math B-ASJ. Predictive validity of mathematics mock examination results of senior and junior high school students' performance in WASSCE and BECE in Ghana. Asian research. 2017a;3(4):1-8:32328 ISSN: 2456-477X:Article no. ARJOM.

[50] Adamu N. Impact of learning treatment on the performance of students in social studies in junior secondary schools in Taraba State, Nigeria [masters degree thesis]. Zaria, Nigeria: School of Postgraduate Studies, Ahmadu Bello University; 2015.

[51] Adediwura AA, Tayo B. Educ res Rev.. Perception of teacher's knowledge, attitude and teaching skills as predictor of academic performance in Nigerian Secondary Schools. 2007;2(7):165-71.

[52] Bosson-Amedenu S, Hinson J, Nimoh V. Basic school grade retention in Ghana withApplications of cox proportional hazard model and Mann–Whitney's U TestApplied. Mathematics. 2019;9(2):59-65DOI. DOI: 10.5923/j.am.20190902.03

[53] Senyefia B, Emmanuel T, Nimoh V, Eunice O. A study of cooperative strategies that enhance individual mathematical achievement among college of education students in Ghana. Teach Educ Curriculum Stud. 2021;6(1):28-32.
DOI: 10.11648/j.tecs.20210601.16

[54] Bosson-Amedenu S, Danku DK, Frank O. Comparative analysis of school life expectancy in two Randomly Selected Basic Schools in Ghana: using life table functions and survival analysis. Sci J Appl Math Stat. 2019;7(2):8-14.
DOI: 10.11648/j.sjams.20190702.11

[55] Mupa P, Tendeukai IC; 2015. Factors contributing to ineffective teaching and learning in primary schools: why are schools in decadence? J Educ Pract. Available from: http://www.iiste.org ISSN 2222-1735 [paper] ISSN 2222-288X [online]. Vol. 6(19).

[56] Fernandez C. Probl Educ 21st century. Training. Knowledge base for teaching and pedagogical content knowledge: Some useful Models and Implications for Teachers'. 2014;60:79-100.

[57] Delvin M, Kift S, Nelson K. Effective teaching and support of students from low socioeconomic status backgrounds: practical advice for teaching staff. Resources for Australian higher education. Australian government office for learning and teaching. 2012;11.

[58] Sawchuck S. EWA research brief studies say about teacher effectiveness. London: SAGE Publications; 2011.

[59] Osei-Asibey E, Wiah NE, Nortey ENN. Neutrosophic-principal component analysis of causes of performance gap among private and public school students in the basic education certificate examination. Asian J Probab Stat. 2022;20(3):132-49. Article no.AJPAS.93512 ISSN: 2582-0230.

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