A Fuzzy-Based Control System for Customers' Admission in a Two-Station Queue Network

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Authors' contributions

This work was carried out in collaboration among all authors. Authors ATA and ADG performed the simulations in the study. All authors read and approved the final manuscript.

Original Research Article

Abstract

The study proposes a fuzzy-based control of admission of customers in a queue network with two stations in tandem. Each of the stations has individual arrival streams which may either be accepted or rejected. Class i arrivals occur in a Poisson stream with constant rate λ_i, $i = 1, 2$. Successive services in each station j are independent and exponentially distributed, with mean $1/\mu_j$ in station j, $j =1, 2$, irrespective of the customer's class. The objective of the study is to decide an optimal admission policy based on the state of the queue such that profit is maximized. The state of the system is described by (z_1, z_2), where z_i is the number of customers in station i, and $i = 1, 2$. The tool adopted is a fuzzy process which determines this policy using the fuzzy input values, s and λ giving a corresponding decision, *dec.* which is either a '1' or '0' representing 'Admit' or 'Reject' respectively. The membership functions of arrivals were defined and implemented using fuzzy rules

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to derive a fuzzy output of decision which either 'Admit' or 'Reject' an arrival. Numerical results show a considerable improvement in the control of customers' admission and it was concluded that the proposed method is efficient in the control of customers' admission in queue network.

1 Introduction

Queuing theory is the mathematical study of waiting lines. Queue lengths and waiting times can be depicted via the use of a queuing model. Queuing theory plays an important role in our daily life. It is not possible to exactly determine the arrival and departure of customers when the number and types of facilities as well as the essential of the customers are not known. Queuing theory techniques, in particular, can help us to determine suitable number and type of service facilities to be provided to different types of customers [1].

There are many applications of queuing theory. This include traffic flow, programming patients in hospital, facility design in bank and other institutions, programming of service facilities in a repair and maintenance in workshop, programming of limited transport fleet to a large number of users, programming of reconstruction of used engines and assemblies of aircrafts, missile system, transport fleet, among others [2].

It is desirable to have customers' serviced within the shortest possible time in any service system. However, it is almost possible to meet up with all customers' service requirements at all times as these preferences for services changes with time. This is inevitable in a world of technology where service needs and customers' characteristics are dynamic in nature. In other to ensure that customers are serviced within the shortest possible time, it is imperative to have a system which can accommodate and service a sizeable number of customers.

The need for service providers to cope with dynamic and varying customer needs with limited resources has become an issue of great concerns in recent times. This is because service providers have to find effective control mechanisms to manage their revenues as well as customer satisfaction in order to make the best use of their service capacities. In literature, these service systems are modeled as multi-class queuing systems with admission controls.

The optimality of trunk reservation policy for a multi-class loss queuing system in which the rewards that customer classes pay for being on their classes is significant as acceptance decisions on individual customer classes have threshold structures, with respect to the number of customers in the system [3]. Consequently, if customer class i is accepted when there are n customers in the system, then class i should also be accepted when the system is less crowded. The admission control studies which consider class-dependent service rates have focused on providing heuristic policies [4]. The most common approaches of these studies include linear programming techniques [5] and asymptotic analyses [6].

The waiting times of customers awaiting service in any system is one of the important service quality indicators, which has significant consequential effects on revenues or customer satisfaction levels. Consequently. waiting times can affect customer choice in selecting service providers. In essence, for a specific service provider, waiting times can be a determinant of the demand intensity of their system [6].

The effects of system congestion on customer behaviour in a queue system had been studied by [7] in which the objective of the study was to minimize a weighted difference between the average expected waiting time of those that enter, and the acceptance rate of customers. Similar studies to this include [8] and [9] which discussed the potentials of systems to obtain admission rewards when customers behave greedily based on congestion levels. These studies fail to consider heterogeneity among arrivals. Similarly, [10] studied congestion-related costs through the abandonment of customers in a single-class multi-server model for controlling the admission decisions of arriving customers.

Admission control in a single-server model with retrials where holding costs are used as means to incorporate congestion sensitivity of customers was considered by [11]. Similarly [12] investigated the callback option to mitigate congestion in call centres. The study was modeled such that arriving customers were routed to an

offline queue to be called back later when they accept the callback offer else customers were routed to the online queue in which they incur congestion-related waiting time costs [13] considers a Make-to-Stock queuing model with impatience when unsatisfied demands are backlogged. The control consists of both an admission decision to the system and an admission in service decision. There are no ordering costs while threshold policies were optimal using the propagation of structural properties.

The congestion effect through class-dependent holding costs in the admission control problem of a multi-class queue model was considered by [14]. The model used a continuous-time Markov decision process formulation and relative bias functions in their policy iteration algorithm for obtaining the optimal policy of an *M/M/c/N* queue with class and congestion-dependent admission rewards. [15] considers a parametric admission in a retrial queue with impatience on retrials and introduces a Smoothed Rate Truncation method in order to work with models with bounded transition rates.

Stochastic controlled queuing models, have been largely studied in the literature as a result of its wide applications in networking, resources allocation, inventory control, etc. [16]. This approach had been used in solving a variety of optimization problems including admission control systems [17]; optimal scheduling [18] or optimal routing between queues [19] so as to minimize deadline misses; scheduling in order to minimize long run costs [20]; inventory control in Make-to-Order systems [21]; optimal control of the service rates [22]; admission in service involving slotted models [23]. The concept of optimality is an important concept in customer admission and management in any queue system. Consequently, there is a direct relationship between the admission control system as well as queue size and server vacations. This is corroborated in [24], [25] and [26]. These studies established the fact that a significant relationship exist in the customers' admission policies as well as queue size and servers' vacations.

The problem of dynamic control of admission of customers had been solely based on network metrics and this does not adequately meet the need and flexibility of a dynamic queue system. This is because the nature and service requirements of customers in contemporary business environments are changing on daily basis and this requires a large degree of flexibility on the admission control system. This is the reason the study is proposing a fuzzy-based admission control system which can adequately cater for these challenges.

Fuzzy set theory is a paradigm shift which helps to resolve classical and non-classical problems in a more convenient way than crisp systems by softening set boundaries. Fuzzy queues were first proposed by R. J. Lie and E. S. Lee in 1989 [27]. A Poisson arrival queueing system is a fairly reasonable appropriation where the arrival and service rates are really more realistic than probabilistic. However, in many practical situations the parameters of arrival rate (λ) and service rate (μ) are frequently fuzzy and cannot be expressed in exact terms [28].

Optimality in queue control is an aspect of mathematical modeling which have been found to have multi-dimensional applications [29]. Of focus, the present-day advancement in business requirements and expansion in the expectation of customers from essential service providers have made it necessary to have a system which regulates admission into a queue system with the sole aim of rendering efficient and timeless services.

The study is aimed at designing a system that dynamically manages customers' admission in a queue network. This is necessary to ensure that the buffer is not over-congested with customers awaiting service turns which could result in queue instability, customers' dropping and deficient customer management system. Particularly, the system adopted a two-station queue network structure with independent queue capacities.

2 Materials and Methods

2.1 Problem definition

We consider a continuous-time controlled queuing model in which customers arrive according to a Poisson process with a constant intensity. The model has two queues in tandem, each having independent input of arriving customers that may either be admitted or rejected. The service admission decision is made by a controller. Once admitted in service by the controller, the service begins instantly and it is not interrupted. The model is shown in Fig. 1.
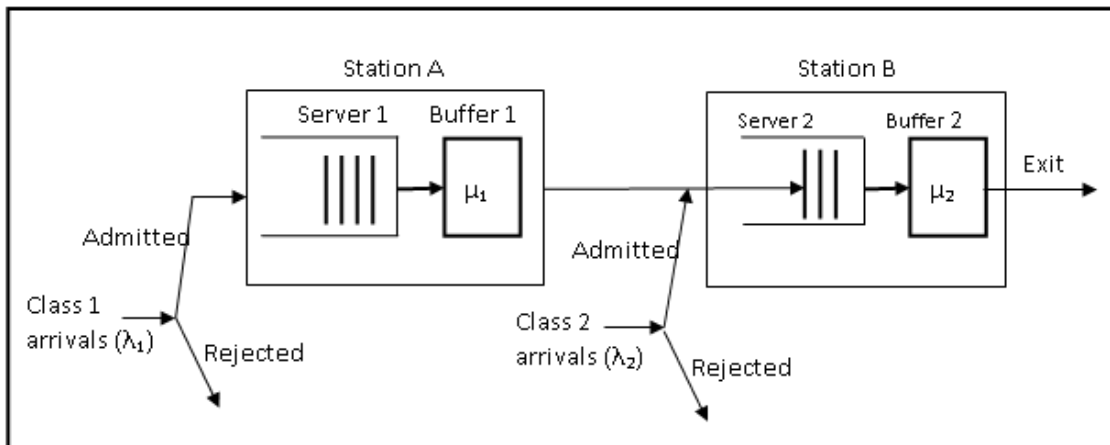
Fig. 1. Proposed customers' admission system in a two-station queue network

The capacity of the server in each station is exponential while the buffer has an unlimited queueing capacity. Class1 customers seek admission to station A and later to station B after being served in station A while Class 2 customers seek admission to station B only. However, what is common to both classes of customers is that they both exit the system after being served in station B. Class i arrivals occur in a Poisson stream with constant rate λ_i where $i = 1, 2$. Services in each station j are independent and exponentially distributed, with mean $1/\mu_j$ in station j, where $j = 1, 2$, irrespective of the customer's class.

If the system has a fixed reward r_i for each admitted customer of class i and pays a holding cost h_j for each customer per time unit in station j. In this case, it is possible to decide the optimal admission policy, based on the state of the system in order to optimize profit. In this case, the state transitions depend on the current state while the times between two successive admissions are no longer exponential random variables because of the possibility of non-admittance of some arrivals.

2.2 Modeling customers' admission and rejection as a dynamic fuzzy control system

Modeling a continuous-time dynamic queue system in the conventional approach is to describe the physical system by, for example, a non-linear ordinary differential equation of the form as given in (1):

$$\begin{cases} \dot{x}(t) = f(x(t), \ u(t)), & t_0 \leq t < \infty \\ x(t_0) = x_0 \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state-vector, $u(t) \in R^m$ $(m \leq n)$ is the control vector, $f: R^n \times R^m \to R^n$ is a non-linear integer function while x_0 is the initial state of the system. This mathematical model is well defined on $[t_0, \infty)$ because for each initial state x_0 and each control input $u(t)$, there exists a unique solution $x(t)$. The mathematical model expressed in (1) represents a dynamic process via its explicit or implicit solution function: $x(t) = x(t; u(t), x0)$. At any instant $t = t^* \in [t_0, \infty)$, the system is described by the relation that if the state vector x is equal to $x(t^*)$ and the control vector u is equal to $u(t^*)$, then the derivative of the state vector can be expressed as given in (2):

$$x = \frac{dx}{dy}\bigg|_{t = t^*} \tag{2}$$

In this case, both the state and the control vectors, i.e. x and u respectively have "fuzzy values," instead of crisp values. This implies that these values at any instant are located within certain subsets with membership values. The derivative of the state, $\dot{x}$ has a fuzzy value at each instant which makes both the state and the control possible. By standard, a function $y = y(t)$ which assumes a fuzzy value at instant $t = \tilde{t}$ is described such that if I

is an interval with $\mu_\square$ being the membership function defined on it, then the fuzzy value $y(\tilde{t})$ is located in I with a corresponding membership value measured by $\mu_\square$ which is a membership value equal to $\mu_I(y(\tilde{t}))$.

Consider a control system with state $x = x(t)$ and control input $u = u(t)$ defined on the time domain $[0, \infty)$ in which the system dynamics are described by the derivative of the state $\dot{x}$ which is a function of both x and u. If the relation between $\dot{x}$ and both (x, u) is exactly known, then mathematical modeling techniques and the systems control theory can be applied. The exact modeling $\dot{x} = ax + bu$ simply implies that at any instant $t = t^*$ $\in [0, \infty)$, if $x = x(t^*)$ and $u = u(t^*)$, then this is (3):

$$\dot{x} = \frac{dx(t)}{dt}\bigg|_{t\,=\,\mathrm{t}^*} \tag{3}$$

2.3 Design of the fuzzy controller

A fuzzy control system was designed to manage the admission and rejection of customers in the system.

2.3.1 Fuzzy inputs and rules

The system is described by (z_1, z_2), where z_1 is the number of customers in station i, where $i = 1, 2$. In order to prevent a situation whereby an arriving class 1 customer is rejected even when the system is empty or an arriving class 2 customer is rejected even when station 2 is empty, it is assumed that the reward at each station for every customer is greater than the corresponding expected holding cost. Consequently,

$$r_1 > \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} \tag{4}$$

$$r_2 > \frac{h_2}{\mu_2} \tag{5}$$

The system receives a reward for admitting customers while also incurring a cost for holding customers. Considering (4) and (5) above, it is optimal for the system to keep the server in station B busy provided there is a continuous arrival of customers and no customer is held in queue. It is important to observe that this condition is temporal as a result of the memoryless property of the exponential distribution whereby neither the inter-arrival nor the service times can be conditioned on the present observable state. Similarly, if the system is highly rewarded for admitting customers and incurs a low cost for holding class 1 customers, then the system accepts class 1 customers easily.

The fuzzy inputs are $s_i = 0, 1, \ldots,$ where $i = 1, 2$ of customers in the buffer of station i and the customer arrival rates $\lambda_j \in [0, \infty)$, $j = 1, 2$ of class j. The fuzzy outputs are decisions, $dec_j = 1, 0$. When an arriving class j customer is admitted, it is a 1, while it is a 0 if it is rejected. The fuzzy inputs include "zero", "fairly positive", "positive" and "highly positive" represented as "NE", "FP", "PO" and "HP" respectively. A four fuzzy sets for each of the four inputs were chosen and the complete rule base consists of 256 (i.e. 4^4) rules combinations. However, only 15 combinations of the rules produce output decision ADMIT while the other 241 combinations produces the output decision REJECT. However, all rule combinations that produce output decision ADMIT and only 10 rule combinations that produces output decision REJECT were recorded. The fuzzy rule base is as shown in Table 1.

In Table 1, ADMIT implies that an arriving customer of class j is admitted into station j. In this case, there are 15 combinations that produces ADMIT as output while 241 combinations produces REJECT as output decision. The membership functions for the fuzzy inputs s_i, $i = 1, 2$, and λ_j, $j = 1, 2$, are depicted shown in Fig. 2.

Table 1. All rule combinations with ADMIT as output and only 10 rule combinations with REJECT decision

| S_A	S_B	λ_1	λ_1	*Decision (dec)*
NE	NE	NE	NE	ADMIT
NE	NE	NE	FP	ADMIT
NE	NE	NE	PO	ADMIT
NE	NE	FP	HP	ADMIT
NE	NE	FP	FP	ADMIT
NE	NE	PO	NE	ADMIT
NE	FP	NE	NE	ADMIT
NE	FP	NE	FP	ADMIT
NE	FP	FP	NE	ADMIT
NE	PO	NE	NE	ADMIT
FP	NE	NE	NE	ADMIT
FP	NE	NE	FP	ADMIT
FP	NE	FP	NE	ADMIT
FP	FP	NE	NE	ADMIT
PO	NE	NE	NE	ADMIT
NE	NE	NE	NE	REJECT
NE	FP	NE	FP	REJECT
NE	FP	NE	PO	REJECT
NE	FP	FP	FP	REJECT
NE	PO	PO	NE	REJECT
FP	NE	NE	FP	REJECT
FP	FP	NE	FP	REJECT
FP	NE	FP	HP	REJECT
FP	PO	NE	NE	REJECT
PO	NE	NE	HP	REJECT
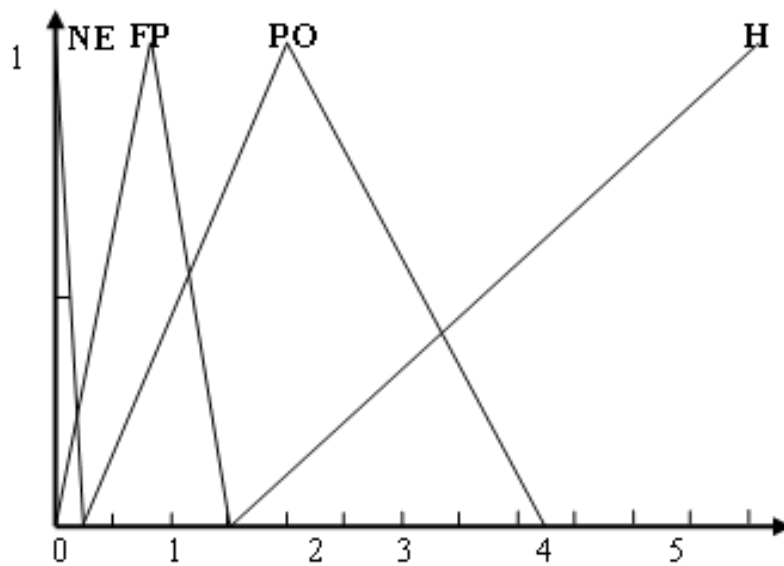
Fig. 2. Membership functions for the fuzzy input, *s*

Similarly, the membership for the fuzzy input, λ is given in Fig. 3.

The fuzzy output, *dec.* is shown in Fig. 4.

The universes of discourse for the fuzzy inputs s_i and λ_j are [0, ∞] while for the fuzzy output *dec. is* [0, 1].
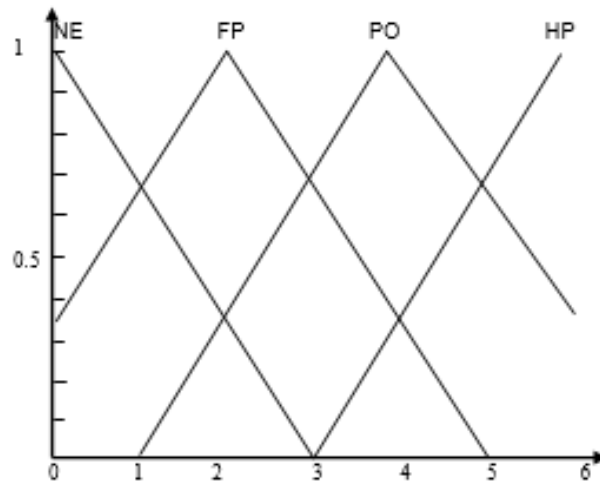
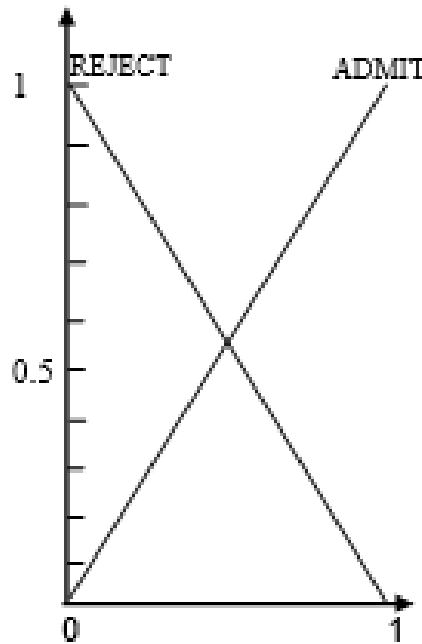**Fig. 3. Membership for the fuzzy input λ**

Fig. 4. Fuzzy output, *dec*

2.3.2 Membership functions applicable in the control of class 1 arrivals

Since it is necessary to determine the relationships between the fuzzy inputs s_i and λ_j, as well as fuzzy output *dec.*, it is assumed that $z_1 + z_2 > 0$. The scenario given by the rule:

If s_1 is HP while s_2 is zero (NE) while each of λ_1 and λ_2 is NE, then dec_1 is REJECT.

In this case, it is necessary to decide whether to admit the last class 1 customer when there are $s_1 \geq 0$ customers already in queue 1, one customer is in server 1 while all other input variables are zero, if E_1 is the mean holding cost the last customer incurs in queue 1. Similarly, E_2 is the mean holding cost from the time service starts at server 1 until exit from server 2. In this case, the customer is admitted only if its reward compensates its expected holding cost which is as expressed in (6):

$$r_1 \geq E_1 + E_2. \tag{6}$$

Since there are $s_1 + 1$ customers in station A, then $E_1 = h_1(s_1 + 1) / \mu_1$. Consequently, E_2 is computed as follows. Server 1 starts to service the customer just when server 2 starts to service the previous customer. In this case, the state of the system is $(z_1, z_2) = (1, 1)$. The system continues to state $(0, 2)$ if server 1 is done with service before server 2. The system moves from state $(0, 2)$ or $(1, 0)$ to state $(0, 1)$ and lastly to $(0, 0)$. It is obvious from these transitions that the sojourn time in state $(1, 1)$ is a random variable with exponential distribution and mean $1/(\mu_1 + \mu_2)$. As a result, the mean holding cost from server 1 to exit becomes (7):

$$E_2 = \frac{h_1}{\mu_1 + \mu_2} \tag{7}$$

$$E_2 = \frac{h_1}{\mu_1 + \mu_2} + \frac{\mu_1}{\mu_1 + \mu_2} x \ (\text{mean cost from } (0, 2) \text{ to departure})$$

$$E_2 = \frac{h_1}{\mu_1 + \mu_2} + \frac{\mu_1}{\mu_1 + \mu_2} x \ (\text{mean cost from } (1, 0) \text{ to departure})$$

$$E_2 = \frac{h_1}{\mu_1 + \mu_2} + \frac{\mu_1}{\mu_1 + \mu_2} \frac{2h_2}{\mu_2} + \frac{\mu_1}{\mu_1 + \mu_2} \left(\frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} \right)$$

$$E_2 = \frac{h_1 \left(1 + \frac{\mu_2}{\mu_1} \right) + h_2 \left(1 + \frac{2\mu_1}{\mu_2} \right)}{\mu_1 + \mu_2}$$

Substituting $E_1 = h_1 (s_1 + 1)/\mu_1$ and the above into (5) and solving for s_1 gives (8):

$$s_1 \leq s_1, dec_1 = \frac{\mu_1}{h_1} \left(r_1 - \frac{h_1}{\mu_1} - \frac{h_2}{\mu_2} \right) - \frac{\mu_1}{h_1} \frac{h_2}{\mu_2} \frac{\mu_1}{\mu_1 + \mu_2} - 1 \tag{8}$$

However, if the number of customers in queue 1 is greater than the threshold s_1, then the dec_1 is REJECT.

2.3.3 Membership Functions Applicable in the Control of Class 2 Arrivals

It is necessary to specify the numerical settings of s_i and λ_j for the fuzzy output dec_2 with the assumption that $z_2 > 0$. Considering the rule scenario:

If s_1 is HP and s_2 is NE while each of λ_1 and λ_2 is 0, then dec_2 is REJECT.

It is necessary to determine a condition for s_1 under which a class 2 customer is REJECTED when each of λ_1, λ_2, and s_2 is zero. If it is assumed $z_2 > 0$, then there is a customer in server 2. If this customer is accepted, the state of the system at time zero will be $(s_1 + 1, 2)$, else the system starts from state $(s_1 + 1, 1)$.

Let $F(i, j)$ be the total expected holding cost from state (i, j) to state $(0, 0)$. If both i and j are greater than 0, then the system moves from state (i, j) to $(i - 1, j + 1)$ with probability $\mu_1 /(\mu_1 + \mu_2)$ or to state $(i, j - 1)$. When $i = 0$ ($j = 0$), the system will visit state $(0, j -1)$ with probability 1. Consequently, $F(i, j)$ can be computed as given in (9):

$$
F(i,j) \begin{cases} j\dfrac{h_2}{\mu_2} + F(0, j-1) & if \ \ i = 0 \\[2ex] i\dfrac{h_1}{\mu_1} + F(i-1,1) if \ \ j = 0 \\[2ex] \dfrac{ih_1 + jh_2 + \mu_1 F(i-1, j+1) + \mu_2 F(i, j-1) \ if \ i,j > 0}{\mu_1 + \mu_2} \end{cases} \tag{9}
$$

Consequently, the only condition under which an arriving class 2 customer is rejected is given in (10):

$$
r_2 - F(s_1 + 1, 2) < - \ F(s_1 + 1, 1) \tag{10}
$$

If the condition depicted in (6) holds for some value s_1, dec_2 then it will also hold for every $s_1 \geq s_1, dec_2$. This implies that the fuzzy set HP for s_1 with membership grade 1.0 in the fuzzy rule base for dec_2 is at:

$$
s_1, dec_2 = \min \left\{ \ s_1 : F(s_1 + 1, 2) - F(s_1 + 1, 1) > r_2 \ \right\} \tag{11}
$$

If a case described by the rule: *if s_1 is NE and s_2 is HP while each of λ_1 and λ_2 is NE, then dec_2 is REJECT*. It is important to determine a condition for s_2 under which a class 2 arrival is rejected when $x_1 = s_1 = \lambda_1 = \lambda_2 = 0$, while the number of existing customers in station B is $x_2 = s_2 + 1$. This is the optimal decision provided the expected holding cost for the new customer is greater than the corresponding reward. This can be expressed as $r_2 < (s_2 + 2)h_2/\mu_2$. In this case, the fuzzy set HP for s_2 with membership grade 1.0 in the fuzzy rule base for dec_2 is fixed at:

$$
s_2, dec_2 = \frac{\mu_2}{h_2} r_2 - 2 \tag{12}
$$

As for the fuzzy input λ_1, if we consider the rule: *if each of s_1 and s_2 is NE while λ_1 is HP and λ_2 is NE, then dec_2 is REJECT*. Consequently, an arriving class 2 customer is REJECTED while $x_i = 1$, $s_i = 0$, $i = 1, 2$. In this case, only class 1 arrivals are ADMITTED.

3 Results and Discussion

Consideration is given to the proposed queue network structure in Fig. 1 with parameters $\mu_1 = 1$, $\mu_2 = 1.5$, $\lambda_1 = 0.5$, $\lambda_2 = 0.5$, $h_1 = 1$, $h_2 = 1$, $r_1 = 10$, $r_2 = 6$. The optimal policy for dec_1 is determine using the fuzzy logic rule in Table 1. The logic processes adopted are as follows:

a. The scaling were determined and the factors for the fuzzy inputs s_1, s_2, λ_1, and λ_2 in the rule base for dec_1;
b. The algorithm begin from an initial state $s_1 = s_2 = 0$;
c. With the current s_1 and s_2 and the given λ_1 and λ_2 as inputs, the decision is made using fuzzification, fuzzy inference as well as de-fuzzification;
d. The decision dec_1 is plotted in the two-dimensional plane of s_1 and s_2;
e. Proceed to step (f) if $dec_1 = 0$; else set $s_2 = s_2 + 1$ and proceed to step (c); and
f. If $dec_1 = 0$, then stop; else set $s_2 = s_2 + 1$ and proceed to step (c).

Step (c) in the logical processes can be illustrated with an example. Assume that $s_1 = s_2 = 2$, then s_1 should be scaled down to 1.7 while s_2 is scaled down to 0.9. From Fig. 3, s_1 corresponds to NE with grade 0.43, FP with grade 0.90 and PO with grade 0.23, while s_1 corresponds to NE with grade 0.69 and FP with grade 0.64. Similarly, $\lambda_1 = 0.5$ is NE with grade 1.0 and $\lambda_2 = 0.5$ is FP with grade 0.67 and PO with grade 0.67. The inputs s_1, s_2, λ_1 and λ_2 have 3, 2, 1, and 2 fuzzy sets respectively. Consequently, this gives 3 x 2 x 1 x 2 = 12 fuzzy decisions for fuzzy output dec_1. Going by the fuzzy rule base, the 12 fuzzy decisions dec_1 are expressed as follows:

i.   *If s_1 is NE with grade 0.43, s_2 is NE with grade 0.69, λ_1 is NE with grade 1.00 and λ_2 is FP with grade 0.67, then dec.$_1$ is ADMIT with grade 0.43;*

ii. *If s_1 is NE with grade 0.43, s_2 is NE with grade 0.69, λ_1 is NE with grade 1.00 and λ_2 is PO with grade 0.67, then dec.$_1$ is ADMIT with grade 0.43;*

iii. *If s_1 is NE with grade 0.43, s_2 is FP with grade 0.64, λ_1 is NE with grade 1.00 and λ_2 is FP with grade 0.67, then dec.$_1$ is ADMIT with grade 0.43;*

iv. *If s_1 is NE with grade 0.43, s_2 is FP with grade 0.64, λ_1 is NE with grade 1.00 and λ_2 is PO with grade 0.67, then dec.$_1$ is REJECT with grade 0.43;*

v.   *If s_1 is FP with grade 0.90, s_2 is NE with grade 0.69, λ_1 is NE with grade 1.00 and λ_2 is FP with grade 0.67, then dec.$_1$ is ADMIT with grade 0.67;*

vi. *If s_1 is FP with grade 0.90, s_2 is NE with grade 0.69, λ_1 is NE with grade 1.00 and λ_2 is PO with grade 0.67, then dec.$_1$ is REJECT with grade 0.67;*

vii. *If s_1 is FP with grade 0.90, s_2 is FP with grade 0.64, λ_1 is NE with grade 1.00 and λ_2 is FP with grade 0.67, then dec.$_1$ is REJECT with grade 0.64;*

viii. *If s_1 is FP with grade 0.90, s_2 is FP with grade 0.64, λ_1 is NE with grade 1.00 and λ_2 is PO with grade 0.67, then dec.$_1$ is REJECT with grade 0.64;*

ix. *If s_1 is PO with grade 0.23, s_2 is NE with grade 0.69, λ_1 is NE with grade 1.00 and λ_2 is FP with grade 0.67, then dec.$_1$ is REJECT with grade 0.23;*

x.   *If s_1 is PO with grade 0.23, s_2 is NE with grade 0.69, λ_1 is NE with grade 1.00 and λ_2 is PO with grade 0.67, then dec.$_1$ is REJECT with grade 0.23;*

xi. *If s_1 is PO with grade 0.23, s_2 is FP with grade 0.64, λ_1 is NE with grade 1.00 and λ_2 is FP with grade 0.67, then dec.$_1$ is REJECT with grade 0.23; and*

xii. *If s_1 is PO with grade 0.23, s_2 is FP with grade 0.64, λ_1 is NE with grade 1.00 and λ_2 is PO with grade 0.67, then dec.$_1$ is REJECT with grade 0.23.*

The peak values and heights of the fuzzy decisions, *dec. from Fig. 3* are e_1=1, e_2=1, e_3=1, e_4=0, e_5=1, e_6=0, e_7=0, e_8=0, e_9=0 e_{10}=0 e_{11}=0 e_{12}=0. Similarly, $f_1 = 0.43$, $f_2 = 0.43$, $f_3 = 0.43$, $f_4 = 0.43$, $f_5 = 0.67$, $f_6 = 0.67$, $f_7 = 0.64$, $f_8 = 0.64$, $f_9 = 0.23$, $f_{10} = 0.23$, $f_{11} = 0.23$, $f_{12} = 0.23$. Using the height method of de-fuzzification, the crisp output *dec.** can be expressed as:

$$dec.^* = \frac{\sum_{i=1}^{12} e_i f_i}{\sum_{i=1}^{12} f_i} = 0.4$$

Since *dec.** < 0.5, the decision, *dec.$_1$* is REJECT which means that the server at station A will not admit an arriving class 1 customer. The outputs for *dec.$_1$* is indicated in Fig. 5.
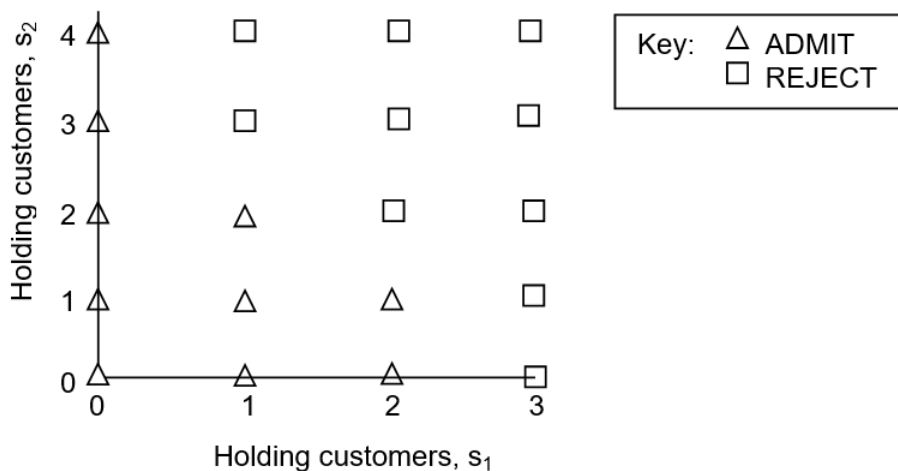

Fig. 5. Outputs for *dec.$_1$*

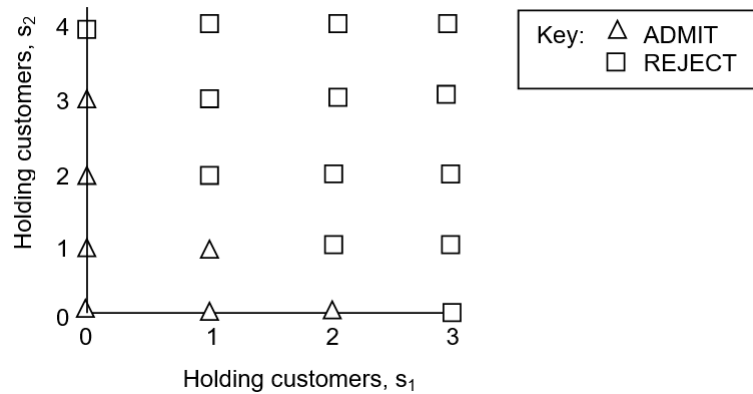Similarly, the outputs for $dec._2$ is indicated in Fig. 6.

Fig. 6. Outputs for *dec.*₂

4 Conclusion

The proposed model was able to manage the admission of arrivals in a two-server queue using a fuzzy-based policy such that the performance of the system is optimal. The tool adopted is a fuzzy process which determines this policy using the fuzzy input values, s and λ giving a corresponding decision, dec. Numerical results show a considerable improvement in the control of customers' admission and it was concluded that the proposed method is effective in the control of customers' admission in a queue network.

Acknowledgements

Competing Interests

Authors have declared that no competing interests exist.

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