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Comb-Based Filter Design Using Sharpening Technique and Certain Palindromic Polynomial

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Abstract

This paper presents a novel approach to improve aliasing rejection in comb-based decimation filters. The method is established on certain palindromic polynomials with all zeros on the unit circle and the sharpening technique. As a result, aliasing rejection and the passband characteristic are improved. The method is illustrated with various examples and compared with the methods from the literature.

Keywords

Comb Filter, Aliasing, Passband Droop, Palindromic Polynomial, Sharpening

1. Introduction

This paper presents how some results from mathematics may be useful for digital filter design. More precisely, we consider the problem of improving the frequency characteristic of comb filters. This filter has all its coefficients equal to unity, making it the simplest decimation filter [1]. However, its frequency characteristic provides low attenuation in bands around comb zeros called folding bands, where aliasing occurs. The passband characteristic is not flat and has a passband droop, which may deteriorate the decimated signal. Usually, the aliasing rejection may be increased by cascading two or more combs. The number of the cascaded combs is called the order of comb. However, the increase of the comb order increases the comb passband droop.

Consequently, the comb order is typically less than 6. Different techniques have been proposed to increase comb aliasing rejection. The Rotated Sinc (RS) filter was proposed in [2]. However, the RS filter requires two multipliers. Since the comb filter is a multiplier-less filter, the preferred methods to improve the comb filter should also be multiplier-less methods. Authors in [3] [4] used combs of different lengths to improve comb aliasing rejection.

Similarly, the cosine filters are introduced in [5]. Different authors adopted results in mathematics to improve comb aliasing rejection. The cyclotomic polynomials are elaborated in [6]. Likewise, the authors in [7] [8] considered certain symmetric polynomials to derive simple multiplier-less filters that introduce the additional zeros into comb folding bands, thus improving the attenuation in those bands. Based on Kaiser sharpening polynomials, the sharpening technique is presented in [9] to improve the comb passband and stopband simultaneously. Recently, some new sharpening polynomials have been introduced to increase comb aliasing rejection [10] [11]. The corresponding compensator designs for a high passband droop in the sharpened comb filters are also presented. A two-stage structure where sharpening is performed in the second stage, and the sharpening coefficients are obtained using linear programming, is proposed in [12].

This paper proposes a method to increase comb aliasing rejection while keeping a low passband droop. The former is achieved by using the simple filter derived from certain palindromic polynomials, which introduce additional zeros into comb folding bands, thus increasing the width and attenuation in those bands.

The sharpening technique further improves aliasing rejection and decreases passband droop.

The novelty of this paper is a new simple comb-based multiplier-less decimator design established on the mathematical results, resulting in high aliasing rejection in all folding bands and a low droop in a narrow passband.

The rest of the paper is organized in the following way. The mathematical background is elaborated in Section 2. The proposed method is introduced in Section 3 and illustrated with various examples. The comparisons with the techniques from the literature are provided in Section 4. Finally, the concluding remarks are given in Section 5.

2. Mathematical Background

2.1. Using a Certain Palindromic Polynomial [7] [13]

We consider a symmetric polynomial $f(z)$ with even degree $n = 2m$, $m > 0$,

$$f(z) = a_0 + a_1 z + \dots + a_n z^n, \tag{1}$$

where $a_k = a_{n-k}$, for all k, $k = 0, \dots, n$.

According to the result in [13], it is useful to uniquely associate a polynomial $g(u)$ to the polynomial $f(z)$ as

$$f(z) = z^{n/2} g(u), \tag{2}$$

where

$$u = z + z^{-1}. \tag{3}$$

Theorem in [13] stated that all zeros of the polynomial (1) are all on the unit circle if all zeros of the polynomial $g(u)$ are all real and in the interval [−2, 2].

This useful result is explored in [7] to modify the system function of the cascaded comb,

$$H^2(z) = 1 + 2z^{-1} + \ldots + Mz^{-(M-1)} + \ldots + z^{-2(M-1)}, \tag{4}$$

to separate double zeros in (4). To this end, the comb system function (4) is expressed in terms of the polynomial (1) as

$$H^2(z) = f(z)/z^{2(M-1)}, \tag{5}$$

where:

$$f(z) = 1 + 2z^1 + \ldots + Mz^{(M-1)} + \ldots + z^{2(M-1)}. \tag{6}$$

According to work in [13], the middle coefficient in the palindromic polynomial (1) is crucial for the position of the zeros of the polynomial on the unit circle. To this end, the authors in [7], using the Theorem introduced in [13], proposed to change the middle coefficient in (6) to M-a, where $a = 2^{-k}$, where k is an integer:

$$f_m(z) = 1 + 2z^1 + \ldots + (M\text{-}a)z^{(M-1)} + \ldots + z^{2(M-1)}. \tag{7}$$

The system function of the so-called modified comb filter is obtained from (7):

$$H_m(z) = f_m(z)/z^{2(M-1)}. \tag{8}$$

We will take the value $k = 1$, leading to an $a = 1/2$. The system function of the proposed filter in [7] is given as:

$$G(z) = H^{K-2}(z)H_m(z), \tag{9}$$

where K is the order of the equivalent comb filter.

The zeros of the filters (9) and (4) are all distributed on the unit circle. However, the zeros in (9) are all single, while those in (4) are all double. As a result, the folding bands in the filter (9) are wider and provide higher aliasing rejection than in the equivalent comb filter.

2.2. Using Sharpening Technique

The sharpening technique uses the amplitude change function (ACF), which is a polynomial relationship of the form $H_o = f(H)$ between the amplitudes of the overall and the prototype filters characteristics, H_o and H, respectively [14].

To improve the prototype filter characteristic in both the passband and stopband, the ACF has to be horizontal near the passband and stopband, i.e., to have zero derivatives at these points, denoted as m and l, respectively. The following sharpening polynomial, with $m = l = 1$, is used in [9] to improve the comb passband and stopband simultaneously: $Sh\{H\} = 3H^2 - 2H^3$, where $Sh\{H\}$ means the sharpening of H.

We will use the same sharpening polynomial in this work.

3. Proposed Method

We propose here to sharpen a modified comb $H_m(z)$ with the system function given in (8):

$$Sh\{H_m(z)\} = 3H_m^2(z) - 2H_m^3(z) = H_m^2(z)[3z^{-N} - H_m(z)], \tag{10}$$

where a delay of N is necessary to add the value of 3 in the middle of $H_m(z)$ and thus keep the linearity of the phase [9].

The system function of the proposed filter is given as:

$$H_p(z) = Sh\{H_m(z)\}H^{K_1}(z). \tag{11}$$

We will consider the narrow passband defined with the passband edge $\omega_p = \pi/(8M)$. The method is illustrated in the following example.

Example 1: We consider $M = 12$ and $K_1 = 1$. The equivalent order of the comb filter is $K = 4$.

The overall magnitude responses of the proposed filter and the equivalent comb are contrasted in **Figure 1**. The zoom in the passband is also shown. The proposed filter has better aliasing rejection and lesser passband droop.

In the next example, we show that the passband droop in the proposed filter does not depend on M but only on K_1.

Example 2: Consider $M = 32$, $K = 4$, and $K_1 = 1$. The magnitude responses of the proposed and the equivalent comb filter, along with the passband zoom, are shown in **Figure 2.** Note that the passband droops for $M = 32$ and $M = 12$ are equal since $K_1 = 1$.

The considered values of K_1 here are equal to 1, 2, and 3, corresponding to the orders of equivalent comb filters of $K = 4$, 5, and 6. The values of the passband droop for different values of K_1 are given in **Table 1**.

The principal features of the proposed filter are the following:

- All folding bands are wider than the equivalent comb folding bands.

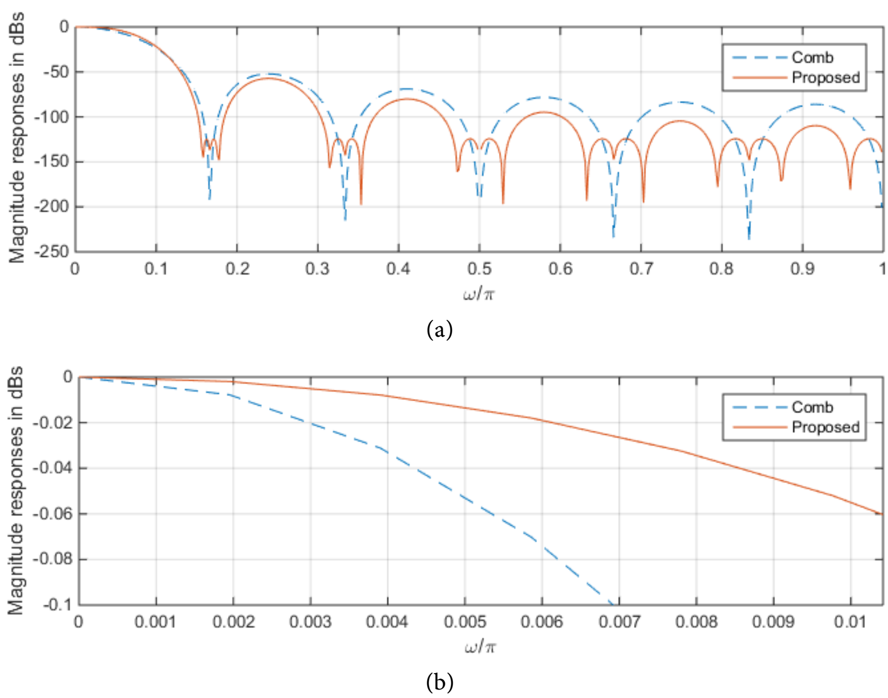


Figure 1. Magnitude response of the proposed filter and equivalent comb, $M = 12$, $K_1 = 1$, $K = 4$. (a) Overall magnitude responses; (b) Passband zoom.

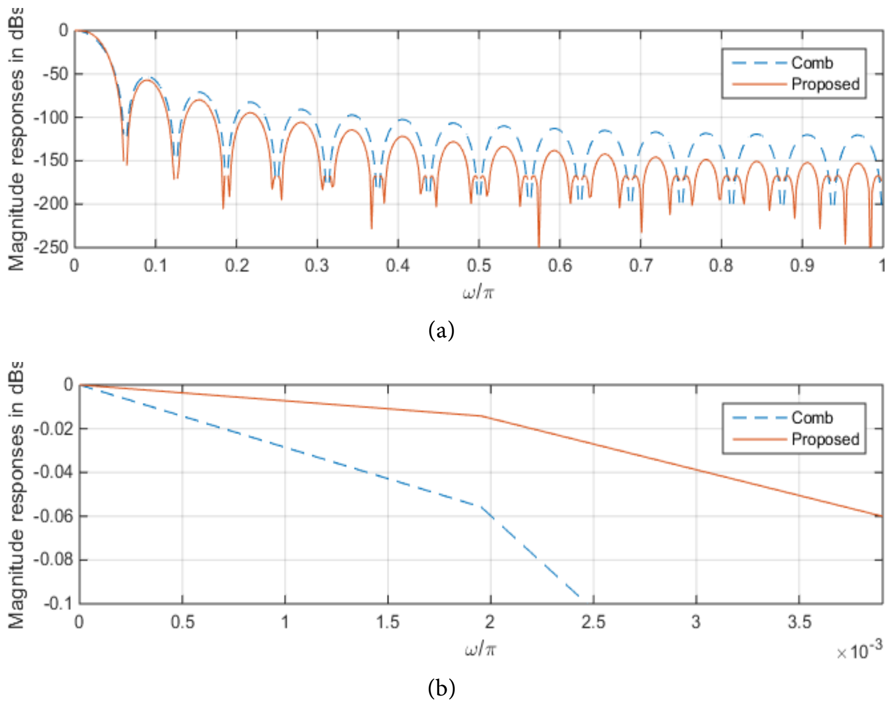


Figure 2. Magnitude responses in Example 2. (a) Overall magnitude responses; (b) Passband zoom.

Table 1. Passband doop in dBs.

K_1	Passband droop in dBs
1	0.06
2	0.1145
3	0.17

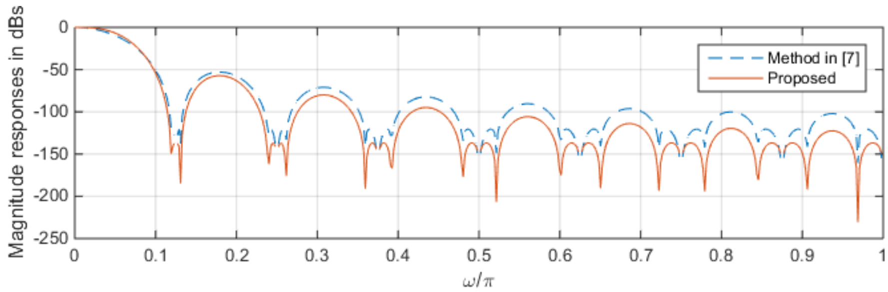
- The corresponding passband droop is significantly decreased compared to the equivalent comb.
- Maximum passband droop does not depend on M but only on K_1. If one wants to decrease the passband droop, the simple comb compensator from the literature can be used.
- The proposed filter is a multiplier-less filter.

4. Some Comparisons

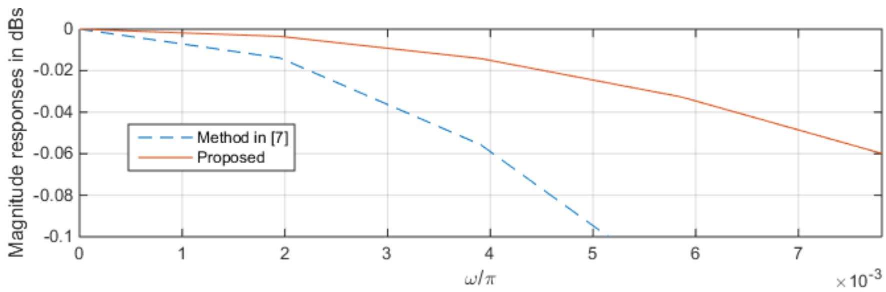
We first compare the proposed method with the methods in [7] and [9] since the proposed method is related to those two methods.

4.1. Comparison with the Method in [7]

In this comparison, $M = 16$, and the equivalent comb order $K = 4$. In the proposed method, $K_1 = 1$. The overall magnitude responses and the passband zoom are shown in **Figure 3**. The proposed method provides better aliasing rejection and a decreased comb passband droop.



(a)

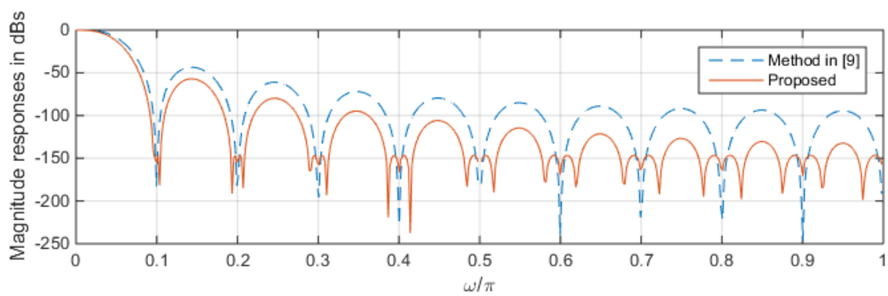


(b)

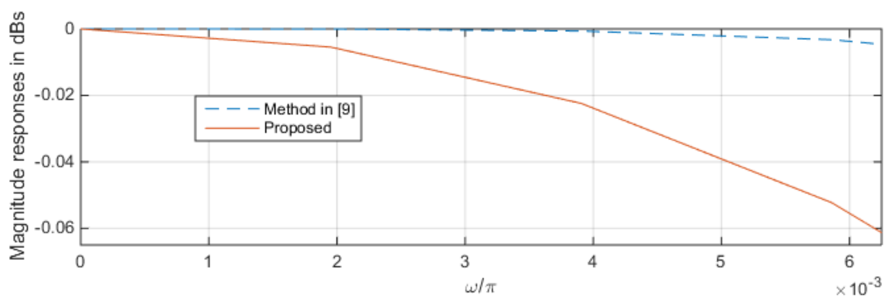
Figure 3. Comparison with method in [7]. (a) Overall magnitude responses; (b) Passband zoom.

4.2. Comparison with the Method in [9]

To compare with the method in [9], we consider $M = 20$, $K_s = 2$, and $K_1 = 1$, where K_s is the order of comb in the sharpening polynomial, so the equivalent comb filter has an order of $K = 6$.We can observe in **Figure 4** that the proposed



(a)



(b)

Figure 4. Comparison with the method in [9]. (a) Overall magnitude responses; (b) passband zoom.

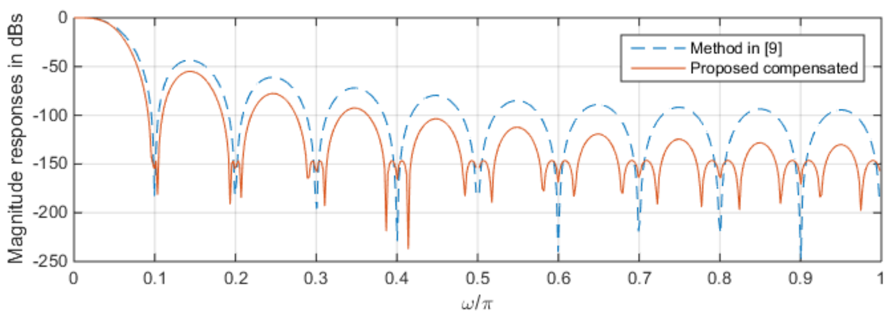
method provides much betteralias rejection, while the method in [9] exhibits better passband characteristics.

We can observe that the proposed method provides much better alias rejection while the method in [9] exhibits better passband characteristics. However, as mentioned before, a simple comb compensator can be added at a low rate, i.e., after decimation in the proposed filter. We used the comb compensator presented in [15] with the parameters $A = 2^{-3} - 2^{-8}$; $B = 2^{-3} + 2^{-5}$, requiring 11 adders at a low rate. The magnitude responses of the proposed compensated filter and the filter from [9] are contrasted in **Figure 5**. Note that the compensated filter provides high aliasing rejection and a low passband droop with a maximum passband deviation of 0.007 dB. The maximum passband deviation in [9] is 0.0045 dB.

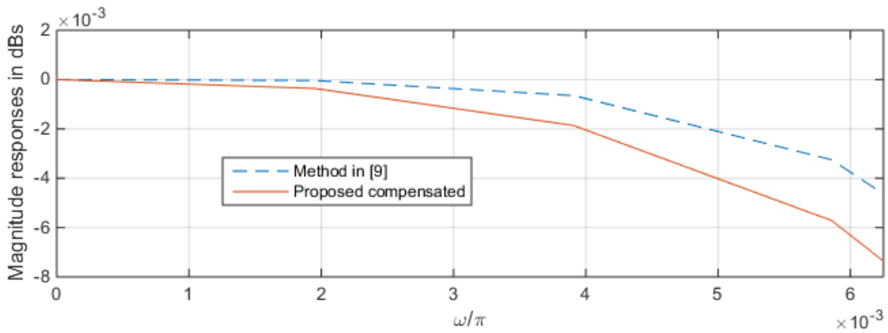
4.3. Comparison with the Method in [10]

The compensated sharpened comb is introduced in [10]. Various sharpened combs and the corresponding compensators for $M = 32$ are presented in Table I [10].

From Table I in [10], we pick up $M = 32$, the sharpened comb: $\mathrm{Sh}\{H\} = -2^{-7}H^2 + H^4$, and the corresponding compensator. The proposed method $K_1 = 1$ and the same compensator used in Section 4.2 is applied. The magnitude responses of the proposed compensated filter and the filter in [10] are contrasted in **Figure 6**. The proposed compensated filter provides much better aliasing rejection, while the passbands are similar. The maximum passband deviation in the filter from [10] is 0.001 dB, while in the proposed method is 0.007 dB.

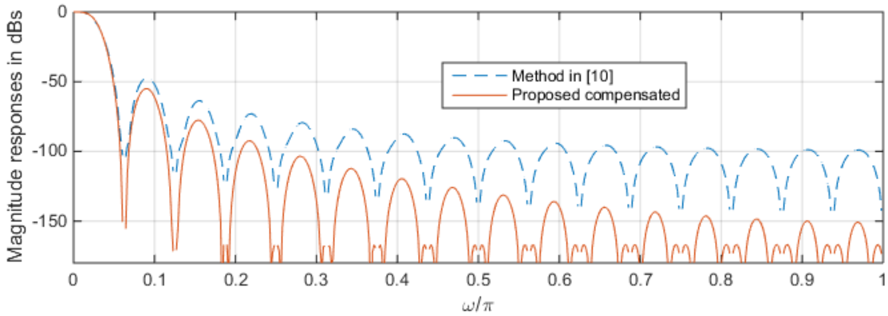


(a)

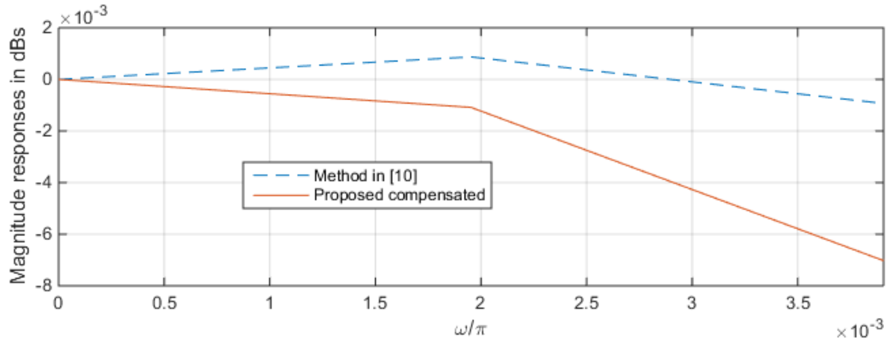


(b)

Figure 5. Comparison compensated proposed filter with the method in [9]. (a) Overall magnitude responses; (b) Passband zoom.



(a)



(b)

Figure 6. Comparison of the compensated proposed filter and filter in [10]. (a) Overall magnitude responses; (b) Passband zoom.

5. Conclusion

This paper presents how certain mathematic results can be useful for improving the magnitude characteristic of the comb decimation filter. Two results are specifically applied here: the Theorem of the zeros on the unit circle and the sharpening technique. In that way, the obtained comb-based filter has a high aliasing rejection and a low passband droop. The passband droop can be further increased by cascading a comb compensator at a low rate. Any comb compensator from the literature can be used. The proposed method is very simple since only one parameter of design K_1 has only three possible values.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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