



Validating AU Microscopii d with Transit Timing Variations

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Abstract

AU Mic is a young (22 Myr), nearby exoplanetary system that exhibits excess transit timing variations (TTVs) that cannot be accounted for by the two known transiting planets nor stellar activity. We present the statistical “validation” of the tentative planet AU Mic d (even though there are examples of “confirmed” planets with ambiguous orbital periods). We add 18 new transits and nine midpoint times in an updated TTV analysis to prior work. We perform the joint modeling of transit light curves using EXOFASTv2 and extract the transit midpoint times. Next, we construct an $O-C$ diagram and use Exo-Striker to model the TTVs. We generate TTV log-likelihood periodograms to explore possible solutions for d’s period, then follow those up with detailed TTV and radial velocity Markov Chain Monte Carlo modeling and stability tests. We find several candidate periods for AU Mic d, all of which are near resonances with AU Mic b and c of varying order. Based on our model comparisons, the most-favored orbital period of AU Mic d is 12.73596 ± 0.00793 days ($T_{C,d} = 2458340.55781 \pm 0.11641$ BJD), which puts the three planets near 4:6:9 mean-motion resonance. The mass for d is $1.053 \pm 0.511 M_{\oplus}$, making this planet Earth-like in mass. If confirmed, AU Mic d would be the first known Earth-mass planet orbiting a young star and would provide a valuable opportunity in probing a young terrestrial planet’s atmosphere. Additional TTV observations of the AU Mic system are needed to further constrain the planetary masses, search for possible transits of AU Mic d, and detect possible additional planets beyond AU Mic c.

Unified Astronomy Thesaurus concepts: Exoplanet astronomy (486); Exoplanet dynamics (490); Exoplanet systems (484); Exoplanets (498)



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1. Introduction

AU Microscopii (TOI-2221, TIC 441420236, HD 197481, GJ 803) is a fundamental system proven to be quite viable for study of planetary formation and orbital dynamics of young systems given its youthfulness (22 ± 3 Myr; Mamajek & Bell 2014), proximity (9.71 pc; Gaia Collaboration et al. 2021), and relative brightness ($m_V = 8.81$ mag). It is highly active (Butler et al. 1981; Kundu et al. 1987; Cully et al. 1993; Tsikoudi & Kellett 2000; Feinstein et al. 2022; Gilbert et al. 2022) and reported to have the largest number of flare events among the Kepler and Transiting Exoplanet Survey Satellite (TESS) targets to date (Ilin & Poppenhaeger 2022). Ilin & Poppenhaeger (2022) examined TESS data for any flares caused by star-to-planet interaction (SPI) between AU Mic b and the host star; they did not find any and concluded that most flares were attributed to stellar activity, but they have not ruled this phenomenon out yet. The aforementioned heightened stellar activity of AU Mic had made it a challenging target for radial velocity (RV) and transit observations (Addison et al. 2021; Cale et al. 2021; Gilbert et al. 2022; Wittrock et al. 2022; Zicher et al. 2022).

AU Mic hosts two transiting planets (Plavchan et al. 2020; Martioli et al. 2021; Gilbert et al. 2022) and a large dust disk (Fajardo-Acosta et al. 2000; Zuckerman 2001; Song et al. 2002; Kalas et al. 2004; Liu et al. 2004; Plavchan et al. 2005; Strubbe & Chiang 2006; MacGregor et al. 2013; Grady et al. 2020; Arnold et al. 2022; Olofsson et al. 2022; Vizgan et al. 2022). Gallenne et al. (2022) searched for additional companions in the inner-disk region (0.4–2.4 au) with high-angular-resolution observations through the Very Large Telescope (VLT)/SPHERE and with combined data from VLT/NACO, Very Large Telescope Interferometer (VLTI)/PIONIER, and VLTI/GRAVITY, but did not find any brighter than $K_s \approx 11.2$ mag, which in turn caps the upper planetary mass limit at $12.3 \pm 0.5 M_J$. Szabó et al. (2021, 2022) probed the AU Mic system with the CHAracterising ExOPlanet Satellite (CHEOPS) and did a joint TESS + CHEOPS transit timing variation (TTV) analysis, which found the timing of the transits during summer of 2022 to be 30–85 minutes later than predicted using the linear ephemeris available at the time. Wittrock et al. (2022) did a TTV and photodynamical analysis of the AU Mic systems and detected a TTV excess that cannot be accounted for with the presence of both planets b and c and the stellar activities of AU Mic; thus, a nontransiting hypothetical planet, d, between AU Mic b and c was proposed. This would have made AU Mic among the few systems that have a nontransiting planet between its adjacent transiting planets; other systems include HD 3167 d (Christiansen et al. 2017), Kepler-20 g (Buchhave et al. 2016), Kepler-411 e (Sun et al. 2019), and TOI-431 c (Osborn et al. 2021). Kane et al. (2022) explored the orbital dynamics of the AU Mic system by injecting the hypothetical planet d; they found that it lies at the very edge of instability, and that d’s eccentricity will vary between 0.0 and 0.3 even on short timescales.

This paper is a continuation of the work done in Wittrock et al. (2022), with an emphasis now placed on validating the tentative planet AU Mic d through the TTV method. In Section 2, we highlight the new light curves in addition to the old data sets we include for TTV analyses. Next, we summarize the steps taken in modeling the observed transits using the EXOFASTv2 package (Eastman et al. 2019) in Section 3. Section 4 presents our modeling of the extracted TTVs using

the Exo-Striker package (Trifonov 2019) and presents a novel technique, called a TTV log-likelihood periodogram, which searches for parameters that maximize the log-likelihood. In Section 5, we perform the RV vetting of our TTV analysis to check for consistency between the RV results and those of the TTV analysis. We discuss the results in Section 6, and present our conclusion in Section 7.

2. Data from Observations

We incorporated a total of 54 data sets from four years of AU Mic’s photometric and Rossiter–McLaughlin (R-M) observations with various facilities, of which 45 are of AU Mic b and nine are of AU Mic c (Tables 1 and 2). The following transits had been presented in the previous works: the R-M observations (Martioli et al. 2020; Palle et al. 2020), the TESS and one of the Spitzer observations (Plavchan et al. 2020; Martioli et al. 2021; Gilbert et al. 2022), the CHEOPS observations (Szabó et al. 2021, 2022), and the Brierfield, the Las Cumbres Observatory (LCO) South African Astronomical Observatory (SAAO) and Siding Spring Observatory (SSO) prior to 2021, the PEST, and two of the Spitzer observations (Wittrock et al. 2022). This paper introduces 18 new and unpublished observations from the Antarctic Search for Transiting Exoplanets (ASTEP), the Las Cumbres Observatory Global Telescope (LCOGT) network, and the Mount Kent Observatory (MKO) CDK700 and adds nine midpoint times from CHEOPS observations. Therefore, this section will describe only the new observations that were not included in Wittrock et al. (2022).

2.1. CHAracterising ExOPlanet Satellite Photometry

CHAracterising ExOPlanet Satellite Photometry (CHEOPS) is a space-based telescope whose mission is to search for transits of known exoplanets and recover their radii more accurately, which will then place constraints on atmospheric and interior modeling and formation processes (Rando et al. 2020; Benz et al. 2021). Szabó et al. (2021, 2022) observed seven transits for AU Mic b and two transits for AU Mic c. The CHEOPS light curves have been processed and modeled separately, as described in Szabó et al. (2021, 2022), and this paper only incorporates the transit midpoint times from those works.

2.2. Ground-based Photometry

All of the ground-based observations listed in Table 1 have been coordinated through the TESS Follow-up Observing Program (TFOP) Working Group (WG).³³ Along with the ones mentioned in Wittrock et al. (2022), we added 13 new follow-up photometric transit observations, including one from ASTEP, four from the LCO Cerro Tololo Interamerican Observatory (CTIO) 1.0 m, three from LCO SAAO 1.0 m, four from LCO SSO 1.0 m, and one from LCO Teide Observatory (TO) 1.0 m. These light curves are available on ExoFOP-TESS³⁴ (Akeson et al. 2013). The follow-up observation schedules were conducted with the online version of the TAPIR package (Jensen 2013). AstroImageJ (AIJ; Collins et al. 2017) had been utilized to process the ground-based light curves and then create a subset table containing only BJD_TDB, normalized detrended flux, flux uncertainty, and

³³ <https://tess.mit.edu/followup>

³⁴ <https://exofop.ipac.caltech.edu/tess>

Table 1
List of AU Mic’s Photometric and R-M Observation Data Incorporated for Transit and TTV Analyses

Planet	Telescope	Date (UT)	Filter	Exposure Time (s)	No. of Images	Obs. Dur. (min)	Transit Coverage	References		
b	ASTEP 0.4 m	2021-07-09	R_c	9	636	377	Full	New		
		2021-07-26	R_c	9	724	407	Full			
		2021-08-12	R_c	9	801	550	Full			
c	ASTEP 0.4 m	2021-07-21	R_c	9	1294	752	Full	New		
		2021-08-09	R_c	9	951	560	Full			
b	Brierfield 0.36 m	2020-08-13	I	16	398	379	Full	1		
b	CFHT (SPIRou)	2019-06-17	955–2515 nm	122.6	116	302.8	egress	2		
b	CHEOPS	2020-07-10	CHEOPS	15	7 194	938.1	Full	3		
		2020-08-21	CHEOPS	15	8844	658.3	Full			
		2020-09-24	CHEOPS	3	12,222	1066.1	Full			
				2021-07-26	CHEOPS	3	9366	666.9	Full	4
				2021-08-12	CHEOPS	3	11,746	687.3	Full	
				2021-08-29	CHEOPS	3	9338	687.3	Full	
				2021-09-06	CHEOPS	3	9002	687.3	Full	
		c	CHEOPS	2021-08-09	CHEOPS	3	14,406	878.5	Full	4
				2021-08-28	CHEOPS	3	12,698	865.8	Full	
		b	IRTF (iSHELL)	2019-06-17	2.18–2.47 nm	120	47	105.2	Egress	2
b	LCO CTIO 1.0 m	2021-06-14	Pan-STARRS z_s	15	452	342	Full	New		
		2021-07-01	Pan-STARRS z_s	15	440	340	Full			
		2021-08-04	Pan-STARRS z_s	15	362	274	Egress			
c	LCO CTIO 1.0 m	2021-10-04	Pan-STARRS z_s	15	412	311	Ingress	New		
b	LCO SAAO 1.0 m	2020-05-20	Pan-STARRS Y	35	99	262	Egress	1		
		2020-05-20	Pan-STARRS z_s	15	333	266	Egress			
		2020-06-06	Pan-STARRS z_s	15	266	218	Egress			
		2020-06-23	Pan-STARRS z_s	15	223	183	Egress			
		2020-09-07	Pan-STARRS z_s	15	211	172	Ingress			
		2021-07-18	Pan-STARRS z_s	15	259	212	Ingress			
		2021-08-03	Pan-STARRS z_s	15	317	252	Ingress			
		2021-09-23	Pan-STARRS z_s	15	456	344	Full			
		2020-10-11	Pan-STARRS z_s	15	311	266	Ingress and egress			
		2020-04-25	Pan-STARRS Y	35	40	104	Egress		1 and new	
2020-04-25	Pan-STARRS z_s	15	212	172	Egress					
2020-08-13	Pan-STARRS z_s	15	379	312	Full					
2020-09-16	Pan-STARRS z_s	15	408	340	Full					
2020-10-03	Pan-STARRS z_s	15	248	219	Egress					
2021-06-22	Pan-STARRS z_s	15	351	281	Full	New				
2021-08-12	Pan-STARRS z_s	15	406	343	Full					
2021-08-29	Pan-STARRS z_s	15	448	340	Full					
2021-09-15	Pan-STARRS z_s	15	295	277	Egress					
b	LCO TO 1.0 m	2021-09-23	Pan-STARRS z_s	15	300	231	Egress	New		
c	MKO CDK700 0.7 m	2021-08-28	r'	16	74	87	Egress	New		
b	PEST 0.30 m	2020-07-10	V	15	1 143	556	Full	1		
b	Spitzer (IRAC)	2019-02-10	4.5 μm	0.08	3020	475.7	Full	1		
		2019-02-27	4.5 μm	0.08	3377	475.7	Egress			
		2019-09-09	4.5 μm	0.08	6002	990.9	Full			
b	TESS ^b	2018-07-26	TESS	120	329	718.0	Full	5		
		2018-08-12	TESS	120	296	708.0	Full			
		2020-07-10	TESS	20	2132	719.7	Full			

Table 1
(Continued)

Planet	Telescope	Date (UT)	Filter	Exposure Time (s)	No. of Images	Obs. Dur. (min)	Transit Coverage	References
c	TESS ^b	2020-07-19	TESS	20	2137	719.7	Full	5
		2020-07-27	TESS	20	2120	719.7	Full	
		2018-08-11	TESS	120	342	718.0	Full	
		2020-07-09	TESS	20	2138	719.7	Full	
		2020-07-28	TESS	20	2133	719.7	Full	
b	VLT (ESPRESSO)	2019-08-07	378.2–788.7 nm	200	88	359	Full	6

Notes. All ground-based photometric observations listed here were organized via the TESS Follow-up Observing Program Working Group (TFOPWG).^a

^a <https://tess.mit.edu/followup>.

^b ~12 hr snippets of the ~27 days duration TESS Cycles 1 and 3 light curves were extracted for our analysis, centered approximately on each transit.

References. (1) Wittrock et al. (2022); (2) Martioli et al. (2020); (3) Szabó et al. (2021); (4) Szabó et al. (2022); (5) Gilbert et al. (2022); (6) Palle et al. (2020).

Table 2
List of Facilities Utilized for Photometric and Rossiter–McLaughlin Follow-up Observations of AU Mic

Telescope	Instrument	Location	Aperture (m)	Pixel Scale (arcsec)	Resolution (pixels)	FOV (arcmin)	References
ASTEP 400	FLI Proline 16800E	Concordia Research Station, Antarctica	0.4	0.93	4096 × 4096	63.5 × 63.5	1
Brierfield	Moravian 16803	Bowral, New South Wales	0.36	0.732	4 096 × 4 096	50.0 × 50.0	2
CFHT	SPIRou	Maunakea, Hawai‘i	3.58	3
CHEOPS	0.32	1.11	1024 × 1024	19 × 19	4
IRTF	iSHELL	Maunakea, Hawai‘i	3.2	5
LCO CTIO	Sinistro	Cerro Tololo, Chile	1.0	0.389	4096 × 4096	26.5 × 26.5	6
LCO SAAO	Sinistro	Sutherland, South Africa	1.0	0.389	4096 × 4096	26.5 × 26.5	6
LCO SSO	Sinistro	Mount Woorut, New South Wales	1.0	0.389	4096 × 4096	26.5 × 26.5	6
LCO TO	Sinistro	Mount Teide, Tenerife, Canary Islands	1.0	0.389	4096 × 4096	26.5 × 26.5	6
MKO CDK700	U16	Mount Kent, Queensland	0.7	0.401	4096 × 4096	27.4 × 27.4	...
PEST	SBIG ST-8XME	Perth, Western Australia	0.3048	1.23	1 530 × 1 020	31 × 21	7
Spitzer	IRAC	...	0.85	1.22	256 × 256	5.2 × 5.2	8
VLT	ESPRESSO	Cerro Paranal, Chile	8.2	9

References. (1) <https://astep.oca.eu>; (2) <https://www.brierfieldobservatory.com>; (3) <https://www.cfht.hawaii.edu>; (4) <https://cheops.unibe.ch>; (5) <http://irtfweb.ifa.hawaii.edu>; (6) <https://lco.global/observatory>; (7) <http://pestobservatory.com>; (8) <https://www.spitzer.caltech.edu>; (9) <https://www.eso.org/public/teles-instr/paranal-observatory/vlt>.

detrending parameter columns from the ground-based light curves (e.g., airmass, position centroid, FWHM, etc.). We use these detrending parameter columns for EXOFASTv2 modeling and extraction of midpoint times to assess the impact of systematic trends in the ground-based light curves on the modeled transit midpoint time posterior distributions (Section 3). The choice of detrending parameters were modified for some ground transit observations from Wittrock et al. (2022) to improve signal rms and minimize the uncertainty in the timing of the transits; see Table 3 for a complete list of detrending parameters applied to each transit observation for this paper.

2.2.1. Antarctic Search for Transiting Exoplanets (FLI Proline 16800E) Photometry

ASTEP 400, part of the Antarctic Search for Transiting Exoplanets (ASTEP) program and located at the Concordia Research Station, Antarctica, is a 0.4 m telescope that has been utilized for transiting exoplanet searches (Guillot et al. 2015; Mékarnia et al. 2016). The data collected with ASTEP 400 were processed on site with an IDL-based aperture photometry pipeline (Abe et al. 2013; Mékarnia et al. 2016). Although five AU Mic transit observations were made with ASTEP 400, the photometric conditions during four of those nights were of suboptimal quality, so only the first transit observation of AU Mic b is included in this paper.

2.2.2. Las Cumbres Observatory Global Telescope (Sinistro) Photometry

The Las Cumbres Observatory Global Telescope (LCOGT; Brown et al. 2013) network participated in collecting the transits of the AU Mic systems through four different 1.0 m LCO Ritchey–Chrétien Cassegrain telescopes equipped with Sinistro;³⁵ Pan-STARRS z_s was used with an exposure time of 15 s for all new LCO observations.

The third night from LCO CTIO was impacted by poor sky conditions. The third transit observation from LCO SAAO was affected by intermittent clouds. Additionally, the 2020 October 11 night from LCO SAAO fortuitously observed the egress of planet c while intending to observe the transit of planet b; this transit was not included in the previous analysis of Wittrock et al. (2022) but is now included in this work. The first night from LCO SSO was impacted by poor sky conditions and tracking. All LCOGT light curves have been reduced and detrended with AIJ, then a subset table generated from each light curve.

2.2.3. Mount Kent Observatory CDK700 (U16) Photometry

The light curve from the Mount Kent Observatory CDK700 has been reduced and detrended with AIJ, then a subset table generated from it. However, this observation was found to have missed AU Mic c’s predicted transit calculated by the EXOFASTv2 package. Since we are employing the `reject-flatmodel` option to improve the convergence of our transit models, this set is dropped from the analysis (see Section 3 regarding the use of `rejectflatmodel`).

3. EXOFASTv2 Transit Modeling

We model the 33 photometric transits of AU Mic b and five photometric transits of AU Mic c using the EXOFASTv2 package (Eastman et al. 2013, 2019); CHEOPS and R-M observations are not included in this transit modeling step. EXOFASTv2 utilizes the Markov chain Monte Carlo (MCMC) technique to estimate the posterior probabilities and determine the statistical significance of our detections and the confidence intervals in their corresponding transit midpoint times. Since EXOFASTv2 is written in IDL, it uses the differential evolution MCMC algorithm (Ter Braak 2006) instead of `emcee` (Foreman-Mackey et al. 2013) for sampling purposes (Eastman et al. 2013). The detrending parameters flare (Spitzer), sky (Spitzer and PEST), Sky/Pixel_T1 (LCOGT), and SKY (ASTEP) are treated as additive, while the remaining detrending parameters are treated as multiplicative. See Table 3

³⁵ <https://lco.global/observatory>

Table 3
Detrending Parameters Incorporated into EXOFASTv2 Modeling of AU Mic b Transits

Telescope	Date (UT)	Filter	Detrending Parameter(s)	Note
TESS	2018-07-26	TESS	...	a
TESS	2018-08-11	TESS	...	a
TESS	2018-08-12	TESS	...	a
Spitzer	2019-02-10	4.5 μm	$x, y, \text{noise/pixel}, \text{FWHM}_x, \text{FWHM}_y, \text{sky}, \text{linear}, \text{quadratic}$	b
Spitzer	2019-02-27	4.5 μm	$x, y, \text{noise/pixel}, \text{FWHM}_x, \text{FWHM}_y, \text{sky}, \text{linear}, \text{quadratic}, \text{flare}$	b
Spitzer	2019-09-09	4.5 μm	$x, y, \text{noise/pixel}, \text{FWHM}_x, \text{FWHM}_y, \text{sky}, \text{linear}, \text{quadratic}, \text{Gaussian}$	b
LCO SSO	2020-04-25	z'	AIRMASS	c
LCO SSO	2020-04-25	y	AIRMASS	c
LCO SAAO	2020-05-20	z'	AIRMASS	c
LCO SAAO	2020-05-20	y	AIRMASS	c
LCO SAAO	2020-06-06	z'	AIRMASS	c
LCO SAAO	2020-06-23	z'	AIRMASS, Width_T1	c
TESS	2020-07-09	TESS	...	a
TESS	2020-07-10	TESS	...	a
PEST	2020-07-10	V	comp_flux, dist_center, fwhm, airmass, sky	d
TESS	2020-07-19	TESS	...	a
TESS	2020-07-27	TESS	...	a
TESS	2020-07-28	TESS	...	a
Brierfield	2020-08-13	I	Meridian_Flip, tot_C_cnts, X(FITS)_T1, Y(FITS)_T1	c
LCO SSO	2020-08-13	z'	AIRMASS, Width_T1	c
LCO SAAO	2020-09-07	z'	Sky/Pixel_T1, Width_T1	c
LCO SSO	2020-09-16	z'	AIRMASS, Width_T1	c
LCO SSO	2020-10-03	z'	Sky/Pixel_T1, Width_T1	c
LCO SAAO	2020-10-11	z'	AIRMASS	c
LCO CTIO	2021-06-14	z'	tot_C_cnts, Sky/Pixel_T1, Width_T1	c
LCO SSO	2021-06-22	z'	tot_C_cnts, Y(FITS)_T1, Width_T1	c
LCO CTIO	2021-07-01	z'	tot_C_cnts, Sky/Pixel_T1, Width_T1	c
ASTEP	2021-07-09	R	SKY	e
LCO SAAO	2021-07-18	z'	AIRMASS, Sky/Pixel_T1	c
LCO SAAO	2021-08-03	z'	X(FITS)_T1, Y(FITS)_T1, Sky/Pixel_T1	c
LCO CTIO	2021-08-04	z'	AIRMASS	c
LCO SSO	2021-08-12	z'	Y(FITS)_T1, Sky/Pixel_T1, Width_T1	c
LCO SSO	2021-08-29	z'	tot_C_cnts, X(FITS)_T1, Width_T1	c
LCO SSO	2021-09-15	z'	AIRMASS, X(FITS)_T1, Y(FITS)_T1	c
LCO SAAO	2021-09-23	z'	Sky/Pixel_T1, Width_T1	c
LCO TO	2021-09-23	z'	tot_C_cnts, Width_T1	c
LCO CTIO	2021-10-04	z'	AIRMASS, X(FITS)_T1, Width_T1	c

Notes. The flare (Spitzer), sky (Spitzer and PEST), Sky/Pixel_T1 (LCOGT), and SKY (ASTEP) were implemented as additives; the remaining detrending parameters were implemented as multiplicative. See Section 2 for details on detrending parameters used for each observation. Since both Pan-STARRS Y and Pan-STARRS z_s are not available among the filters in EXOFASTv2, y and z' (Sloan z) were used as respective approximate substitutes. (a) See Gilbert et al. (2022) for details on the detrending parameters applied to TESS data. (b) See Wittrock et al. (2022) for details on the detrending parameters applied to Spitzer data. (c) Detrending parameters generated from AIJ (Collins et al. 2017). (d) Detrending parameters generated from the PEST pipeline (<http://pestobservatory.com/the-pest-pipeline>). (e) Detrending parameters generated from IDL-based aperture photometry pipeline (Abe et al. 2013; Mékarnia et al. 2016).

for a full list of nights included for EXOFASTv2 analysis and their corresponding detrending parameters. Since both Pan-STARRS Y and Pan-STARRS z_s are not available among the filters in EXOFASTv2, y and z' (Sloan z) were used as respective approximate substitutes.

We use MIST for evolutionary models (Choi et al. 2016; Dotter 2016) and have EXOFASTv2 ignore the Claret and Bloemen limb-darkening tables (Claret & Bloemen 2011) since AU Mic is a low-mass red dwarf. Table 4 provides a list of priors for EXOFASTv2. The logarithmic version of stellar mass and orbital period were used because they are the fitted priors in EXOFASTv2. The purpose of TTVs and depth offset priors are to place constraints on the variation of transit timing and depth of all light curves; any transit depth variability was not explored for this paper. We set MAXSTEPS = 7500 and NTHIN = 25 and include the rejectflatmodel option for all light curves with NTEMPS = 8 to aid in faster

convergence. The reason for allowing the rejectflatmodel option to become active for this work is that both AU Mic b and c are confirmed transiting planets with well-established orbital periods, so we “reject” any flat models to help narrow the vast range of possible outcomes. After EXOFASTv2 completes the transit modeling, it generates the transit models (Figure 1), median posteriors (Tables 5, 6, 7, and 8), and midpoint times (Table 11).

4. Exo-Striker Transit Timing Variation Modeling

In this section, we present our $O-C$ diagram from EXOFASTv2, the super-period of AU Mic b’s TTVs, and the TTV dynamical modeling with Exo-Striker. We explore three different scenarios: a system with two planets, b and c; a system with three planets, where planet d is interior to b; and a system with three planets, where planet d is between b and c.

Table 4
Stellar, Planetary, and Transit Priors for EXOFASTv2 Modeling

Prior	Unit	Input		References
		AU Mic b	AU Mic c	
$\log_{10}\left(\frac{M_*}{M_{\odot}}\right)$...	$\mathcal{N}(-0.301, 0.026)$		1
R_*	R_{\odot}	$\mathcal{N}(0.75, 0.03)$		2
T_{eff}	K	$\mathcal{N}(3700, 100)$		3
Age	Gyr	$\mathcal{N}(0.022, 0.003)$		4
T_C	BJD_TDB	$\mathcal{N}(2458330.39080, 0.00058)$	$\mathcal{N}(2458342.2239, 0.0019)$	5
$\log_{10}\left(\frac{\text{Period}}{\text{days}}\right)$...	$\mathcal{N}(0.92752436, 0.00000031)$	$\mathcal{N}(1.2755182, 0.0000012)$	5
R_p/R_*	...	$\mathcal{N}(0.0512, 0.0020)$	$\mathcal{N}(0.0340, 0.0034)$	5
TTV offset	days	$\mathcal{U}(-0.02, 0.02)$...
Depth offset	...	$\mathcal{U}(-0.01, 0.01)$...

Notes. \mathcal{N} denotes the Gaussian priors, and \mathcal{U} denotes the uniform priors. TTVs and depth offsets are arbitrary and applied as constraints to all transits. The logarithmic version of stellar mass and orbital period were used because they are the fitted priors in EXOFASTv2. The equivalent evolutionary point (EEP) was set to 1 but is allowed to float freely, so it is not included in the prior table above.

References. (1) Plavchan et al. (2020); (2) White et al. (2019); (3) Plavchan et al. (2009); (4) Mamajek & Bell (2014); (5) Gilbert et al. (2022)

Additionally, we develop a novel technique, a TTV log-likelihood periodogram, which aids us in exploring the TTV parameters that maximize the log-likelihood.

4.1. O–C Diagram and Transit Timing Variation Super-period

In the previous section, we ran the EXOFASTv2 package to model the transits and generate the observed midpoint times (Table 11). CHEOPS and R-M data were not included in transit modeling, but CHEOPS midpoint times from Szabó et al. (2021, 2022) and R-M midpoints times from Martioli et al. (2020) and Palle et al. (2020) are now added to the list of midpoint times generated from EXOFASTv2. We calculate the expected midpoint times using the planets’ EXOFASTv2-generated periods and T_C . Then, we use both the calculated and the observed midpoint times to create the O–C diagram of both AU Mic b and c (Figure 2).

Relative to the space-based transit midpoint times, most ground-based photometric transits have larger timing uncertainties and some larger scatter. Some of the notable outliers in transit midpoint time of AU Mic b (regardless of timing uncertainty) include ESPRESSO, PEST 0.3 m, Brierfield 0.36 m, ASTEP 0.4 m, and at least one each of LCO CTIO, SAAO, and SSO 1.0 m. Many of AU Mic b’s transits observed in 2021 are considerably later (~ 20 minutes) than those observed from 2020 given the previously measured ephemerides.

AU Mic b’s TTVs appear to have a quasi-sinusoidal pattern. This indicates that there is potentially a “super-period” embedded in the TTV observations. To model the super-period, we construct a sinusoidal model that includes a linear trend to account for the apparent drift in the TTVs:

$$y(t) = A \sin(Bt + C) + Dt + E, \quad (1)$$

where t is the time since the first transit of b, and A , B , C , D , and E are the unknown coefficients. The coefficients are optimized using the `scipy.optimize.curve_fit`³⁶ (Table 9), and we use Equation (1) to model the super-period onto AU Mic b’s O–C diagram (Figure 3).

We can use that super-period to estimate the period of AU Mic d using Equation (5) from Lithwick et al. (2012):

$$P^j \equiv \frac{1}{|j/P' - (j-1)/P|}, \quad (2)$$

where j is an integer that represents the j : $j-1$ mean-motion orbital resonant (MMR) chain, P is the orbital period of the inner planet, P' is the orbital period of the outer planet, and P^j is the super-period of the TTVs. Since the 3:2 period ratio is the most common pairing for the MMR (Fabrycky et al. 2014), we assume $j=3$ and will not consider other j values for this problem. From the coefficient B , we obtain the super-period $P^3 = 1186.44931 \pm 153.24946$ days. From Equation (2), we end up with the estimated periods $P_d = 5.62927 \pm 0.00177$ days or 5.65482 ± 0.00178 days (assuming AU Mic d is orbiting interior to AU Mic b), and $P_d = 12.65156 \pm 0.00594$ days or 12.73779 ± 0.00603 days (assuming AU Mic d is orbiting between AU Mic b and c); the absolute value brackets in the denominator of Equation (2) allow us to have two potential solutions for both the inner d and middle d scenarios. Additionally, we note a statistically significant (3.2σ) nonzero linear drift of 0.03052 ± 0.00964 min day⁻¹ in the TTVs of AU Mic b.

This approach, in using AU Mic b’s TTV super-period, is meant to provide a starting point for estimating the orbital period of AU Mic d that could drive the observed excess TTVs of AU Mic b as reported by Wittrock et al. (2022). Additionally, numerous compact systems have planets in near-MMR chains—e.g., HD 158259 (Hara et al. 2020), TRAPPIST-1 (Gillon et al. 2016, 2017; Luger et al. 2017), V1298 Tau (David et al. 2019a), and several Kepler systems (Lissauer et al. 2011; Fabrycky et al. 2014)—and most Kepler systems with measured TTVs have planets in near-MMR chains (Lithwick et al. 2012; also see Steffen et al. 2013 for example), so it is not unreasonable or unwarranted to assume that AU Mic d might be near-MMR pairing with b and with c.

Wittrock et al. (2022) generated the massless no-TTV two-planet model with `Exo-Striker` as a control test on the presence and statistical significance of TTVs, but it also serves as

³⁶ https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.curve_fit.html

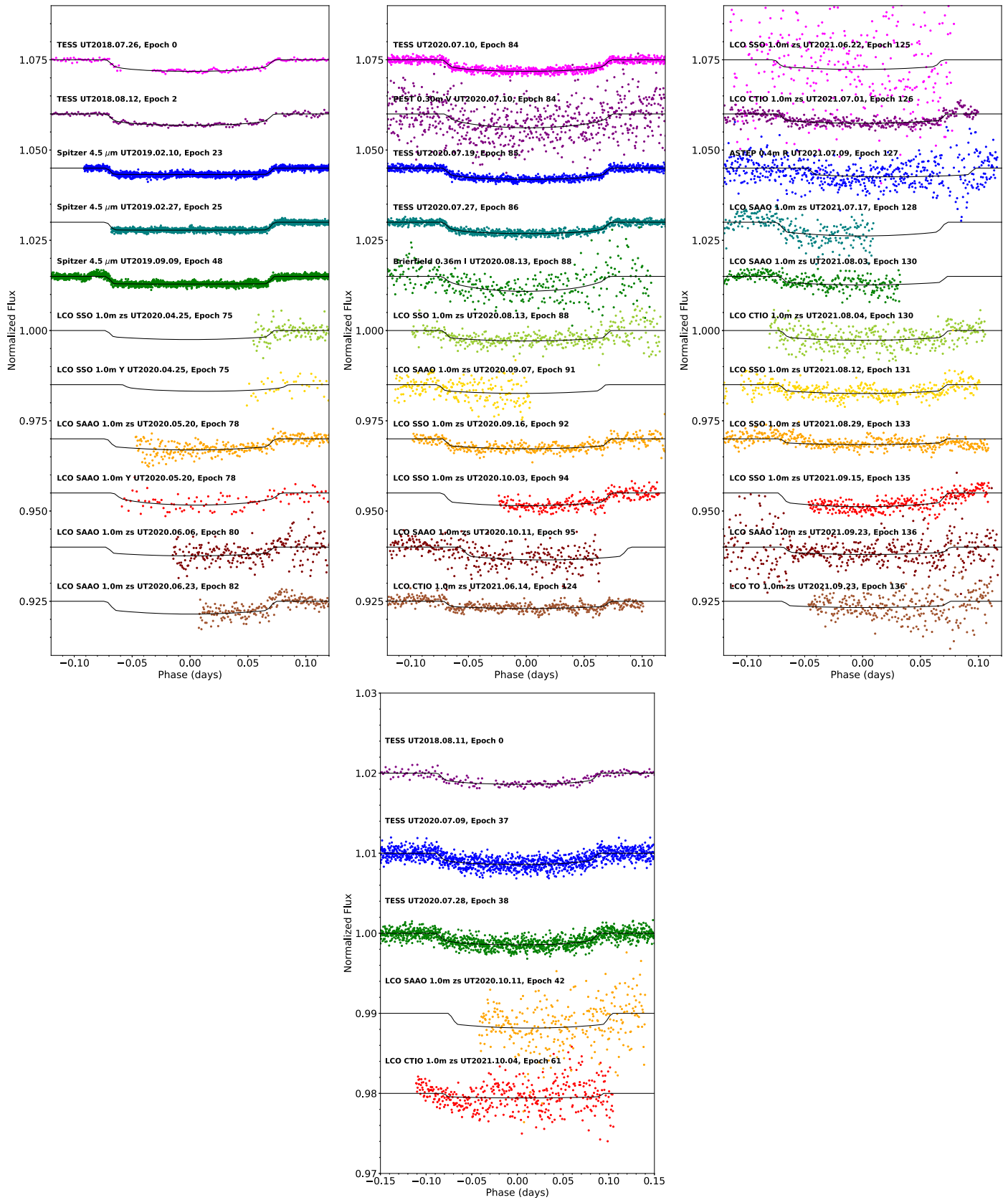


Figure 1. Comparison between ground-based + Spitzer + TESS transits (multicolors) and EXOFASTv2’s best-fit model (black) for AU Mic b (first three columns in the top row) and c (last column in the bottom row). Each transit is labeled with the name of telescope, the date of observation in UT, and the epoch, which refers to the number of transits since the first transit of b and c, respectively.

Table 5
EXOFASTv2-generated Median Values and 68% Confidence Interval for AU Mic System

Posterior	Description	Unit	Output	
			AU Mic b	AU Mic c
M_*	Stellar mass	M_\odot	0.510 $^{+0.028}_{-0.027}$	
R_*	Stellar radius	R_\odot	0.744 $^{+0.023}_{-0.021}$	
L_*	Stellar luminosity	L_\odot	0.0916 $^{+0.011}_{-0.0098}$	
ρ_*	Stellar density	g cm^{-3}	1.75 $^{+0.14}_{-0.16}$	
$\log g$	Surface gravity	$\log_{10}(\text{g cm}^{-3})$	4.404 $^{+0.026}_{-0.031}$	
T_{eff}	Effective temperature	K	3678 $^{+90}_{-88}$	
[Fe/H]	Metallicity	dex	0.23 $^{+0.24}_{-0.30}$	
[Fe/H] ₀	Initial metallicity ^a	dex	0.17 $^{+0.22}_{-0.28}$	
Age	...	Gyr	0.0201 $^{+0.0025}_{-0.0024}$	
EEP	Equal evolutionary phase ^b	...	162.0 \pm 2.9	
P_{orb}	Orbital period	days	8.4630177 $^{+0.000052}_{-0.000050}$	18.858970 $^{+0.000051}_{-0.000052}$
M_p	Planetary mass ^c	M_J	0.053 $^{+0.019}_{-0.012}$	
R_p	Planetary radius	R_J	0.353 \pm 0.013	
T_C	Time of conjunction ^d	BJD_TDB	2458330.39168 \pm 0.00053	2458342.2240 $^{+0.0019}_{-0.0018}$
T_T	Time of minimum projected separation ^e	BJD_TDB	2458330.39169 \pm 0.00053	2458342.2240 $^{+0.0019}_{-0.0018}$
T_0	Optimal conjunction time ^f	BJD_TDB	2458525.04109 $^{+0.00052}_{-0.00051}$	2458342.2240 $^{+0.0019}_{-0.0018}$
a	Semimajor axis	au	0.0649 \pm 0.0012	
e	Eccentricity	...	0.081 $^{+0.17}_{-0.058}$	
i	Inclination	deg	89.57 $^{+0.28}_{-0.31}$	
ω	Argument of periastron	deg	-202 $^{+44}_{-120}$	
T_{eq}	Equilibrium temperature ^g	K	600 $^{+17}_{-16}$	
τ_{circ}	Tidal circularization timescale	Gyr	161 \pm 76	
K	RV semi-amplitude ^c	m s^{-1}	8.4 $^{+3.0}_{-1.9}$	
R_p/R_*	0.0488 \pm 0.0010	
a/R_*	18.79 $^{+0.50}_{-0.59}$	
δ	Transit depth (R_p/R_*) ²	...	0.002379 $^{+0.000100}_{-0.000099}$	
δ_I	Transit depth in I	...	0.0039 $^{+0.0025}_{-0.0011}$	
δ_R	Transit depth in R	...	0.0039 $^{+0.0031}_{-0.0012}$	
$\delta_{z'}$	Transit depth in z'	...	0.00310 \pm 0.00023	
$\delta_{4.5\mu\text{m}}$	Transit depth in 4.5 μm	...	0.00239 \pm 0.00010	
δ_{TESS}	Transit depth in TESS	...	0.00308 \pm 0.00017	
δ_V	Transit depth in V	...	0.00344 $^{+0.0012}_{-0.00057}$	
δ_y	Transit depth in y	...	0.0038 $^{+0.0027}_{-0.0010}$	
τ	Ingress/egress transit duration	days	0.00691 $^{+0.00028}_{-0.00020}$	
T_{14}	Total transit duration	days	0.14553 $^{+0.00031}_{-0.00028}$	
T_{FWHM}	FWHM transit duration	days	0.13859 \pm 0.00019	
b	Transit impact parameter	...	0.134 $^{+0.096}_{-0.089}$	
b_S	Eclipse impact parameter	...	0.136 $^{+0.095}_{-0.090}$	
τ_S	Ingress/egress eclipse duration	days	0.00711 $^{+0.00052}_{-0.00054}$	
$T_{S,14}$	Total eclipse duration	days	0.1486 $^{+0.0099}_{-0.010}$	
$T_{S,\text{FWHM}}$	FWHM eclipse duration	days	0.1415 $^{+0.0095}_{-0.0097}$	
$\delta_{S,2.5\mu\text{m}}$	Blackbody eclipse depth at 2.5 μm	ppm	0.62 $^{+0.15}_{-0.13}$	
$\delta_{S,5.0\mu\text{m}}$	Blackbody eclipse depth at 5.0 μm	ppm	23.6 $^{+2.8}_{-2.5}$	
$\delta_{S,7.5\mu\text{m}}$	Blackbody eclipse depth at 7.5 μm	ppm	69.6 $^{+5.6}_{-5.2}$	
ρ_p	Planetary density ^c	g cm^{-3}	1.48 $^{+0.52}_{-0.34}$	
$\log g_p$	Surface gravity ^c	...	3.02 $^{+0.13}_{-0.11}$	
Θ	Safronov number	...	0.0380 $^{+0.013}_{-0.0085}$	
$\langle F \rangle$	Incident flux	$10^9 \text{ erg s}^{-1} \text{ cm}^{-2}$	0.0288 $^{+0.0034}_{-0.0033}$	
T_P	Time of periastron	BJD_TDB	2458322.77 $^{+0.83}_{-2.3}$	
T_S	Time of eclipse	BJD_TDB	2458334.56 $^{+0.52}_{-0.93}$	
T_A	Time of ascending node	BJD_TDB	2458328.28 $^{+0.29}_{-0.47}$	
T_D	Time of descending node	BJD_TDB	2458332.42 $^{+0.26}_{-0.44}$	
V_c/V_e	i	...	0.980 $^{+0.022}_{-0.033}$	
$\sqrt{\left(1 - \frac{R_p}{R_*}\right)^2 - b^2}$	Transit chord	...	1.0401 $^{+0.0074}_{-0.017}$	
sign	j	...	0.51 $^{+0.41}_{-0.26}$	
$e \cos \omega$	-0.011 $^{+0.097}_{-0.17}$	

Table 5
(Continued)

Posterior	Description	Unit	Output	
			AU Mic b	AU Mic c
$e \sin \omega$	$0.010^{+0.033}_{-0.036}$	$0.030^{+0.047}_{-0.037}$
$M_p \sin i$	Minimum mass ^c	M_J	$0.053^{+0.019}_{-0.012}$	$0.0247^{+0.010}_{-0.0065}$
M_p/M_*	Mass ratio ^c	...	$0.000099^{+0.000036}_{-0.000023}$	$0.000046^{+0.000019}_{-0.000012}$
d/R_*	Separation at mid-transit	...	$18.19^{+0.96}_{-1.3}$	$30.5^{+1.8}_{-2.0}$
P_T	A priori nongrazing transit probability	...	$0.0523^{+0.0042}_{-0.0026}$	$0.0317^{+0.0023}_{-0.0018}$
$P_{T,G}$	A priori transit probability	...	$0.0577^{+0.0046}_{-0.0029}$	$0.0338^{+0.0024}_{-0.0019}$
P_S	A priori nongrazing eclipse probability	...	$0.05024^{+0.0045}_{-0.00099}$	$0.0297^{+0.0021}_{-0.0014}$
$P_{S,G}$	A priori eclipse probability	...	$0.0554^{+0.0050}_{-0.0011}$	$0.0316^{+0.0023}_{-0.0015}$
$u_{1,I}$	Linear limb-darkening coefficient in I	...		$0.80^{+0.47}_{-0.48}$
$u_{1,R}$	Linear limb-darkening coefficient in R	...		$0.80^{+0.53}_{-0.56}$
$u_{1,z'}$	Linear limb-darkening coefficient in z'	...		$0.477^{+0.094}_{-0.10}$
$u_{1,4.5\mu m}$	Linear limb-darkening coefficient in $4.5 \mu m$...		$0.0076^{+0.012}_{-0.0057}$
$u_{1,TESS}$	Linear limb-darkening coefficient in TESS	...		$0.467^{+0.057}_{-0.058}$
$u_{1,V}$	Linear limb-darkening coefficient in V	...		$0.63^{+0.46}_{-0.42}$
$u_{1,y}$	Linear limb-darkening coefficient in y	...		$0.76^{+0.52}_{-0.47}$
$u_{2,I}$	Quadratic limb-darkening coefficient in I	...		$-0.10^{+0.49}_{-0.40}$
$u_{2,R}$	Quadratic limb-darkening coefficient in R	...		$-0.13^{+0.41}_{-0.40}$
$u_{2,z'}$	Quadratic limb-darkening coefficient in z'	...		$-0.174^{+0.12}_{-0.078}$
$u_{2,4.5\mu m}$	Quadratic limb-darkening coefficient in $4.5 \mu m$...		0.067 ± 0.038
$u_{2,TESS}$	Quadratic limb-darkening coefficient in TESS	...		$0.186^{+0.092}_{-0.091}$
$u_{2,V}$	Quadratic limb-darkening coefficient in V	...		$0.15^{+0.45}_{-0.47}$
$u_{2,y}$	Quadratic limb-darkening coefficient in y	...		$-0.08^{+0.47}_{-0.41}$

Notes. See Table 3 in Eastman et al. (2019) for a detailed description of all parameters. Since both Pan-STARRS Y and Pan-STARRS z_s are not available among the filters in EXOFASTv2, y and z' (Sloan z) were used as respective approximate substitutes. Additionally, the Claret and Bloemen limb-darkening tables (Claret & Bloemen 2011) default option has been disabled since AU Mic is a low-mass red dwarf.

^a The metallicity of the star at birth.

^b Corresponds to static points in a star’s evolutionary history. See Section 2 in Dotter (2016).

^c Uses measured radius and estimated mass from Chen & Kipping (2017).

^d Time of conjunction is commonly reported as the “transit time.”

^e Time of minimum projected separation is a more correct “transit time.”

^f Optimal time of conjunction minimizes the covariance between T_C and period.

^g Assumes no albedo and perfect redistribution.

^h Depends on the tidal Q factor.

ⁱ The velocity at T_C of an assumed circular orbit divided by the velocity of the modeled eccentric orbit.

^j The sign of the solution to the quadratic mapping from V_c/V_e to e .

a useful snapshot on whether AU Mic b’s TTVs are behaving linearly over time. The massless planets’ $O-C$ diagram (Figure 5 from Wittrock et al. 2022) and the statistical comparison between the massless two-planet model and the non-massless two-planet model—e.g., reduced chi-square $\chi^2_{\text{red}} = 8.7 \times 10^8$ versus 38, respectively, and log-likelihood $\ln \mathcal{L} = -6.1 \times 10^9$ versus -75 , respectively (Wittrock et al. 2022)—show very clear indications that a linear trend does not fit very well with AU Mic b’s observed TTVs.

In the meanwhile, given the relatively sparse observations of AU Mic c, we cannot draw any meaningful interpretation of what c’s TTV super-period could be. Thus, the potential super-period in AU Mic c’s observed TTVs is not explored in this paper.

4.2. Exo-Striker Dynamical Modeling Preparations and Processes

The $O-C$ diagram from Wittrock et al. (2022) displayed the apparent deviation of the TTVs from a linear ephemeris;

this was believed to be attributed to the yet-to-be-confirmed planet AU Mic d even after accounting for the impacts from stellar activity, such as flaring and spot crossings. With the new data from Section 2 added to this work, we model the TTVs of AU Mic b and c using the Exo-Striker package (Trifonov 2019), with our focus now on validating AU Mic d. Exo-Striker is capable of applying MCMC via emcee (Foreman-Mackey et al. 2013) to determine the statistical significance of our TTV measurements and the confidence in their corresponding dynamical model posteriors.

For our Exo-Striker modeling, we incorporate the priors for the host star and the planets from Table 10 and the midpoint time priors from Table 11. Exo-Striker uses only the Simplex algorithm for any TTV models. We use the N -body algorithm for all model fittings and MCMC runs, and we set the dynamical model time steps to 0.01 days. For our model fittings, we use the following scipy minimizer algorithms: the truncated Newton algorithm³⁷ as a primary minimizer and the

³⁷ <https://docs.scipy.org/doc/scipy/reference/optimize.minimize-tn.html>

Table 6
EXOFASTv2-generated Median Values and 68% Confidence Interval for Follow-up Observations of AU Mic Transits (Part I)

Planet	Telescope	Date (UT)	Filter	σ^2 (Added Variance)	TTV ^a (days)	$T\delta V^b$	F_0^c
b	TESS	2018-07-26	TESS	0.000000010 ^{+0.000000012} _{-0.000000011}	-0.00242 ^{+0.00065} _{-0.00066}	0.0020 ± 0.0011	0.999994 ± 0.000030
c	TESS	2018-08-11	TESS	0.000000006 ^{+0.000000012} _{-0.000000010}	-0.0007 ± 0.0020	0.0030 ± 0.0029	0.999992 ± 0.000030
b	TESS	2018-08-12	TESS	0.000000003 ^{+0.000000017} _{-0.000000015}	-0.00063 ± 0.00075	0.0021 ± 0.0011	1.000012 ± 0.000039
b	Spitzer	2019-02-10	4.5 μm	0.0000000700 ^{+0.000000056} _{-0.000000054}	0.00336 ^{+0.00053} _{-0.00054}	-0.0063 ± 0.0012	1.000028 ^{+0.00034} _{-0.00033}
b	Spitzer	2019-02-27	4.5 μm	0.0000000384 ^{+0.000000043} _{-0.000000041}	0.00413 ^{+0.00052} _{-0.00054}	-0.0024 ± 0.0011	1.000312 ± 0.000027
b	Spitzer	2019-09-09	4.5 μm	0.0000001237 ^{+0.000000051} _{-0.000000058}	0.00082 ^{+0.00054} _{-0.00053}	-0.0025 ± 0.0012	0.999765 ± 0.000027
b	LCO SSO	2020-04-25	z'	0.00000275 ^{+0.00000033} _{-0.00000029}	-0.0047 ^{+0.0016} _{-0.0014}	-0.0033 ^{+0.0054} _{-0.0044}	0.99999 ± 0.00014
b	LCO SSO	2020-04-25	y	0.00000066 ^{+0.00000013} _{-0.00000093}	0.003 ± 0.010	0.0005 ^{+0.0065} _{-0.0070}	1.00009 ^{+0.00048} _{-0.00044}
b	LCO SAAO	2020-05-20	z'	0.00000188 ^{+0.00000017} _{-0.00000016}	-0.0048 ^{+0.0018} _{-0.0016}	0.0040 ^{+0.0023} _{-0.0024}	1.00034 ± 0.00017
b	LCO SAAO	2020-05-20	y	0.00000141 ^{+0.00000054} _{-0.00000043}	0.0088 ^{+0.0065} _{-0.0041}	0.0005 ^{+0.0051} _{-0.0053}	0.99984 ^{+0.00038} _{-0.00037}
b	LCO SAAO	2020-06-06	z'	0.00000673 ^{+0.00000069} _{-0.00000062}	-0.0079 ± 0.0032	0.0000 ^{+0.0053} _{-0.0055}	0.99995 ^{+0.00035} _{-0.00034}
b	LCO SAAO	2020-06-23	z'	0.00000189 ^{+0.00000024} _{-0.00000021}	-0.0019 ± 0.0014	0.0075 ^{+0.0018} _{-0.0029}	0.99982 ^{+0.00016} _{-0.00019}
c	TESS	2020-07-09	TESS	-0.000000003 ^{+0.000000016} _{-0.000000015}	-0.0011 ^{+0.0029} _{-0.0026}	0.0034 ± 0.0029	1.000009 ± 0.000024
b	TESS	2020-07-10	TESS	0.000000005 ^{+0.000000014} _{-0.000000013}	-0.00393 ^{+0.00063} _{-0.00067}	0.0020 ± 0.0011	1.000002 ± 0.000021
b	PEST	2020-07-10	V	0.0000163 ^{+0.00000013} _{-0.00000012}	-0.0116 ^{+0.0033} _{-0.0034}	0.00879 ^{+0.0020} _{-0.0020}	0.99931 ^{+0.00020} _{-0.00021}
b	TESS	2020-07-19	TESS	-0.000000005 ^{+0.000000012} _{-0.000000011}	-0.00326 ^{+0.00064} _{-0.00066}	0.0020 ± 0.0011	0.999998 ± 0.000020
b	TESS	2020-07-27	TESS	-0.000000001 ^{+0.000000012} _{-0.000000011}	-0.00217 ^{+0.00063} _{-0.00066}	0.0019 ^{+0.0011} _{-0.0010}	0.999994 ± 0.000020
c	TESS	2020-07-28	TESS	0.000000010 ^{+0.000000014} _{-0.000000013}	0.0008 ^{+0.0029} _{-0.0026}	0.0033 ± 0.0029	1.000004 ± 0.000022
b	Brierfield	2020-08-13	I	0.0000171 ^{+0.00000015} _{-0.00000013}	0.0074 ^{+0.0039} _{-0.0039}	0.0047 ^{+0.0035} _{-0.0035}	0.99996 ^{+0.00031} _{-0.00034}
b	LCO SSO	2020-08-13	z'	0.00000406 ^{+0.00000035} _{-0.00000032}	-0.0035 ^{+0.0018} _{-0.0021}	0.0013 ^{+0.0034} _{-0.0035}	1.00003 ^{+0.00026} _{-0.00025}
b	LCO SAAO	2020-09-07	z'	0.00000688 ^{+0.00000077} _{-0.00000067}	0.0060 ^{+0.0090} _{-0.0072}	-0.0038 ^{+0.0049} _{-0.0040}	0.99972 ^{+0.00043} _{-0.00034}
b	LCO SSO	2020-09-16	z'	0.000001191 ^{+0.00000011} _{-0.000000100}	-0.00314 ^{+0.00093} _{-0.00097}	-0.0009 ± 0.0021	1.00000 ± 0.00013
b	LCO SSO	2020-10-03	z'	0.00000125 ^{+0.00000016} _{-0.00000014}	-0.0013 ± 0.0011	0.0071 ^{+0.0017} _{-0.0021}	1.00225 ^{+0.00016} _{-0.00017}
b and c	LCO SAAO	2020-10-11	z'	0.00000613 ^{+0.00000063} _{-0.00000056}	-0.002 ^{+0.017} _{-0.013}	0.0044 ^{+0.0031} _{-0.0036}	1.00301 ^{+0.00045} _{-0.00048}
b	LCO CTIO	2021-06-14	z'	0.000000740 ^{+0.000000075} _{-0.000000068}	0.01230 ^{+0.00099} _{-0.0010}	-0.0076 ^{+0.0018} _{-0.0015}	1.000026 ^{+0.00092} _{-0.00086}
b	LCO SSO	2021-06-22	z'	0.0001177 ^{+0.000010} _{-0.0000095}	-0.0086 ^{+0.013} _{-0.0070}	-0.0012 ^{+0.0070} _{-0.0060}	0.99832 ^{+0.00076} _{-0.00074}
b	LCO CTIO	2021-07-01	z'	0.000000980 ^{+0.000000089} _{-0.000000082}	0.0139 ^{+0.0011} _{-0.0010}	0.0009 ± 0.0019	1.00003 ^{+0.00012} _{-0.00011}
b	ASTEP	2021-07-09	R	0.00001316 ^{+0.00000087} _{-0.00000081}	-0.0130 ^{+0.020} _{-0.0045}	-0.0050 ^{+0.0036} _{-0.0030}	0.99998 ^{+0.00021} _{-0.00020}
b	LCO SAAO	2021-07-17	z'	0.00000535 ^{+0.00000054} _{-0.00000049}	0.0131 ^{+0.0022} _{-0.0018}	0.0087 ^{+0.0010} _{-0.0020}	0.99904 ^{+0.00020} _{-0.00021}
b	LCO SAAO	2021-08-03	z'	0.00000215 ^{+0.00000022} _{-0.00000020}	0.0179 ^{+0.0014} _{-0.0018}	-0.0039 ^{+0.0045} _{-0.0039}	0.99990 ^{+0.00028} _{-0.00023}
b	LCO CTIO	2021-08-04	z'	0.00000574 ^{+0.00000047} _{-0.00000043}	0.0078 ^{+0.0024} _{-0.0030}	0.0017 ^{+0.0030} _{-0.0031}	1.00031 ± 0.00026
b	LCO SSO	2021-08-12	z'	0.00000264 ^{+0.00000024} _{-0.00000022}	0.0132 ^{+0.0015} _{-0.0016}	-0.0029 ^{+0.0032} _{-0.0033}	0.99992 ^{+0.00021} _{-0.00020}
b	LCO SSO	2021-08-29	z'	0.00000197 ^{+0.00000016} _{-0.00000015}	0.01952 ^{+0.00075} _{-0.00072}	-0.00973 ^{+0.00042} _{-0.00020}	0.999574 ^{+0.00088} _{-0.00087}
b	LCO SSO	2021-09-15	z'	0.00000194 ^{+0.00000020} _{-0.00000018}	0.01954 ^{+0.00034} _{-0.00069}	0.00914 ^{+0.0063} _{-0.0012}	0.99952 ± 0.00015
b	LCO SAAO	2021-09-23	z'	0.0000181 ^{+0.00000014} _{-0.00000013}	0.0124 ^{+0.0030} _{-0.011}	-0.0040 ^{+0.0044} _{-0.0038}	1.00029 ^{+0.00031} _{-0.00029}
b	LCO TO	2021-09-23	z'	0.0000106 ^{+0.00000012} _{-0.00000010}	0.0169 ^{+0.0022} _{-0.0035}	0.0001 ^{+0.0061} _{-0.0064}	0.99959 ^{+0.00053} _{-0.00048}
c	LCO CTIO	2021-10-04	z'	0.00000273 ^{+0.00000022} _{-0.00000020}	-0.0105 ^{+0.015} _{-0.0066}	-0.0077 ^{+0.0031} _{-0.0017}	1.00047 ^{+0.00018} _{-0.00015}

Notes. See Table 3 in Eastman et al. (2019) for a detailed description of all parameters. Since both Pan-STARRS Y and Pan-STARRS z_s are not available among the filters in EXOFASTv2, y and z' (Sloan z) were used as respective approximate substitutes.

^a Transit timing variation.

^b Transit depth variation.

^c Baseline flux.

Nelder–Mead algorithm³⁸ as a secondary minimizer, with the configurations of both minimizers set at default, including one consecutive integration and 5000 integration steps. We manually fit the model to the data using the previously mentioned minimizer algorithms, then each time we find a possible best-fit model, we proceed to perform MCMC computations by adopting MCMC parameters as best $\ln\mathcal{L}$ and with 1000 burn-in steps, 10,000 main steps for the two-planet dynamical models and 8 000 main steps for the three-planet dynamical models, and 196 walkers for the two-planet dynamical models and 441 walkers for the three-planet

dynamical models. The three aforementioned scenarios are explored and presented in the following sections.

4.3. Two-planet Dynamical Modeling

We explored a best-fit scenario for a two-planet model for the purpose of obtaining the statistical significance of TTVs and comparing the results with those of the three-planet models. We share our maximum-likelihood two-planet model with its MCMC $O-C$ diagram (Figure 4), posteriors (Table 12), and MCMC corner plot (Figure 5). The $O-C$ diagram from Figure 4 does exhibit the super-period that was obvious in Figure 3. However, the planets' inferred masses are very small ($K < 0.07 \text{ m s}^{-1}$) and their eccentricities relatively large ($e > 0.2$), neither of which are in agreement with those

³⁸ <https://docs.scipy.org/doc/scipy/reference/optimize.minimize-neldermead.html>

Table 7
EXOFASTv2-generated Median Values and 68% Confidence Interval for Follow-up Observations of AU Mic b (Part II)

Planet	Telescope	Date (UT)	Filter	C_0^a	C_1^a	M_0^b	M_1^b	M_2^b
b	TESS	2018-07-26	TESS
c	TESS	2018-08-11	TESS
b	TESS	2018-08-12	TESS
b	Spitzer	2019-02-10	4.5 μm	-0.000148 ± 0.000046	...	-0.00097 ± 0.00036	-0.0105 ± 0.0014	0.0121 ± 0.0014
b	Spitzer	2019-02-27	4.5 μm	-0.000271 ± 0.000046	0.000077 ± 0.000031	$0.00021^{+0.00040}_{-0.00041}$	0.00037 ± 0.00044	$-0.00208^{+0.00092}_{-0.00089}$
b	Spitzer	2019-09-09	4.5 μm	-0.00051 ± 0.00014	...	-0.00129 ± 0.00032	0.00280 ± 0.00027	0.0020 ± 0.0017
b	LCO SSO	2020-04-25	z'	$-0.00009^{+0.00038}_{-0.00035}$
b	LCO SSO	2020-04-25	y	$-0.00005^{+0.0010}_{-0.00099}$
b	LCO SAAO	2020-05-20	z'	0.00104 ± 0.00031
b	LCO SAAO	2020-05-20	y	$-0.00034^{+0.00070}_{-0.00073}$
b	LCO SAAO	2020-06-06	z'	$0.00000^{+0.00070}_{-0.00065}$
b	LCO SAAO	2020-06-23	z'	$-0.00032^{+0.00049}_{-0.00052}$	$-0.00007^{+0.00056}_{-0.00055}$...
c	TESS	2020-07-09	TESS
b	TESS	2020-07-10	TESS
b	PEST	2020-07-10	V	-0.0043 ± 0.0025	...	$0.00017^{+0.00076}_{-0.00074}$	0.00026 ± 0.00080	$0.0007^{+0.0012}_{-0.0011}$
b	TESS	2020-07-19	TESS
b	TESS	2020-07-27	TESS
c	TESS	2020-07-28	TESS
b	Brierfield	2020-08-13	I	0.0036 ± 0.0055	-0.0021 ± 0.0018	$0.0032^{+0.0050}_{-0.0051}$
b	LCO SSO	2020-08-13	z'	-0.00003 ± 0.00039	$0.00001^{+0.00027}_{-0.00026}$...
b	LCO SAAO	2020-09-07	z'	-0.00021 ± 0.00060	...	$-0.00056^{+0.00097}_{-0.00089}$
b	LCO SSO	2020-09-16	z'	0.00002 ± 0.00035	$0.00001^{+0.00032}_{-0.00033}$...
b	LCO SSO	2020-10-03	z'	-0.00153 ± 0.00019	...	-0.00134 ± 0.00026
b and c	LCO SAAO	2020-10-11	z'	$0.00073^{+0.00082}_{-0.0012}$
b	LCO CTIO	2021-06-14	z'	-0.00003 ± 0.00017	...	0.00002 ± 0.00037	-0.00001 ± 0.00022	...
b	LCO SSO	2021-06-22	z'	$0.0014^{+0.0020}_{-0.0019}$	0.0016 ± 0.0016	$-0.0006^{+0.0021}_{-0.0022}$
b	LCO CTIO	2021-07-01	z'	-0.00004 ± 0.00018	...	0.00002 ± 0.00049	0.00001 ± 0.00017	...
b	ASTEP	2021-07-09	R	-0.00001 ± 0.00044
b	LCO SAAO	2021-07-17	z'	$-0.00088^{+0.00065}_{-0.00067}$...	$-0.00100^{+0.00068}_{-0.00067}$
b	LCO SAAO	2021-08-03	z'	$-0.00002^{+0.00048}_{-0.00043}$...	$-0.00033^{+0.00057}_{-0.00056}$	0.00004 ± 0.00035	...
b	LCO CTIO	2021-08-04	z'	$-0.00022^{+0.00046}_{-0.00045}$
b	LCO SSO	2021-08-12	z'	$0.00032^{+0.00056}_{-0.00057}$...	-0.00000 ± 0.00023	$-0.00020^{+0.00030}_{-0.00029}$...
b	LCO SSO	2021-08-29	z'	-0.00301 ± 0.00050	0.00131 ± 0.00022	-0.00131 ± 0.00048
b	LCO SSO	2021-09-15	z'	0.00081 ± 0.00033	-0.00047 ± 0.00047	$-0.00001^{+0.00037}_{-0.00038}$
b	LCO SAAO	2021-09-23	z'	$-0.00038^{+0.00061}_{-0.00066}$...	$0.00095^{+0.00077}_{-0.00080}$
b	LCO TO	2021-09-23	z'	$0.00052^{+0.00059}_{-0.00060}$	$0.00002^{+0.00071}_{-0.00074}$...
c	LCO CTIO	2021-10-04	z'	$-0.00054^{+0.00070}_{-0.00071}$	0.00009 ± 0.00065	$-0.00070^{+0.00062}_{-0.00064}$

Notes. See Table 3 in Eastman et al. (2019) for a detailed description of all parameters. Since both Pan-STARRS Y and Pan-STARRS z_s are not available among the filters in EXOFASTv2, y and z' (Sloan z) were used as respective approximate substitutes. Additionally, the detrending parameters flare (Spitzer), sky (Spitzer and PEST), Sky/Pixel_T1 (LCOGT), and SKY (ASTEP) were set as additive while the remaining detrending parameters were set as multiplicative.

^a Additive detrending coefficient.

^b Multiplicative detrending coefficient.

Table 8
EXOFASTv2-generated Median Values and 68% Confidence Interval for Follow-up Observations of AU Mic b (Part III)

Planet	Telescope	Date (UT)	Filter	M_3^a	M_4^a	M_5^a	M_6^a	M_7^a
b	TESS	2018-07-26	TESS
c	TESS	2018-08-11	TESS
b	TESS	2018-08-12	TESS
b	Spitzer	2019-02-10	4.5 μm	-0.00618 ± 0.00092	0.00376 ± 0.00064	-0.000281 ± 0.000045	$-0.000316_{-0.000061}^{+0.000062}$...
b	Spitzer	2019-02-27	4.5 μm	$0.00226_{-0.00090}^{+0.00088}$	$0.00061_{-0.00072}^{+0.00071}$	0.00065 ± 0.00022	-0.00065 ± 0.00024	...
b	Spitzer	2019-09-09	4.5 μm	-0.00183 ± 0.00091	-0.0039 ± 0.0013	$0.000004_{-0.000032}^{+0.000031}$	$-0.000486_{-0.000094}^{+0.000095}$	$-0.000761_{-0.000086}^{+0.000088}$
b	LCO SSO	2020-04-25	z'
b	LCO SSO	2020-04-25	y
b	LCO SAAO	2020-05-20	z'
b	LCO SAAO	2020-05-20	y
b	LCO SAAO	2020-06-06	z'
b	LCO SAAO	2020-06-23	z'
c	TESS	2020-07-09	TESS
b	TESS	2020-07-10	TESS
b	PEST	2020-07-10	V	0.0013 ± 0.0014
b	TESS	2020-07-19	TESS
b	TESS	2020-07-27	TESS
c	TESS	2020-07-28	TESS
b	Brierfield	2020-08-13	I	0.0013 ± 0.0024
b	LCO SSO	2020-08-13	z'
b	LCO SAAO	2020-09-07	z'
b	LCO SSO	2020-09-16	z'
b	LCO SSO	2020-10-03	z'
b and c	LCO SAAO	2020-10-11	z'
b	LCO CTIO	2021-06-14	z'
b	LCO SSO	2021-06-22	z'
b	LCO CTIO	2021-07-01	z'
b	ASTEP	2021-07-09	R
b	LCO SAAO	2021-07-17	z'
b	LCO SAAO	2021-08-03	z'
b	LCO CTIO	2021-08-04	z'
b	LCO SSO	2021-08-12	z'
b	LCO SSO	2021-08-29	z'
b	LCO SSO	2021-09-15	z'
b	LCO SAAO	2021-09-23	z'
b	LCO TO	2021-09-23	z'
c	LCO CTIO	2021-10-04	z'

Notes. See Table 3 in Eastman et al. (2019) for a detailed description of all parameters. Since both Pan-STARRS Y and Pan-STARRS z_s are not available among the filters in EXOFASTv2, y and z' (Sloan z) were used as respective approximate substitutes. Additionally, the detrending parameters flare (Spitzer), sky (Spitzer and PEST), Sky/Pixel_T1 (LCOGT), and SKY (ASTEP) were set as additive while the remaining detrending parameters were set as multiplicative.

^a Multiplicative detrending coefficient.

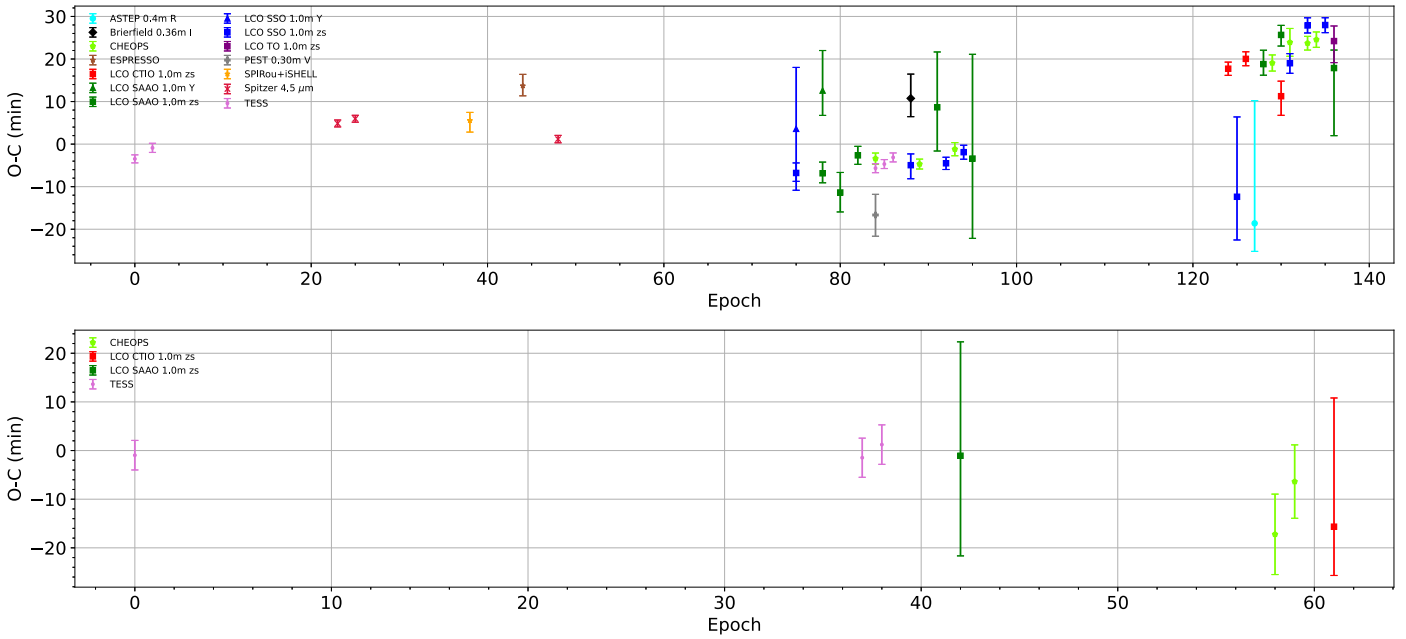


Figure 2. $O-C$ diagrams of AU Mic b (top) and AU Mic c (bottom), using the EXOFASTv2-generated measured midpoint times (Table 11) and the calculated expected midpoint times for all transit data sets from Table 1. The planets’ period and T_C from EXOFASTv2 posteriors were used for the calculation of expected midpoint times. The epoch refers to the number of transits since the first transit of b and c, respectively.

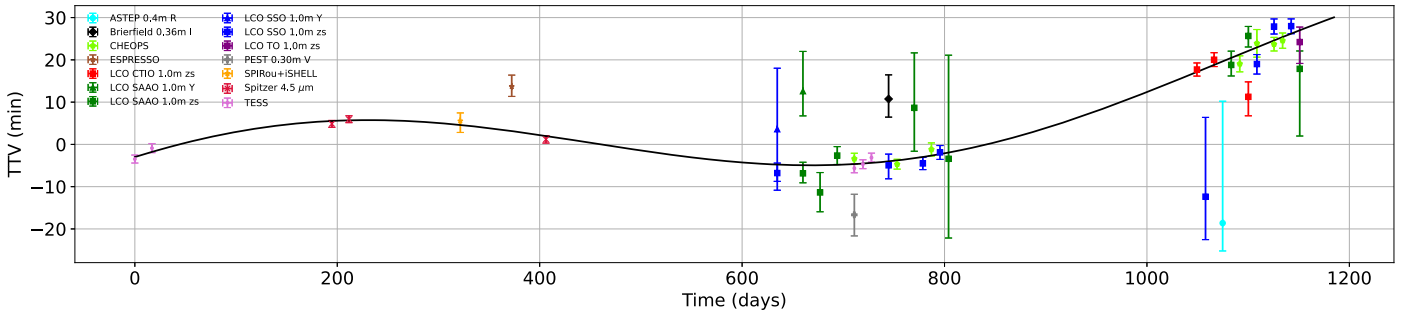


Figure 3. $O-C$ diagram of AU Mic b with the super-period model overlaid (black). The super-period model was generated using the coefficients from Table 9 and Equation (1), and the $O-C$ diagram was generated using the EXOFASTv2-generated measured midpoint times (Table 11) and the calculated expected midpoint times for all transit data sets from Table 1. The planet’s period and T_C from EXOFASTv2 posteriors were used for the calculation of expected midpoint times. The time is with respect to the first transit of b.

Table 9

Coefficients from Equation (1) as Part of Modeling the Super-period of AU Mic b’s TTVs (Figure 3)

Coefficient	Unit	Output
A	min	13.46394 ± 3.06443
B	day^{-1}	0.00504 ± 0.00070
C	...	0.87455 ± 0.28102
D	min day^{-1}	0.03052 ± 0.00964
E	min	-13.31819 ± 4.52818

Notes. The coefficients were solved using the `scipy.optimize.curve_fit`^a.
^a https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.curve_fit.html.

from RV literature (e.g., Cale et al. 2021; Donati et al. 2023). Moreover, the angular momentum deficit (AMD; Laskar 1997, 2000; Laskar & Petit 2017) criteria built within the Exo-Striker package indicated that this model is unstable. We followed this up by testing its stability over 20 Myr with

the N -body simulator rebound (Rein & Liu 2012; Rein & Spiegel 2015).

Since this model’s inferred masses are very small, we performed two stability tests (Figure 6): one with the planets’ original K ’s from Table 12, and another with the planets’ K ’s from Cale et al.’s (2021) RV models. At first glance, the original K ’s case appears to be dynamically stable, but the planets’ already highly eccentric orbits become increasingly eccentric over time, and both planets’ orbits become increasingly misaligned with the system, which would cause both planets to quickly lose their transiting status. The RV K ’s case exhibits signs of chaos in the AU Mic system, with both planets undergoing orbital migrations, with planet c demonstrating greater wobbles in its orbital path. Also, both planets’ eccentricities oscillate rapidly between 0 and 1, and their highly fluctuating inclinations would frequently put both transiting planets in nontransiting configurations. Given that both planets’ observed orbital periods are very consistent and with very small timing uncertainties (Martioli et al. 2020; Plavchan et al. 2020; Cale et al. 2021; Gilbert et al. 2022;

Table 10AU Mic’s Stellar and Planetary Priors for *Exo-Striker* Best-fit and MCMC Modeling

Prior	Unit	Input		
		AU Mic b	AU Mic c	AU Mic d
Mass	M_{\odot}	$\mathcal{N}(0.510, 0.028)$		
Radius	R_{\odot}	$\mathcal{N}(0.744, 0.023)$		
Luminosity	L_{\odot}	$\mathcal{N}(0.0916, 0.011)$		
T_{eff}	K	$\mathcal{N}(3678, 90)$		
$v \sin i$	km s^{-1}	$\mathcal{N}(8.7, 0.2)$		
K	m s^{-1}	$\mathcal{U}(0.0, 10000.0)$	$\mathcal{U}(0.0, 10000.0)$	$\mathcal{U}(0.0, 10000.0)$
P_{orb}	day	$\mathcal{N}(8.463, 0.001)$	$\mathcal{N}(18.859, 0.001)$	$\mathcal{U}(0.0, 100000.0)$
e	...	$\mathcal{U}(0.000, 0.999)$	$\mathcal{U}(0.000, 0.999)$	$\mathcal{U}(0.000, 0.999)$
ω	deg	$\mathcal{U}(0.0, 360.0)$	$\mathcal{U}(0.0, 360.0)$	$\mathcal{U}(0.0, 360.0)$
M_0	deg	$\mathcal{U}(0.0, 360.0)$	$\mathcal{U}(0.0, 360.0)$	$\mathcal{U}(0.0, 360.0)$
i	deg	$\mathcal{N}(89.57, 0.31)$	$\mathcal{N}(89.43, 0.35)$	$\mathcal{U}(0.0, 180.0)$
Ω	deg	$\mathcal{U}(0.0, 360.0)$	$\mathcal{U}(0.0, 360.0)$	$\mathcal{U}(0.0, 360.0)$

Notes. The stellar, orbital period, and inclination priors are taken from EXOFASTv2 posteriors. AU Mic d’s priors apply to three-planet modeling only. $M_0 \equiv$ mean anomaly, and $\Omega \equiv$ longitude of ascending node.

Wittrock et al. 2022; Donati et al. 2023), that the RV models from Cale et al. (2021) and Donati et al. (2023) place both planets’ orbits at much lower eccentricities, and that there have not been any known cases in which either planet “misses” its transit due to being outside the line of sight between the observers and the host star, this two-planet high-eccentricity configuration appears to be nonphysical and highly implausible.

We then turned to an alternative two-planet model with lower eccentricities. We present our low-eccentricity maximum-likelihood two-planet model with its MCMC $O-C$ diagram (Figure 7), posteriors (Table 13), and MCMC corner plot (Figure 8). This time, the AMD criteria indicated that this two-planet low-eccentricity model is stable. However, the inclinations from Table 13 suggest that the planets are misaligned, which contradicts the observed transits of both AU Mic b and c and the inclinations from transit and TTV literature (e.g., Martioli et al. 2021; Gilbert et al. 2022; Wittrock et al. 2022). Additionally, the $O-C$ diagram from Figure 7 does not exhibit the super-period that was very distinctive in the $O-C$ diagram from Figure 3, resulting in the model not converging very well with many observed TTVs, including those from the R-M observations, CHEOPS, and some TESS. Thus, both the two-planet diagram and the statistics information from Table 13 (e.g., $\chi_{\text{red,b}}^2 = 75.69$ and $\chi_{\text{red,c}}^2 = 454.17$) clearly show that the model is of a poor fit, suggesting that we need a third planet to account for the observed TTV excess, a conclusion also reached in Wittrock et al. (2022), but now much more obvious by eye with the additional season of 2021 transit observations.

4.4. Three-planet Dynamical Modeling

The two-planet high-eccentricity case’s high orbital eccentricities and very low inferred masses from Table 12 put this configuration in strong disagreement with the RV models from Cale et al. (2021) and Donati et al. (2023), and this orbital

configuration exhibits potential instability (Figure 6), with both planets’ eccentricities and inclinations undergoing significant fluctuations; these issues suggest that the two-planet high-eccentricity case does not provide a good fit for a two-planet model. The two-planet low-eccentricity case has both transiting planets being misaligned based on the inclinations from Table 13, and Figure 7 and the relatively high χ_{red}^2 and corrected Akaike information criterion (AIC_c ; Sugiura 1978; Hurvich & Tsai 1989; Cavanaugh 1997) from Table 13 (e.g., $\chi_{\text{red,b}}^2 = 75.69$, $\chi_{\text{red,c}}^2 = 454.17$, and $\text{AIC}_c = 1967.84$) demonstrate that the two-planet low-eccentricity model is inadequate in describing the observed TTVs; as such, we consider a three-planet model.

Both Cale et al. (2021) and Wittrock et al. (2022) probed the AU Mic system with RVs and TTVs, respectively, and modeled a tentative planet AU Mic d between the known planets AU Mic b and c with an orbital period of 12.742 and 13.466 days, respectively, the former of which would put the three planets near a 4:6:9 MMR chain. Since no additional transits have been identified for AU Mic, it was proposed that a third planet might be nontransiting. A nontransiting planet between two transiting planets is unusual but not unprecedented: HD 3167 d (Christiansen et al. 2017), Kepler-20 g (Buchhave et al. 2016), Kepler-411 e (Sun et al. 2019), and TOI-431 c (Osborn et al. 2021) are among the confirmed nontransiting planets orbiting between their adjacent transiting planets. We explore two possible scenarios for the three-planet modeling: AU Mic d orbiting interior to AU Mic b, and AU Mic d orbiting between AU Mic b and c. We first investigate the interior orbit case.

4.4.1. Planet d Interior to b

As the 2:3 orbital resonance pair is the most common pairing for mature, compact multiplanet systems (Fabrycky et al. 2014), and since near-resonances are necessary to produce the observed detectable TTVs, we explored the possibility that $P_d = 5.629$ days and modeled the best-fit three-planet configuration using that period as a starting point (Figure 9 and Table 14). The $O-C$ diagram from Figure 9 now displays a super-period in AU Mic b’s TTVs, and the statistics from Table 14 (e.g., $\chi_{\text{red,b}}^2 = 5.21$, $\chi_{\text{red,c}}^2 = 31.23$, and $\text{AIC}_c = -343.02$) demonstrate that the three-planet model has significantly better fitting than both of the two-planet models.

Next, we constructed TTV log-likelihood periodograms using the optimization functions from the *Exo-Striker* library to probe a range of possible orbital periods for AU Mic d using the parameters from Table 14 as the starting point; we generated a set of 4801 orbital periods between 3.5 and 6.5 days corresponding to a step size of 0.0005 days. Since the *Exo-Striker* modeling did not significantly deviate from its initial value during the periodogram run, we adjusted d’s initial inclination prior to each run to brute force a broader exploration of this model parameter. We performed two TTV periodograms with different initial inclinations $i_d = (85^\circ, 90^\circ)$, respectively (Figure 10). From those runs, we came across six most favored inner d periods based on their respective log-likelihoods: 5.08 days ($\ln \mathcal{L} = 208.22$ and 208.84), 5.39 days ($\ln \mathcal{L} = 209.99$ and 209.80), 5.64 days ($\ln \mathcal{L} = 209.65$ and 209.62), 5.86 days ($\ln \mathcal{L} = 214.36$ and 214.33), 6.20 days ($\ln \mathcal{L} = 209.95$ and 208.43), and 6.47 days ($\ln \mathcal{L} = 207.76$ and 211.40).

We perform best-fit modeling and MCMC calculations for each of those best inner d cases and plotted the $O-C$ diagrams

Table 11
AU Mic Planets' Midpoint Time Priors for Exo-Striker Models

Planet	Telescope	Transit N	Date (UT)	Band	T_0 (BJD)
b	TESS	1	2018-07-26	TESS	2458330.38927 ± 0.00039
	TESS	3	2018-08-12	TESS	2458347.31710 ± 0.00053
	Spitzer	24	2019-02-10	4.5 μ m	2458525.04446 ± 0.00016
	Spitzer	26	2019-02-27	4.5 μ m	2458541.97126 ± 0.00014
	SPIRou + iSHELL	39	2019-06-17	^a	2458651.99020 ± 0.00180
	ESPRESSO	45	2019-08-07	378.2–788.7 nm	2458702.77397 ± 0.00178
	Spitzer	49	2019-09-09	4.5 μ m	2458736.61735 ± 0.00014
	LCO SSO	76	2020-04-25	Pan-STARRS Y	2458965.11330 ± 0.00150
	LCO SSO	76	2020-04-25	Pan-STARRS z_s	2458965.12050 ± 0.01000
	LCO SAAO	79	2020-05-20	Pan-STARRS z_s	2458990.50230 ± 0.00170
	LCO SAAO	79	2020-05-20	Pan-STARRS Y	2458990.51580 ± 0.00650
	LCO SAAO	81	2020-06-06	Pan-STARRS z_s	2459007.42520 ± 0.00320
	LCO SAAO	83	2020-06-23	Pan-STARRS z_s	2459024.35730 ± 0.00130
	TESS	85	2020-07-10	TESS	2459041.28124 ± 0.00026
	PEST	85	2020-07-10	V	2459041.27360 ± 0.00340
	CHEOPS	85	2020-07-10	CHEOPS	2459041.28280 ± 0.00060
	TESS	86	2020-07-19	TESS	2459049.74493 ± 0.00024
	TESS	87	2020-07-27	TESS	2459058.20903 ± 0.00024
	Brierfield	89	2020-08-13	I	2459075.14470 ± 0.00390
	LCO SSO	89	2020-08-13	Pan-STARRS z_s	2459075.13380 ± 0.00210
	CHEOPS	90	2020-08-21	CHEOPS	2459083.59700 ± 0.00040
	LCO SAAO	92	2020-09-07	Pan-STARRS z_s	2459100.53230 ± 0.00900
	LCO SSO	93	2020-09-16	Pan-STARRS z_s	2459108.98618 ± 0.00072
	CHEOPS	94	2020-09-24	CHEOPS	2459117.45150 ± 0.00080
	LCO SSO	95	2020-10-03	Pan-STARRS z_s	2459125.91403 ± 0.00090
	LCO SAAO	96	2020-10-11	Pan-STARRS z_s	2459134.37600 ± 0.01700
	LCO CTIO	125	2021-06-14	Pan-STARRS z_s	2459379.81818 ± 0.00069
	LCO SSO	126	2021-06-22	Pan-STARRS z_s	2459388.26030 ± 0.01300
	LCO CTIO	127	2021-07-01	Pan-STARRS z_s	2459396.74582 ± 0.00079
	ASTEP	128	2021-07-09	R	2459405.18200 ± 0.02000
	LCO SAAO	129	2021-07-17	Pan-STARRS z_s	2459413.67100 ± 0.00210
	CHEOPS	130	2021-07-26	CHEOPS	2459422.13420 ± 0.00100
	LCO SAAO	131	2021-08-03	Pan-STARRS z_s	2459430.60180 ± 0.00160
	LCO CTIO	131	2021-08-04	Pan-STARRS z_s	2459430.59180 ± 0.00300
	LCO SSO	132	2021-08-12	Pan-STARRS z_s	2459439.06020 ± 0.00140
	CHEOPS	132	2021-08-12	CHEOPS	2459439.06360 ± 0.00210
	LCO SSO	134	2021-08-29	Pan-STARRS z_s	2459455.99240 ± 0.00089
	CHEOPS	134	2021-08-29	CHEOPS	2459455.98950 ± 0.00070
	CHEOPS	135	2021-09-06	CHEOPS	2459464.45310 ± 0.00090
	LCO SSO	136	2021-09-15	Pan-STARRS z_s	2459472.91850 ± 0.00088
	LCO SAAO	137	2021-09-23	Pan-STARRS z_s	2459481.37450 ± 0.01100
	LCO TO	137	2021-09-23	Pan-STARRS z_s	2459481.37890 ± 0.00340
	c	TESS	1	2018-08-11	TESS
TESS		38	2020-07-09	TESS	2459040.00487 ± 0.00072
TESS		39	2020-07-28	TESS	2459058.86571 ± 0.00066
LCO SAAO		43	2020-10-11	Pan-STARRS z_s	2459134.30000 ± 0.01600
CHEOPS		59	2021-08-09	CHEOPS	2459436.03230 ± 0.00450
CHEOPS		60	2021-08-28	CHEOPS	2459454.89880 ± 0.00380
LCO CTIO		62	2021-10-04	Pan-STARRS z_s	2459492.61030 ± 0.01800

Notes. These midpoint times, with the exception of those from CHEOPS (Szabó et al. 2021, 2022) and R-M observations (Martoli et al. 2020; Palle et al. 2020), were generated from EXOFASTv2 transit modeling.

^a 955–2515 nm (SPIRou) and 2.18–2.47 nm (iSHELL).

(Figures A1 and A2), the output parameters (Tables A1, A2, A3, A4, A5, and A6), and the corner plots (Figures A4, A5, A6, A7, A8, and A9). For all inner d cases, the three-planet model reproduces the TTV super-period of AU Mic b in $O-C$ diagrams. The statistics criteria (e.g., χ^2_{red} and AIC_c values from Tables A1 through A6 versus those from Tables 12 and 13) indicate that these inner d cases are strongly favored over the two-planet cases. Notably, only $P_d = 5.64$ days has AU Mic d being coplanar with the AU Mic system based on its

inclination; the 5.86 and 6.47 days cases have the planet being moderately misaligned with the system, and the 5.08, 5.39, and 6.20 days cases have the planet being highly misaligned with the system.

The AMD criteria in Exo-Striker suggest that all inner d cases are unstable, so we investigate this further. We tested the stability of these three-planet cases over 2 Myr with the N -body simulator rebound and found that all of those configurations are stable (Figure A13). However, the 5.39 days case has all

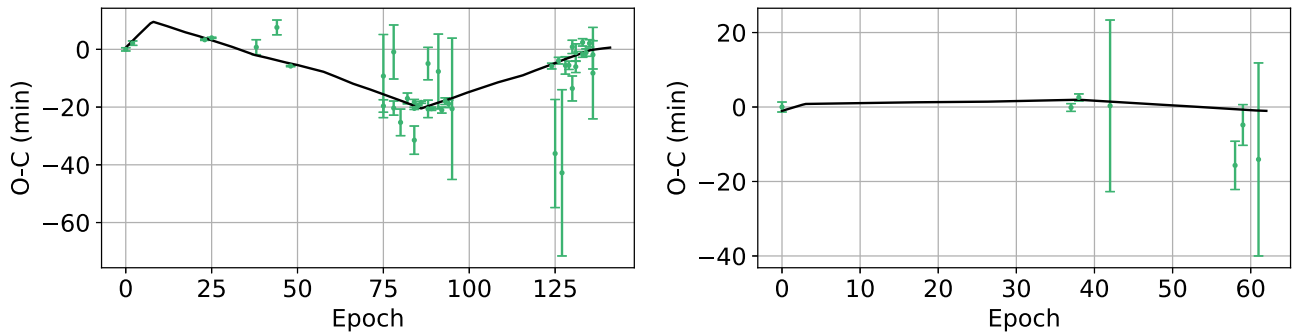


Figure 4. $O-C$ diagrams of AU Mic b (left) and AU Mic c (right), with comparison between TTVs (green) and Exo-Striker-generated MCMC models (black) for a two-planet high-eccentricity configuration.

Table 12
Exo-Striker-generated Best-fit and MCMC Modeling Parameters for Two-planet High-eccentricity Configuration

Parameter	Unit	Best Fit		MCMC	
		AU Mic b	AU Mic c	AU Mic b	AU Mic c
K	m s^{-1}	0.00543	0.05368	0.00385 ± 0.00114	0.06702 ± 0.01389
P_{orb}	day	8.46302	18.85898	8.46401 ± 0.00097	18.85941 ± 0.00060
e	...	0.22188	0.51009	0.21098 ± 0.00952	0.51720 ± 0.00786
ω	deg	172.51565	150.13429	173.20702 ± 0.76278	149.73899 ± 0.35421
M_0	deg	251.90049	13.83210	252.49015 ± 0.71508	13.42102 ± 0.35754
i	deg	88.94778	86.96414	89.00381 ± 0.16286	88.15442 ± 1.22945
Ω	deg	154.51121	0.65309	155.13328 ± 0.77348	1.28668 ± 0.76259
χ^2	...	182.65010	8.66743	187.17573	11.10833
χ_{red}^2	...	6.37725	38.26351	6.60947	39.65681
$\ln \mathcal{L}$...		176.17820		176.09710
BIC	...		-297.87092		-297.70872
AIC_c	...		-312.00346		-311.84126

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding the planets' low K 's generated by Exo-Striker.

three planets' orbits fluctuate toward high-eccentricity configuration, and these planets, especially AU Mic b and d, become highly misaligned for much of the 2 Myr timeline. The 5.86 days case's misalignment issue appears more pronounced, with all three planets becoming quickly misaligned with the system. Although the 5.64 days case has AU Mic d initially being coplanar, rebound has it become misaligned periodically. Lastly, the 6.47 days case has the planets exhibiting the most consistent coplanarity among the inner d cases.

4.4.2. Planet d between b and c

We repeat the steps described in the previous section for the middle d scenario. Based on AU Mic b's TTV super-period and assuming the near-2:3 orbital resonance pair between b and d, we explored the possibility that $P_d = 12.738$ days. Using this orbital period as a starting point, we obtained an initial best-fit model for the three-planet configuration (Figure 11 and Table 15). The $O-C$ diagram from Figure 11 also displays the super-period in AU Mic b's TTVs, and the statistics from Table 15 (e.g., $\chi_{\text{red,b}}^2 = 5.92$, $\chi_{\text{red,c}}^2 = 35.53$, and $\text{AIC}_c = -330.97$) demonstrate that this three-planet model yields a significantly better fit than the two-planet models.

Next, we utilize the TTV periodogram to probe a range of possible orbital periods for AU Mic d using the parameters from Table 15 as the starting point; we generated a set of 8001 orbital periods between 11 and 15 days with 0.0005 days interval in between. We adjusted d's initial inclination prior to each run and did two runs with different initial inclinations

$i_d = (80^\circ, 90^\circ)$, respectively (Figure 12). From those runs, we came across four most favored periods based on their log-likelihood: 11.9 days ($\ln \mathcal{L} = 208.16$) and 14.1 days ($\ln \mathcal{L} = 201.45$) from the initial $i_d = 80^\circ$ case, and 12.6 days ($\ln \mathcal{L} = 209.21$ and 205.70) and 12.7 days ($\ln \mathcal{L} = 211.25$ and 209.79) from both respective inclination cases.

We perform best-fit modeling and MCMC calculations for each of those four best middle d cases and generated the $O-C$ diagrams (Figures 14 and A3), the output parameters (Tables 18, A7, A8, and A9), and the corner plots (Figures 15, A10, A11, and A12). For all of those four cases, the three-planet models adequately account for the TTV super-period of AU Mic b in the $O-C$ diagrams, and the statistics criteria (e.g., χ_{red}^2 and AIC_c values from Tables 18 and A7 through A9 versus those from Tables 12 and 13) indicate that all three-planet middle d cases are strongly favored over the two-planet case. Curiously, planet d's inclination varies considerably among these four cases. For instance, the 12.6 and 12.7 days cases have planet d being relatively coplanar with the AU Mic system, while the 11.9 and 14.1 days cases have it being highly misaligned.

The AMD criteria in Exo-Striker suggest that these three-planet cases might be unstable. As before, we utilized rebound to test the stability of the three-planet cases; through this package, we found that each of these three-planet configurations is stable (Figures 16 and A13). Among the middle d cases, both the 12.6 and 12.7 days cases are the most consistent in maintaining the coplanarity of the planets.

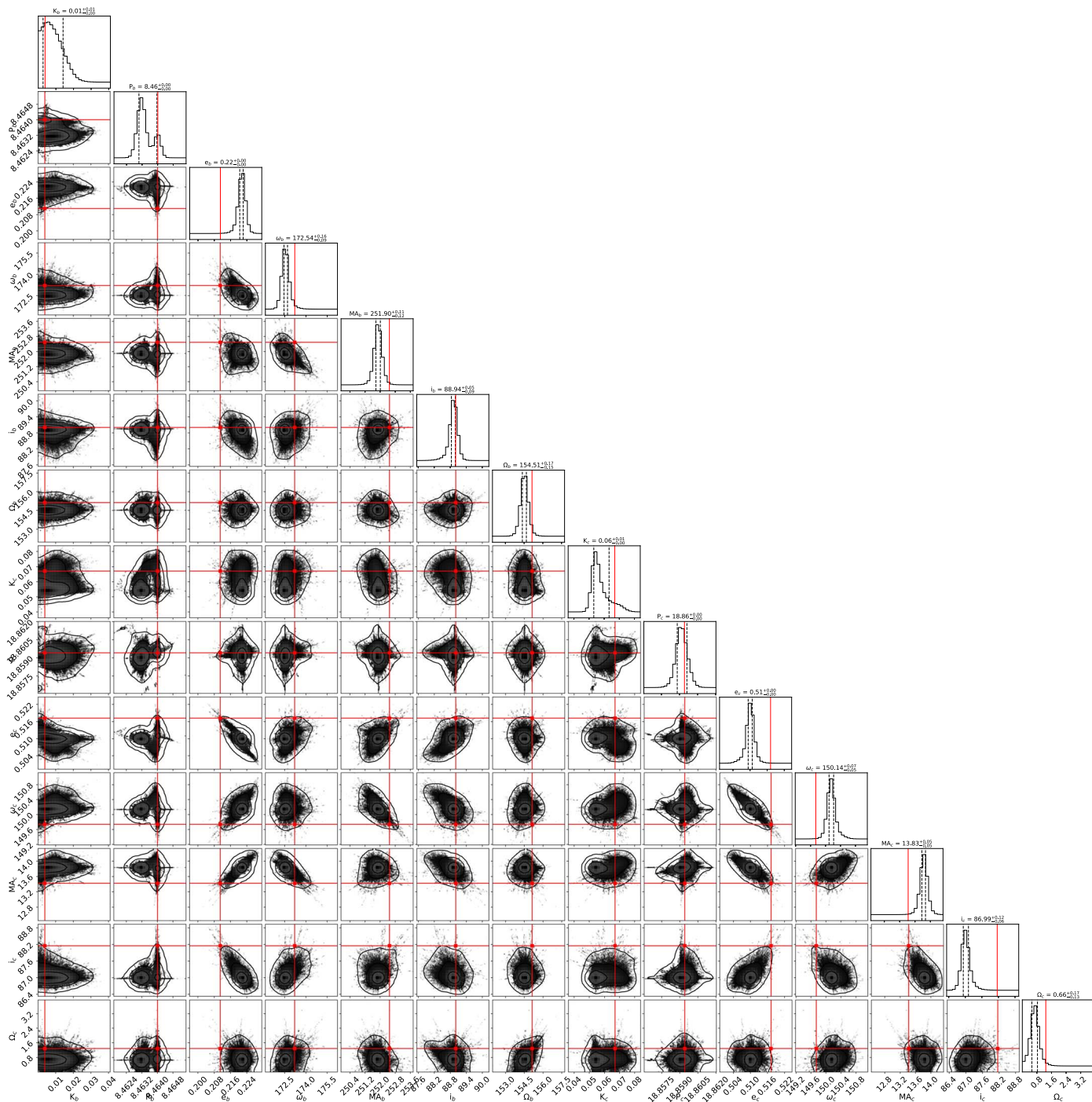


Figure 5. Corner plot of AU Mic b and c’s orbital parameters from Exo-Striker MCMC analysis for a two-planet high-eccentricity configuration.

4.5. Transit Timing Variation Model Comparisons

We perform a model comparison among both the two-planet and three-planet cases, as seen in Table 16. Based on the TTV model information criterion, the $P_d = 12.7$ days configuration is the most favored overall, and the $P_d = 6.47$ days configuration is the most favored among the inner d cases, followed closely by the 6.20 days and 5.86 days cases. All other configurations are not as well favored based on their corresponding Bayesian information criterion (ΔBIC ; Schwarz 1978; Wit et al. 2012) and ΔAIC_c , but they are not

statistically ruled out. The two-planet cases’ ΔBIC and ΔAIC_c are much larger, so both two-planet models can be rejected.

Next, we checked the robustness of the results by including only the high-precision TTVs, namely the TESS + Spitzer + SPIRou + CHEOPS joint modeling, and by performing the best-fit modeling and MCMC calculations, then we did a model comparison of the results (Table 17). This time, the 14.1 days case is the most favored overall, followed closely by the 12.7 days case; the 5.39 days case is the most favored among the inner d cases; and all other cases are disfavored but not statistically ruled out. Again, the two-planet cases are

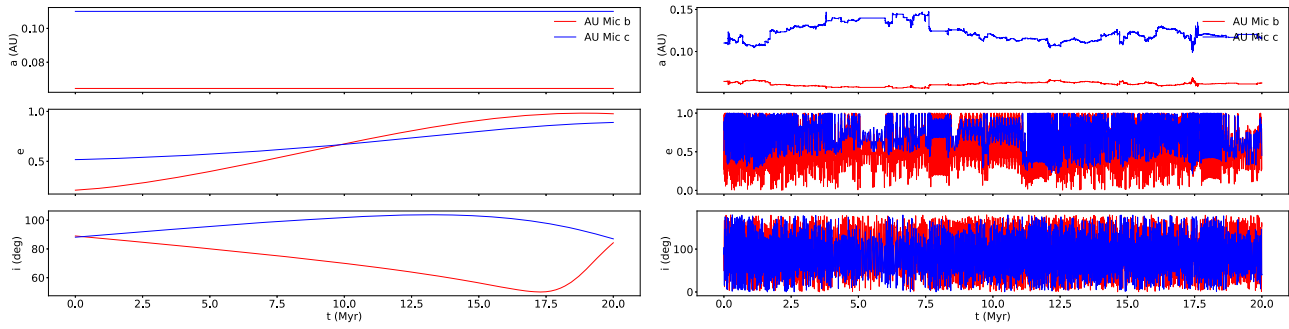


Figure 6. rebound models of the stability of the AU Mic system on a timescale of 20 Myr for a two-planet high-eccentricity configuration with original K 's from Table 12 (left) and with K 's from Cale et al. (2021) RV models (right), due to the original K 's being very small ($<0.07 \text{ m s}^{-1}$). While the original K 's case appears to be stable, both planets' orbits become increasingly eccentric over time, and both planets would quickly become nontransiting due to their increasing orbital misalignment. The RV K 's case exhibits signs of chaos in the AU Mic system, with the planets undergoing orbital migrations but no orbital crossing, their eccentricities being extremely erratic throughout, rapidly oscillating between 0 and 1, and their highly fluctuating inclinations frequently placing both transiting planets in nontransiting configurations.

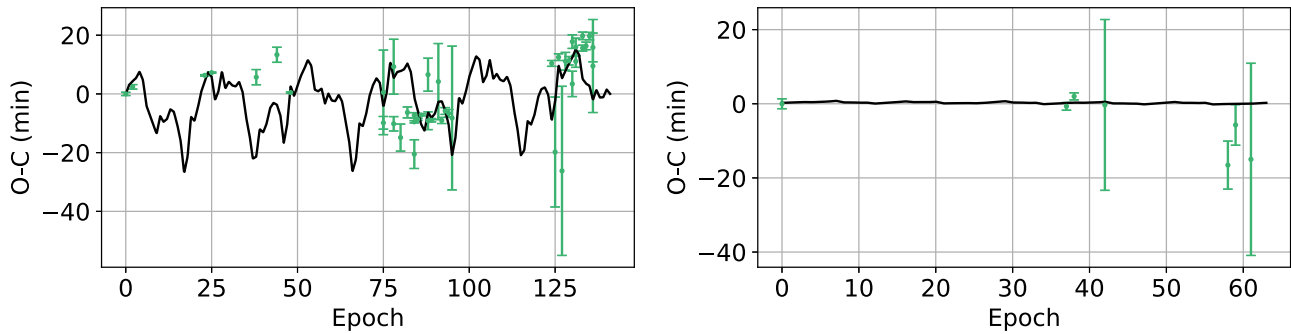


Figure 7. $O-C$ diagrams of AU Mic b (left) and AU Mic c (right), with comparison between TTVs (green) and Exo-Striker-generated MCMC models (black) for a two-planet low-eccentricity configuration.

Table 13
Exo-Striker-generated Best-fit and MCMC Modeling Parameters for Two-planet Low-eccentricity Configuration

Parameter	Unit	Best Fit		MCMC	
		AU Mic b	AU Mic c	AU Mic b	AU Mic c
K	m s^{-1}	2.25722	7.65452	0.42086 ± 0.13722	27.70929 ± 1.41812
P_{orb}	day	8.46265	18.86039	8.46026 ± 0.00011	18.85925 ± 0.00009
e	...	0.00000	0.00000	0.00723 ± 0.00510	0.12867 ± 0.00279
ω	deg	89.92745	229.20188	0.74540 ± 0.24389	212.29306 ± 0.04038
M_0	deg	0.00049	354.88367	90.08325 ± 10.09994	359.95202 ± 0.75854
i	deg	89.56579	89.43523	92.68980 ± 0.30871	85.98426 ± 0.23709
Ω	deg	0.00814	0.00004	160.82677 ± 82.66571	141.76084 ± 81.43847
χ^2	...	4151.71376	11.59895	2259.05984	11.77892
χ^2_{red}	...	138.77709	832.66254	75.69463	454.16775
$\ln \mathcal{L}$...	-1808.30801			-963.74485
BIC	...	3671.10150			1981.97518
AIC_c	...	3656.96896			1967.84264

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding planet b's low K 's generated by Exo-Striker.

statistically ruled out due to their significantly high ΔBIC and ΔAIC_c .

Taking into consideration both model comparisons, the 12.7 days, 14.1 days, 5.39 days, and 6.20 days cases are consistently favored while the 12.6 days, 11.9 days, 5.64 days, 5.08 days, and both two-planet cases are consistently disfavored relative to the other cases. The 5.86 and 6.47 days cases are inconsistent, with both being more relatively favored in the

all-TTVs model comparison and much less so in the high-precision TTVs model comparison.

We did not explore randomly removing individual TTVs to evaluate the robustness of these results and the dependence of our results on particular TTV measurements. Our data set is notably heterogeneous, and some of the earlier observations, particularly the Spitzer transit times, are critical in determining the super-period because they sparsely fill in the TTV curve.

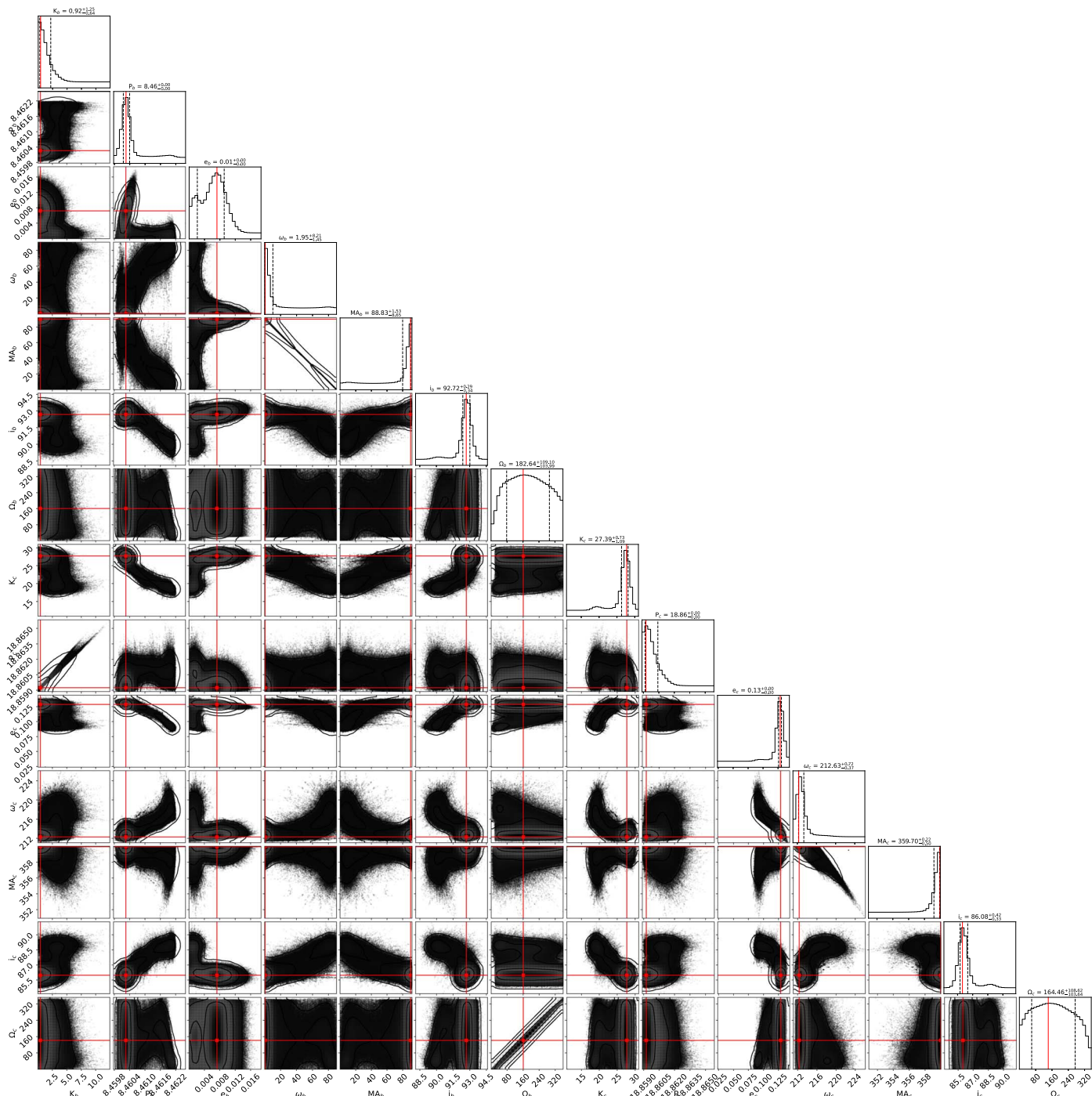


Figure 8. Corner plot of AU Mic b and c’s orbital parameters from Exo-Striker MCMC analysis for a two-planet low-eccentricity configuration.

Thus, as would be expected, our results are highly dependent on these space-based timing measurements. Conversely, for the 2021 season, we have much denser transit timing sampling, and thus our results will be less impacted by the removal of those data sets.

4.6. Stability Analysis of the Ten Cases

With both two-planet cases statistically ruled out and therefore excluded (Tables 16 and 17), we proceeded to test the dynamical stability of the aforementioned ten three-planet configurations by utilizing the SPOCK packages FeatureClassifier and NbodyRegressor (Tamayo et al. 2020).

The FeatureClassifier, hereafter referred to as SPOCK, is a trained model estimating the stability probability after 10^9 orbits of the innermost planet (for AU Mic d in the inner d scenario, 13.9 Myr for 5.08 days, 14.8 Myr for 5.39 days, 15.4 Myr for 5.64 days, 16.0 Myr for 5.86 days, 17.0 Myr for 6.20 days, and 17.7 Myr for 6.47 days; for AU Mic b in the middle d scenario, 23.2 Myr, which is comparable to the age of the system). Subsequently, we weight the parameters by their stability probability to sort out or give weaker weight to unstable systems that would likely have already destroyed themselves over time.

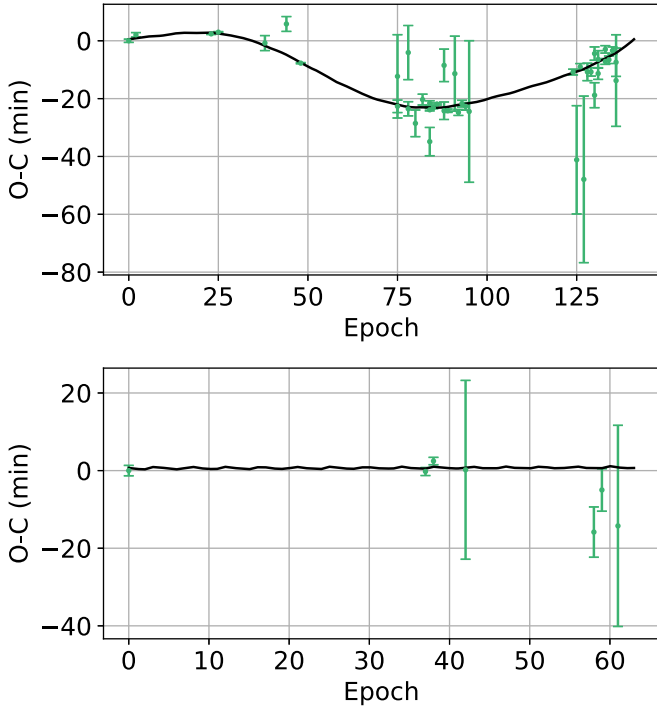


Figure 9. Initial three-planet (with d interior to b) best-fit $O-C$ diagrams of AU Mic b (top) and AU Mic c (bottom), with comparison between TTVs (green) and Exo-Striker-generated MCMC models (black).

Table 14

Exo-Striker-generated Initial Three-planet (with d Interior to b) Best-fit Modeling Parameters

Parameter	Unit	AU Mic b	AU Mic c	AU Mic d
K	m s^{-1}	5.03491	0.51952	0.34401
P_{orb}	day	8.46334	18.86226	5.63749
e	...	0.00000	0.00459	0.00019
ω	deg	98.48619	223.69811	167.66115
M_0	deg	351.49848	0.00066	239.37339
i	deg	89.57446	89.43918	87.32379
Ω	deg	30.38207	31.22049	9.19867
χ^2	...	113.87171	11.06507	...
χ_{red}^2	...	5.20570	31.23419	...
$\ln \mathcal{L}$	209.62069	...
BIC	-337.51315	...
AIC_c	-343.01916	...

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding the planets' low K 's generated by Exo-Striker.

The `NbodyRegressor`, hereafter referred to as `Nbody`, performs an `Nbody` simulation and checks a system for stability after an arbitrarily chosen number of orbits of the innermost planet. Due to the high numerical cost of an `Nbody` simulation over a large number of orbits, we chose a simulation period of 2×10^5 orbits for this part of the analysis (for AU Mic d in the inner d scenario, 2.78 kyr for 5.08 days, 2.95 kyr for 5.39 days, 3.09 kyr for 5.64 days, 3.21 kyr for 5.86 days, 3.40 kyr for 6.20 days, and 3.54 kyr for 6.47 days; for AU Mic b in the middle d scenario, 4.63 kyr). We utilized `Nbody` for two purposes: To notice any deviations that may arise between `SPOCK` and `Nbody` results, and to validate the results from `SPOCK`. `SPOCK` was trained with planetary configurations showing mutual inclinations $\lesssim 10^\circ$. Since higher values for the

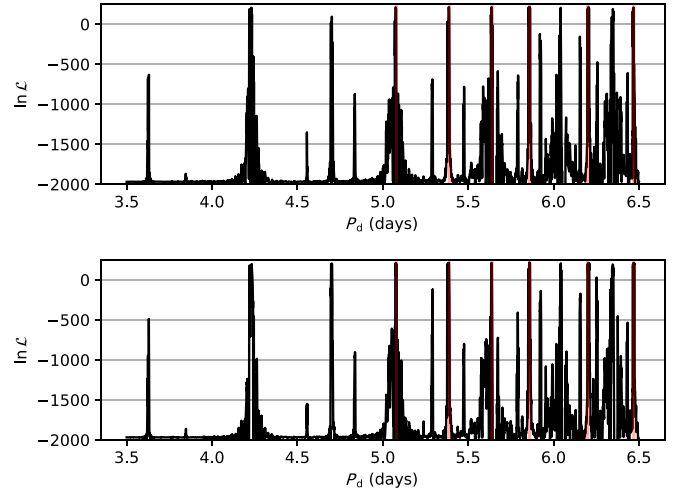


Figure 10. TTV log-likelihood periodograms of AU Mic d's orbital period using the parameters from Table 14 as the starting point and initial inclinations $i_d = 85^\circ$ (top) and 90° (bottom). We obtained the six most favored inner d periods based on their log-likelihood: 5.08 days ($\ln \mathcal{L} = 208.22$ and 208.84), 5.39 days ($\ln \mathcal{L} = 209.99$ and 209.80), 5.64 days ($\ln \mathcal{L} = 209.65$ and 209.62), 5.86 days ($\ln \mathcal{L} = 214.36$ and 214.33), 6.20 days ($\ln \mathcal{L} = 209.95$ and 208.43), and 6.47 days ($\ln \mathcal{L} = 207.76$ and 211.40) from both respective inclination cases. The red lines denote those periods corresponding to best-fitting peaks in the periodograms.

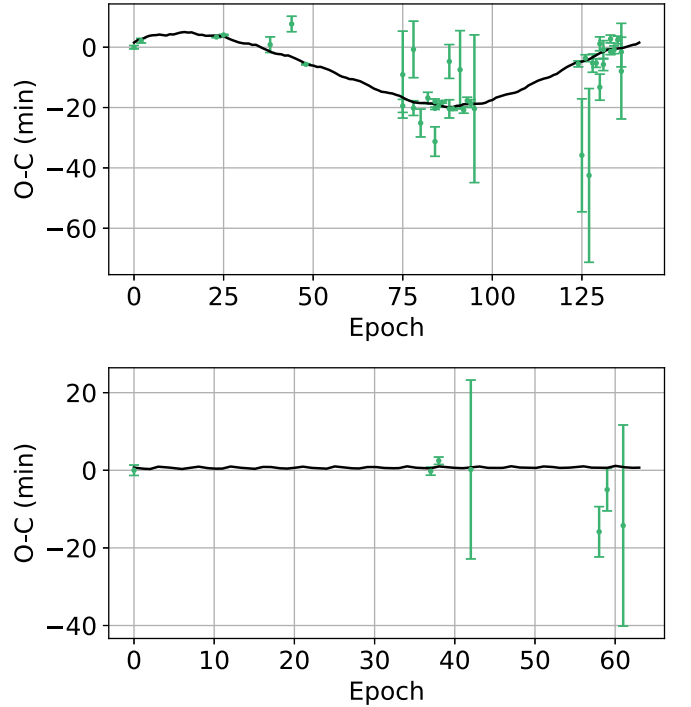


Figure 11. Initial three-planet (with d between b and c) best-fit $O-C$ diagrams of AU Mic b (top) and AU Mic c (bottom), with comparison between TTVs (green) and Exo-Striker-generated MCMC models (black).

mutual inclinations can be found for the cases of 5.08 days, 5.39 days, 6.20 days, and 14.1 days, we considered it useful to use additional analysis methods for comparison. At this point, however, it is important to note that differences and similarities between the results of `SPOCK` and `Nbody` can only serve as an indication and not a proof of `SPOCK`'s reliability, since the time period after which a system is tested for stability differs significantly. A system that becomes unstable after 10^6 orbits

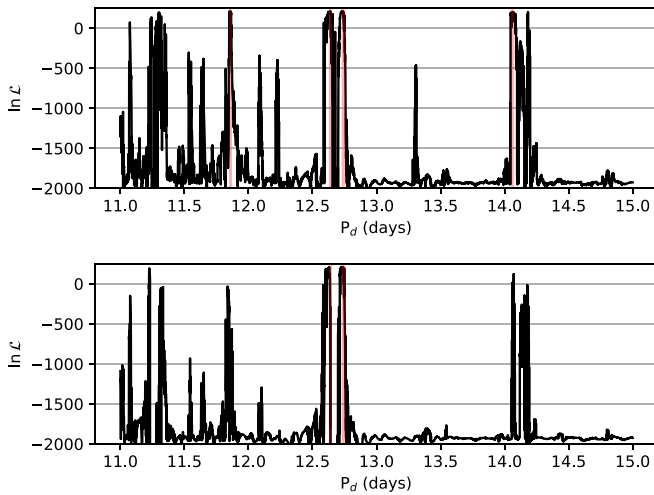


Figure 12. TTV periodograms of AU Mic d’s orbital period using the parameters from Table 15 as the starting point and initial inclinations $i_d = 80^\circ$ (top) and 90° (bottom). We obtained the four most favored periods based on their log-likelihood: 11.9 days ($\ln \mathcal{L} = 208.16$) and 14.1 days ($\ln \mathcal{L} = 201.45$) from the initial $i_d = 80^\circ$ case, and 12.6 days ($\ln \mathcal{L} = 209.21$ and 205.70) and 12.7 days ($\ln \mathcal{L} = 211.25$ and 209.79) from both respective inclination cases. The red lines denote those days corresponding to best-fitting peaks in the periodograms.

Table 15

Exo-Striker-generated Initial Three-planet (with d Between b and c) Best-fit Modeling Parameters

Parameter	Unit	Best Fit		
		AU Mic b	AU Mic c	AU Mic d
K	m s^{-1}	4.85810	1.49734	0.16823
P_{orb}	day	8.46325	18.86214	12.73040
e	...	0.00704	0.00136	0.00283
ω	deg	89.95582	223.96004	166.18027
M_0	deg	0.00002	0.00018	0.00017
i	deg	89.39532	89.41147	81.08949
Ω	deg	0.00002	0.00000	0.00000
χ^2	...	131.05680	11.06507	...
χ^2_{red}	...	5.92174	35.53047	...
$\ln \mathcal{L}$...		203.59374	
BIC	...		-325.45925	
AIC_c	...		-330.96526	

Note. The K ’s listed here are unconstrained, but see Section 6 for discussion regarding the planets’ low K ’s generated by Exo-Striker.

will be classified as stable by Nbody in our case, although SPOCK will probably assign a low probability of stability to the system. Nevertheless, we think it is useful to use Nbody as a comparison to notice significant differences and to look at the stability of the system over different timescales.

We also utilized MEGNO (mean exponential growth factor of nearby orbits; Cincotta & Simó 2000; Cincotta et al. 2003), which is a fast indicator for chaotic orbits, for our orbital simulations. MEGNO can be used similarly to SPOCK to estimate the orbital stability of planetary systems, as done by Tamayo et al. (2020), although it is pointed out that a direct comparison is only possible to a limited extent (see Section 1. D. in Tamayo et al. 2020). Since SPOCK uses MEGNO as one of ten features for internal analysis, we consider a comparison of results useful in this case as well. However, also in this case it is important to emphasize that discrepancies between SPOCK

and MEGNO can only be an indication of inconsistencies of the SPOCK results, since orbital stability and regularity of orbits cannot be used synonymously.

We incorporated the stellar parameters from Table 10 and the planetary parameters from Exo-Striker-generated posteriors (Tables 18 and A1 thru A9). We generated 20,000 random configurations for each of the 10 cases, drawn from the respective Gaussian distributions of the planetary parameters. Next, a simulation object is created from rebound, to which all planetary parameters and the mass of the star of a certain random configuration are passed on. Then, this simulation object is examined through SPOCK for the stability probability after 10^9 orbits of the innermost planet. The same parameters were also used to estimate the stability of the system after 2×10^5 orbits using Nbody. SPOCK determines ten features for the classification of planetary systems, based on which it estimates the stability probability. Since MEGNO is one of them, it can be directly output by SPOCK and used for our analysis. Thus, all three analysis methods use the same orbital parameters and masses.

In Figure 13, the semimajor axis of AU Mic d was then plotted in a histogram for each of these configurations. Since the same number of random configurations was generated for each parameter set, the result is a mixture distribution in which the respective underlying distributions have the same weighting. The values are then weighted with the stability probability in the histogram, resulting in the overlapping histogram for SPOCK, MEGNO, and Nbody. Each histogram thus contains 200,000 values each with and without weighting by the stability probability. Unlike SPOCK, Nbody and MEGNO do not provide continuously distributed stability probabilities. Nbody provides binary stability probabilities, weighting a stable system by 1 and an unstable system by 0. For MEGNO, however, we had to set a limit above which we classify a system as unstable for comparison with the other methods. To reduce the number of systems incorrectly classified as unstable, we use the difference of the mean, μ_{MEGNO} , and the standard deviation, σ_{MEGNO} , of the MEGNO, which can also be output by SPOCK, for the estimation of the orbital stability. If $\mu_{\text{MEGNO}} - \sigma_{\text{MEGNO}} > 2.1$, the system is considered as chaotic and thus unstable and receives weight 0. On the other hand, if the value is smaller, we consider the system stable and it receives weight 1. At this point, it should be emphasized once again that a chaotic system does not necessarily have to lead to instability in the near future.

Figure 13 displays the results from these aforementioned runs. The SPOCK results appear to deviate from the MEGNO and Nbody results for the inner planet d cases, while all three results appear to be consistent for the middle planet d case. For the inner d scenario, the 5.64 days case appears to be the most preferred, although the SPOCK results peak closer to the 5.86 days case. For the middle d scenario, both the 12.6 and 12.7 days cases are clearly preferred over the 11.9 and 14.1 days cases from all three results. Comparisons between the inner d orbits and the middle d orbits should be viewed with caution because of the different simulation durations due to the different innermost orbit. MEGNO and, in particular, Nbody show hardly any visible trends. One reason for the differences between SPOCK and the other methods could be that the training sample was not matched by each parameter set. However, it is more likely that the differences are due to the different number of orbits considered.

Table 16
TTV Model Information Criterion for the AU Mic System

Type	P_d (days)	Relative χ_{red}^2	$\Delta \ln \mathcal{L}$	ΔBIC	ΔAIC_c	
MCMC	12.7	3.95967	0.00000	0.00000	0.00000	
	6.47	3.99117	1.56595	3.13190	3.13190	
	6.20	3.92884	1.67726	3.35452	3.35452	
	5.86	4.08053	1.75398	3.50796	3.50796	
	14.1	4.12141	2.37983	4.75966	4.75966	
	5.39	4.14006	2.78037	5.56074	5.56074	
	12.6	4.33368	5.83824	11.67648	11.67648	
	11.9	4.44417	7.12662	14.25324	14.25324	
	5.08	4.45799	8.78461	17.56922	17.56922	
	5.64	4.55658	9.62125	19.24250	19.24250	
	no d (high e)	5.66526	52.65304	78.06333	69.43680	
	no d (low e)	64.88111	1192.49499	2357.74723	2349.12070	
	Best Fit	12.7	4.05575	0.00000	0.00000	0.00000
		5.86	4.08923	0.56826	1.13652	1.13652
6.20		4.25992	2.95667	5.91334	5.91334	
14.1		4.28073	3.27556	6.55112	6.55112	
5.39		4.30368	3.52700	7.05400	7.05400	
6.47		4.30787	3.53291	7.06582	7.06582	
12.6		4.36285	4.37647	8.75294	8.75294	
11.9		4.43253	5.40495	10.80990	10.80990	
5.64		4.46849	5.87727	11.75454	11.75454	
5.08		4.47444	5.94329	11.88658	11.88658	
no d (high e)		5.46622	39.22908	51.21541	42.58888	
no d (low e)		118.95179	2023.71529	4020.18783	4011.56130	

Note. The numbers of degrees of freedom used to calculate the relative χ_{red}^2 are $N_{\text{dof}} = 14$ for a two-planet model and $N_{\text{dof}} = 21$ for a three-planet model.

Table 17
High-precision (TESS + Spitzer + SPIRou + CHEOPS) TTV Model Information Criterion for the AU Mic System

Type	P_d (days)	Relative χ_{red}^2	$\Delta \ln \mathcal{L}$	ΔBIC	ΔAIC_c	
MCMC	14.1	10.24154	0.00000	0.00000	0.00000	
	12.7	10.88606	1.42079	2.84158	2.84158	
	5.39	11.96840	2.76171	5.52342	5.52342	
	6.20	12.10677	3.84762	7.69524	7.69524	
	5.86	14.15778	6.27514	12.55028	12.55028	
	12.6	14.71546	7.24787	14.49574	14.49574	
	6.47	14.72007	7.58335	15.16670	15.16670	
	11.9	16.53061	9.83512	19.67024	19.67024	
	5.08	16.89976	11.34680	22.69360	22.69360	
	5.64	17.35204	12.42087	24.84174	24.84174	
	No d (high e)	13.55832	40.32199	59.33232	1060.64398	
	No d (low e)	164.49125	601.87497	1182.43828	2183.74994	
	Best Fit	12.7	10.96391	0.00000	0.00000	0.00000
		14.1	11.56410	0.83980	1.67960	1.67960
6.20		13.89642	4.46176	8.92352	8.92352	
5.39		13.94522	4.50562	9.01124	9.01124	
5.86		14.65509	5.64192	11.28384	11.28384	
12.6		14.77799	5.72267	11.44534	11.44534	
6.47		15.66435	7.16597	14.33194	14.33194	
11.9		16.41644	8.26575	16.53150	16.53150	
5.08		16.54831	8.46338	16.92676	16.92676	
5.64		16.68665	8.69418	17.38836	17.38836	
No d (high e)		14.39713	34.30994	47.30822	1048.61988	
No d (low e)		312.57388	1077.13307	2132.95448	3134.26614	

Notes. The numbers of degrees of freedom used to calculate the relative χ_{red}^2 are $N_{\text{dof}} = 14$ for a two-planet model and $N_{\text{dof}} = 18$ for a three-planet model. $N_{\text{dof}} = 18$ is used instead of 21 due to there being 21 data observations in this high-precision model comparison.

To check whether this is indeed the case, we ran 100 N_{body} simulations over 10^7 orbits for each of the ten parameter sets. This is still two orders of magnitude smaller than the 10^9 orbits according to which SPOCK estimates the stability probability,

but a smaller value was chosen because of the numerical effort. In fact, after 10^7 orbits, the systems tended to become unstable more often. The clearest difference occurred in the 5.39 days case. While 85% of the generated configurations were still

Table 18
Exo-Striker-generated Best-fit and MCMC Modeling Parameters for $P_d = 12.7$ Days

Parameter	Unit	Best Fit			MCMC		
		AU Mic b	AU Mic c	AU Mic d	AU Mic b	AU Mic c	AU Mic d
K	m s^{-1}	1.65668	0.45119	0.30658	0.31290 ± 0.26983	0.18450 ± 0.07362	0.45150 ± 0.21874
P_{orb}	day	8.46318	18.86027	12.73174	8.46308 ± 0.00006	18.85969 ± 0.00008	12.73596 ± 0.00793
e	...	0.01013	0.00000	0.00339	0.00577 ± 0.00101	0.00338 ± 0.00164	0.00305 ± 0.00104
ω	deg	89.99210	224.07980	160.53522	88.43038 ± 0.05783	223.28438 ± 1.68357	160.78945 ± 2.59947
M_0	deg	0.00003	0.00003	0.00013	1.58566 ± 1.46718	0.51338 ± 0.41839	2.11220 ± 1.97561
i	deg	89.57141	89.42889	89.53818	89.57917 ± 0.37639	89.22655 ± 0.21654	89.31812 ± 1.15800
Ω	deg	0.00015	0.00000	0.00000	0.31487 ± 0.15366	2.29724 ± 2.17643	0.39499 ± 0.27136
χ^2	...	109.75800	3.80307	...	106.33254	4.53826	...
χ_{red}^2	...	4.73171	28.39027	...	4.61962	27.71770	...
$\ln \mathcal{L}$	215.40728	228.75014	...
BIC	-349.08633	-375.77205	...
AIC _c	-354.59234	-381.27806	...

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding the planets' low K 's generated by Exo-Striker.

stable after 2×10^5 orbits, only 37% were stable after 10^7 orbits. For the 6.47 days case and the 14.1 days case, the number of stable configurations fell by about 9% from 97% to 88% and from 100% to 91%, respectively. For the other cases, only minor effects or, in some cases, no effects at all were observed. However, the results are still consistent with SPOCK, since the 5.39 days case and the 6.47 days case are clearly disfavored here as well. For this reason, we assume that the SPOCK results provide a sound basis for assessing probable parameters for AU Mic d.

To make sure that the configurations' evolving instabilities over the larger timescale match the SPOCK results, we analyzed the fraction of equally classified configurations by SPOCK and Nbody. As SPOCK provides a stability probability instead of a binary stability classification, we have to set a limit above which all configurations are classified as stable in order to be able to compare the results with Nbody and MEGNO. For a first approach, we set this limit to 0.34 because Tamayo et al. (2020) used this limit to analyze the reliability of SPOCK as well. As Tamayo et al. (2021) remark that the chosen limit can influence the false-positive rate, we additionally try the limits 0.2 and 0.5 to make sure that differences in the results are not due to the selected threshold. Therefore, each configuration exceeding a stability probability of 0.34 is classified as stable, and each configuration below this limit is classified as unstable. It is important to mention that this limit is only necessary to directly compare SPOCK with Nbody and MEGNO. To weight the values in the histogram, the stability probabilities are used without a certain stability limit.

Due to the difference in the number of orbits after which the stability probability is predicted (10^9 for SPOCK and 2×10^5 and 10^7 for Nbody), we cannot consider the fraction of different classified configurations as the false-positive or false-negative rate, respectively. Nevertheless, we expect that the number of equally classified configurations should increase for the 10^7 run as the time span in which instabilities can occur grows. For most parameter sets the fraction of different classified systems is clearly below 10% for both the 2×10^5 and the 10^7 runs. Nevertheless, the cases of 5.39 days, 6.20 days, 6.47 days, and 14.1 days are exceptions and show stronger deviating results. For 6.47 days and 14.1 days, the fraction of different classified systems decreased from 13% and 12% to 7% and 5%. For these cases, the number of considered

orbits seems to be the main reason for the differences in the 2×10^5 run, and we assume that the Nbody results will approach closer to SPOCK for a longer Nbody run. However, the cases of 6.20 days and 5.39 days show a different behavior: Both parameter sets show large discrepancies for the 2×10^5 Nbody run (41% for 6.20 days and 46% for 5.39 days). Despite the decrease from 41% to 37% for the 6.20 day case, the fraction of different classified systems is still much larger than for the other parameter sets. For the 5.39 days case, the rate even increases from 46% to 53%, which corresponds to an almost random classification. To check whether the choice of the stability limit causes this effect, we compared the results again for a stability limit of 0.2 and 0.5 for SPOCK. For both limits, the fraction of differently classified configurations was about 50%. Although we cannot exclude the possibility that the parameter set develops instabilities on larger timescales, we would like to mention that both the 5.39 days case as well as the 6.20 days case belong to the sets for which the mutual inclinations do not match the SPOCK training sample. So, it is quite possible that this could be the reason for the deviations.

However, the behavior of MEGNO is particularly striking here. In the comparison with Nbody after 10^7 orbits, the proportion of differently classified systems for the case of 5.39 days was 64%. For the 6.20 days case the proportion is about 19%. Since SPOCK uses, among other features, MEGNO to estimate the stability of planetary configurations, this could be a decisive factor for the deviations. Nevertheless, for most of the parameter sets there is a very good agreement between SPOCK and Nbody, and even for the case of 5.39 days the trend indicates a higher number of unstable systems after a high number of orbits for both methods. This suggests that the trend shown by the SPOCK histogram may well be used as an indication of the stability of the system to at least partly limit the parameter space.

4.7. Most Favored Configuration for AU Mic d

Based on the calculated super-period (Section 4.1), the TTV model comparisons (Section 4.5), and the stability test (Section 4.6), the 12.7 days configuration is the most favored model and thus is presented in this section (Figures 14, 15, and 16, and Table 18), while the other TTV cases are presented in the Appendix. More discussions regarding the results can be found in Section 6.

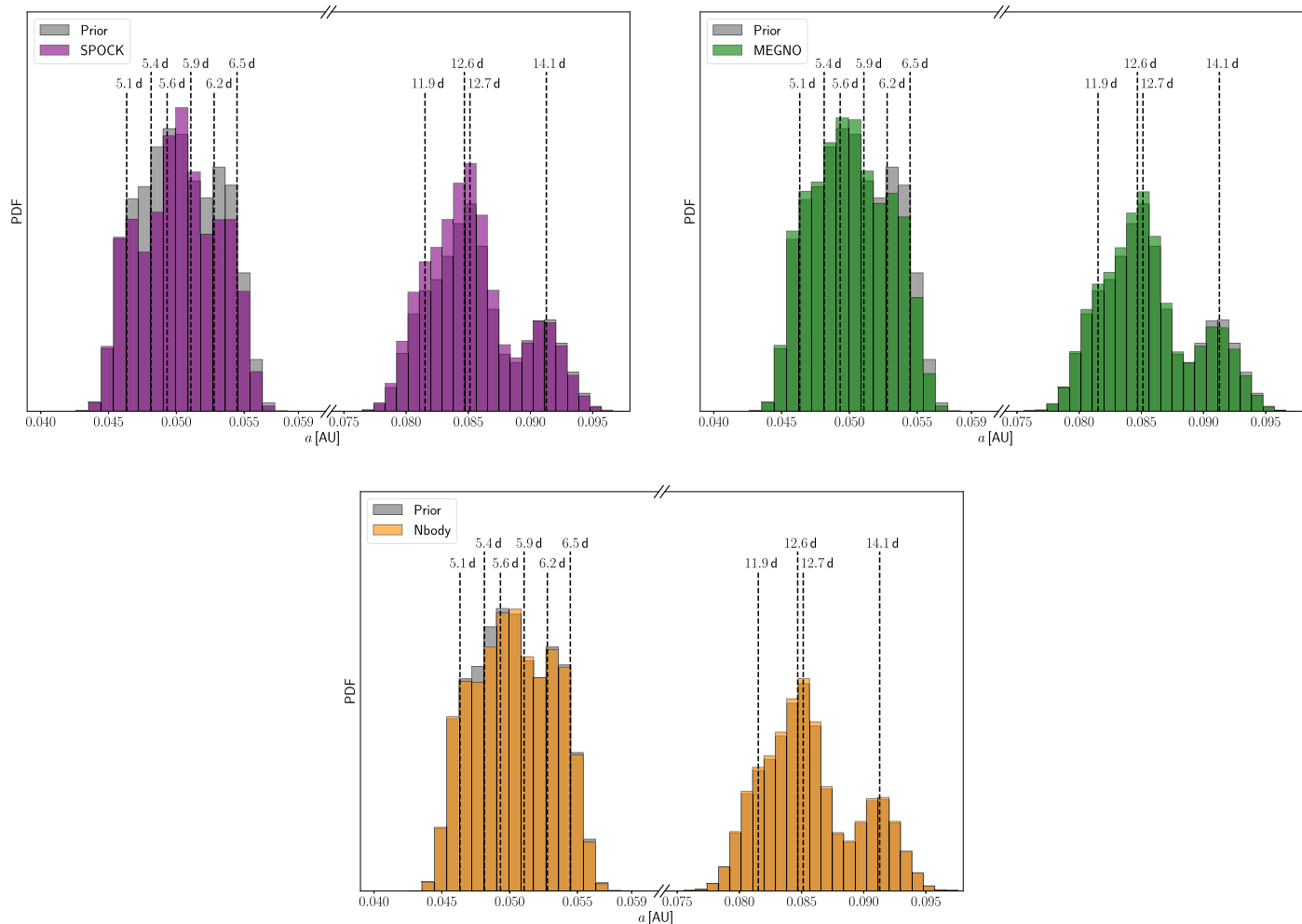


Figure 13. Histograms of the semimajor axis of AU Mic d with SPOCK (top left), MEGNO (top right), and Nbody (bottom), each marked with the corresponding orbital periods of AU Mic d. SPOCK and Nbody estimates the stability probability after 10^9 and 2×10^5 orbits, respectively, of the innermost planet, while MEGNO is a fast indicator for chaotic orbits. For each parameter set, 20,000 random configurations of masses and orbital parameters were generated following the respective Gaussian distributions of planetary parameters from Exo-Striker; each of these random configurations was given its own stability probability by SPOCK, and the semimajor axis of AU Mic d is then plotted in the histogram for each of these configurations. Since the same number of random configurations was generated for each parameter set, the result is a mixture distribution in which the respective underlying distributions have the same weighting. The values were then weighted with the stability probability in the histogram, resulting in the overlapping histogram for SPOCK, MEGNO, and Nbody. Thus, each histogram contains 200,000 values. Nbody uses the same aforementioned simulation object generated by rebound to perform Nbody simulations and outputs either 0 (unstable) or 1 (stable); these outputs are then used for weighting. Nbody is more accurate than SPOCK but is much more computationally intensive over a large simulation time, so both the 200,000-orbit integrations were performed and the pretrained classifier was used. The MEGNO value threshold is set to 2.1, so any configurations that lead to their MEGNO value exceeding this threshold are considered unstable and given a weight of 0 while all other configurations that are considered stable are given a weight of 1. By weighting with the SPOCK or MEGNO outputs, the 5.64 days case appears to be favored over other inner d cases, and both 12.6 and 12.7 days appear to be preferred among the middle d cases.

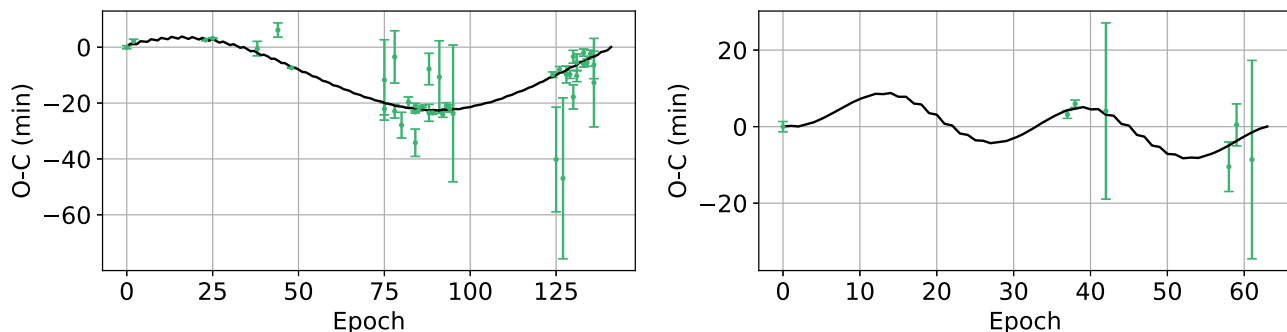


Figure 14. $O-C$ diagrams of AU Mic b (left) and AU Mic c (right), with comparison between TTVs (green) and Exo-Striker-generated MCMC models (black) for $P_d = 12.7$ days.

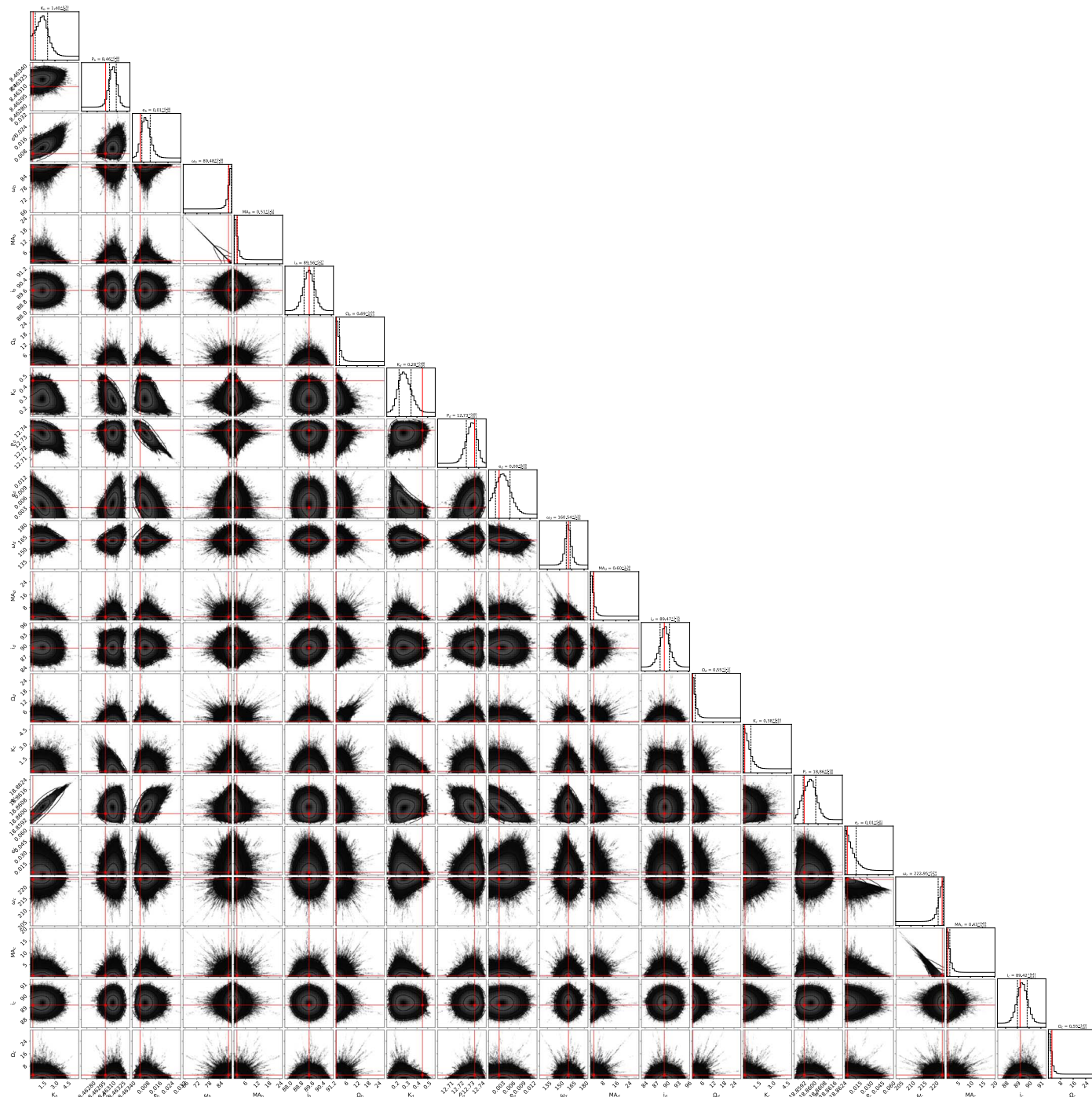


Figure 15. Corner plot of AU Mic b and c’s orbital parameters from Exo-Striker MCMC analysis for $P_d = 12.7$ days.

5. Radial Velocity Vetting of Transit Timing Variation Analysis

Cale et al. (2021) modeled the RVs of the AU Mic system and searched for additional candidate nontransiting planet signals. Their RV models indicated a candidate RV signal between the orbits of AU Mic b and c with an orbital period of 12.742 days but were ruled inconclusive.

We repeat the analysis of Cale et al. (2021) with the addition of the SERVAL-generated (Zechmeister et al. 2018) High Accuracy Radial Velocity Planet Searcher (HARPS) RVs from Zicher et al. (2022) to see if the RVs exhibit consistent behavior with the TTVs. We compute two generalized Lomb–

Scargle (GLS) periodograms, one without HARPS RV data and another with HARPS data included (Figure 17). The planet d’s signal was present in the periodogram without HARPS data; however, after adding HARPS data to the GLS periodogram, the signal vanishes. We also note the 16 days peak in the bottom row of the periodogram is not credible in terms of orbital dynamics stability.

We compute an $\ln \mathcal{L}$ brute-force periodogram (Figure 18) over a range of possible periods for AU Mic d. We do not see any clearly identified peaks consistent with d’s periods derived from the TTVs, but there is additional power in the periodogram, particularly near 12.75 (which is the exact 2:3

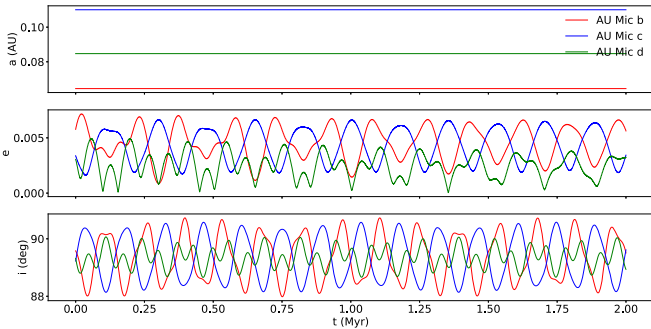


Figure 16. rebound model of the stability of the AU Mic system on a timescale of 2 Myr for $P_d = 12.7$ days.

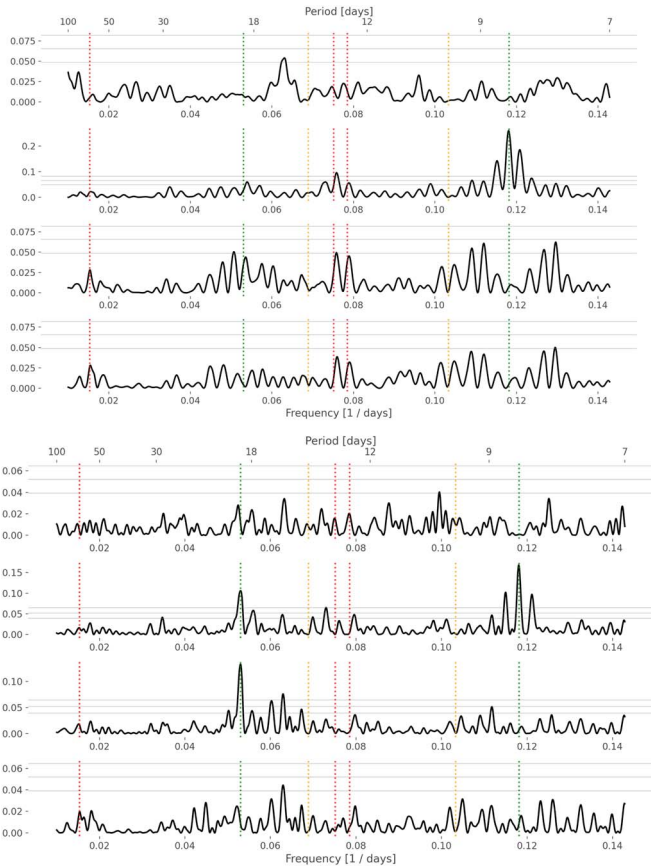


Figure 17. GLS periodograms of the AU Mic system. Top four panels: GLS generated without HARPS data. Bottom four panels: GLS generated with HARPS data. From top to bottom: zero-point corrected, activity corrected, planet b corrected, planet c corrected. The dotted vertical lines, from right to left, are P_b , 2^*P_{rot} , $P_d = 12.74$ days, $P_d = 13.31$ days, 3^*P_{rot} , P_c , $P_{\text{candidate}} = 65.22$ days. AU Mic d’s signal vanishes when HARPS data are added to the GLS periodogram. Also, the 16 days peak in the bottom row of the periodogram is not credible in term of orbital dynamics stability.

orbital resonance and our TTV period) and 13.3 days, indicating our RV model is incomplete and, therefore, our RV analysis is inconclusive in confirming AU Mic d. The 22 days peak would not be dynamically stable, and there is the possibility of a 65 days planetary candidate in the RVs.

We explore the RV model comparison for ten different periods of AU Mic d (Table 19). The three-planet configuration is favored relative to the two-planet configuration for 6.20 days ($\Delta\text{AIC}_c = 104.83$), 5.39 days ($\Delta\text{AIC}_c = 48.64$), 5.64 days ($\Delta\text{AIC}_c = 36.77$), 12.7 days ($\Delta\text{AIC}_c = 23.63$), 6.47 days

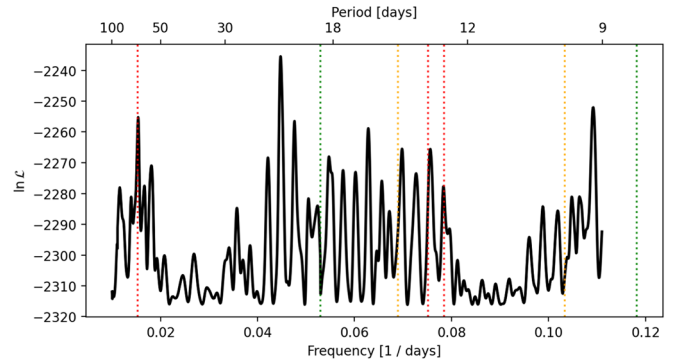


Figure 18. $\ln \mathcal{L}$ brute-force periodogram of AU Mic’s RVs, including models for both AU Mic b and c. AU Mic d’s T_C was allowed to vary. The dotted vertical lines, from right to left, are P_b , 2^*P_{rot} , $P_d = 12.74$ days, $P_d = 13.31$ days, 3^*P_{rot} , P_c , $P_{\text{candidate}} = 65.22$ days. There appears to be no obvious peaks consistent with d’s periods derived from the TTVs, but there is additional power in the periodogram, particularly near 12.75 and 13.3 days. Also note that the 22 days peak would not be dynamically stable, and that there is the possibility of a 65 days planetary candidate in the RVs.

($\Delta\text{AIC}_c = 19.12$), 11.9 days ($\Delta\text{AIC}_c = 12.66$), and 5.08 days ($\Delta\text{AIC}_c = 11.74$) cases, while the two-planet configuration is favored relative to the three-planet configuration for 12.6 days ($\Delta\text{AIC}_c = 9.61$), 14.1 days ($\Delta\text{AIC}_c = 46.22$), and 5.86 days ($\Delta\text{AIC}_c = 69.79$) cases. Based on the log-likelihood, ΔBIC , and ΔAIC_c , the 6.20 days is the most favored one overall and the 12.7 days is the most favored among the three-planet cases. However, the TTV analysis indicated that planet d is of very small mass compared to AU Mic b and c, so it is not surprising that the RV model struggles to detect AU Mic d with any statistical significance.

6. Discussion

We modeled the transits of AU Mic b and c to obtain the midpoint times, which we then use to model the TTVs of the AU Mic system. We attempted both two-planet and three-planet dynamical models and found that the three-planet model adequately describes the observed TTVs, including its super-period. Moreover, we generated TTV periodograms and found ten possible solutions for the period of the nontransiting planet AU Mic d. In the next subsections, we discuss the impact of AU Mic’s stellar activity on the TTVs, the convergence of transit models, the TTV and RV model comparisons, the coplanarity of AU Mic system, the planetary resonant chains, the mass of AU Mic d, the inner d orbits versus middle d orbits, and the validation versus confirmation of AU Mic d.

6.1. Impact of AU Mic’s Stellar Activity on Transit Timing Variations

Wittrock et al. (2022) explored the impacts that stellar flares and spot modulations may have on the TTV profile. Given the 7:4 spin–orbital commensurability between AU Mic and planet b (Szabó et al. 2021) and the long lifetime of AU Mic’s starspots, Wittrock et al. (2022) modeled the TTVs induced by AU Mic’s spot crossing during AU Mic b’s transit and found the effect to be relatively minimal (no more than ~ 2 minutes) compared to TTVs of planet b induced by planets c and d and with no clear pattern. Martioli et al. (2020), Palte et al. (2020), Gilbert et al. (2022), Wittrock et al. (2022), and Szabó et al. (2021, 2022) modeled the stellar flares from SPIRou, ESPRESSO, TESS, Spitzer, and CHEOPS observations,

Table 19
RV Model Information Criterion for the AU Mic System

P_d (days)	Planets	χ^2_{red}	$\ln \mathcal{L}$	ΔBIC	ΔAIC_c	
5.08	b, c, d	4.00978	-2248.46411	0.00000	0.00000	
	b, c	4.10715	-2257.68857	0.85803	11.74349	
	b, d	4.16747	-2271.93853	41.0852	44.70022	
	b	4.14185	-2280.01400	39.64525	54.18589	
	c, d	4.15293	-2283.78303	64.7742	68.38922	
	c	4.24211	-2302.43094	84.47913	99.01977	
	d	4.33652	-2320.10267	131.54986	138.79341	
	-	4.27910	-2327.09888	127.95138	146.16034	
	5.39	b, c, d	3.95174	-2248.37326	0.00000	0.00000
		b, d	4.14993	-2269.88140	37.15265	40.76767
b, c		4.24239	-2276.04763	37.75784	48.64331	
c, d		4.05626	-2275.93760	49.26505	52.88007	
c		4.19304	-2287.77753	55.35401	69.89465	
b		4.10217	-2298.35680	76.51256	91.05320	
d		4.19466	-2305.18164	101.88949	109.13304	
-		4.60850	-2353.45811	180.85153	199.06049	
5.64	b, c, d	3.90634	-2223.31676	0.00000	0.00000	
	b, c	3.95149	-2245.05276	25.88113	36.76659	
	c, d	3.95899	-2248.70048	44.90381	48.51884	
	b, d	3.91049	-2252.70267	52.90820	56.52322	
	b	4.18677	-2279.81918	89.55033	104.09097	
	c	4.24168	-2292.44964	114.81124	129.35188	
	-	4.24294	-2311.41267	146.87368	165.08264	
	d	4.37061	-2332.12947	205.89816	213.14171	
	5.86	b, c	3.95261	-2247.52212	0.00000	0.00000
		b	3.99113	-2267.43656	33.96527	37.62044
b, c, d		4.24926	-2279.06355	80.67377	69.78830	
b, d		4.26988	-2287.46508	91.61319	84.34275	
c, d		4.24476	-2291.16124	99.00551	91.73507	
d		4.24737	-2307.05552	124.93044	121.28852	
c		4.27718	-2315.40526	129.90265	133.55782	
-		4.33599	-2318.39795	130.02440	137.34790	
6.20	b, c, d	3.76222	-2223.32578	0.00000	0.00000	
	b	4.04026	-2266.56250	63.01893	77.55957	
	b, c	4.11199	-2279.09511	93.94778	104.83324	
	b, d	4.19250	-2280.76895	109.02271	112.63774	
	c	4.18665	-2289.96236	109.81864	124.35928	
	c, d	4.20633	-2299.23938	145.96358	149.57860	
	-	4.22326	-2311.14636	146.32301	164.53197	
	d	4.41166	-2331.23825	204.09768	211.34123	
	6.47	b, d	3.82012	-2242.37068	0.00000	0.00000
		b, c, d	4.17966	-2268.82978	58.78184	55.16681
b, c		4.06131	-2281.74255	67.01647	74.28691	
b		4.16875	-2290.60045	78.86866	89.79428	
c, d		4.11485	-2289.43973	94.13811	94.13811	
c		4.28375	-2313.19199	124.05174	134.97736	
-		4.27356	-2315.79337	123.39086	137.98480	
d		4.48229	-2339.34545	188.08591	191.71444	
11.9	b, c, d	4.06977	-2256.91123	0.00000	0.00000	
	b	4.08732	-2266.27497	-4.72704	9.81360	
	b, c	4.13643	-2266.59584	1.77833	12.66379	
	b, d	4.11093	-2270.03862	20.39115	24.00618	
	c	4.29308	-2306.57442	75.87187	90.41251	
	d	4.31650	-2317.55744	109.56516	116.80871	
	-	4.39886	-2321.13976	99.13891	117.34787	
	c, d	4.42181	-2342.39152	165.09695	168.71197	
	12.6	b, c	3.93956	-2242.04753	0.00000	0.00000
		b, c, d	3.95070	-2243.49828	20.49239	9.60693
b, d		4.03554	-2253.46944	34.57107	27.30063	
c, d		4.16405	-2283.63677	94.90575	87.6353	
c		4.15953	-2288.90894	87.85919	91.51436	
b		4.21603	-2299.51198	109.06527	112.72045	
d		4.37735	-2325.24047	172.24951	168.6076	
-		5.01924	-2454.82572	413.82911	421.15261	

Table 19
(Continued)

P_d (days)	Planets	χ^2_{red}	$\ln \mathcal{L}$	ΔBIC	ΔAIC_c	
12.7	b, d	3.82300	-2230.60095	0.00000	0.00000	
	b, c, d	3.93925	-2244.69089	34.04350	30.42848	
	b, c	4.03965	-2259.85878	46.78839	54.05884	
	c, d	4.06094	-2265.50468	69.80745	69.80745	
	b	4.06797	-2276.96998	75.14716	86.07278	
	d	4.15544	-2283.89919	100.73284	104.36137	
	-	4.21856	-2316.20370	147.75097	162.34491	
	c	4.32722	-2330.15944	181.52608	192.45169	
	14.1	b, d	4.00062	-2264.98707	0.00000	0.00000
		b, c	4.08155	-2278.10035	14.49931	21.76975
b		4.32125	-2291.32638	35.08774	46.01336	
c		4.25284	-2293.92168	40.27834	51.20395	
c, d		4.20466	-2290.74089	51.50765	51.50765	
b, c, d		4.35643	-2297.85791	71.60532	67.99030	
d		4.42068	-2319.25242	102.66709	106.29561	
-		4.66740	-2392.62944	231.83023	246.42417	

respectively, which marginalized the effect of flares on the observed TTVs. While `AIJ` and `EXOFASTv2` are not yet capable of jointly modeling the flares during transits, the ground-based observations have relatively lower photometric precision and larger timing uncertainties, so the flares' effect on the TTVs are significantly down-weighted within the photometric precision timing uncertainties (Wittrock et al. 2022).

6.2. Convergence of Transit Models

It can be challenging to get transit model convergence when using standard MCMC methods such as the `emcee` package uses (Foreman-Mackey et al. 2013), especially with a large number of model parameters. However, as mentioned in Section 3, `EXOFASTv2` is written in `IDL` and uses the differential evolution MCMC algorithm instead of the `emcee` package. As part of the convergence criteria, `EXOFASTv2` simultaneously employs two metrics, the Gelman–Rubin statistic and the number of independent draws, to help minimize the individual shortcomings of these convergence criteria (Gelman & Rubin 1992; Goodman & Weare 2010). We were able to use fairly tight priors on most of the parameters (Table 4) and had `rejectflatmodel` switched on to ease the convergence of our transit models. Numerous different walkers, while not fully independent from each other, are found to produce posteriors that are relatively consistent with one another, convincing us that the independent chains do converge. See Eastman et al. (2013) and Eastman et al. (2019) for more information on `EXOFASTv2`'s use of the MCMC algorithm and convergence criteria, and Section 3 of this paper for details on our process with `EXOFASTv2`.

6.3. Dynamically Settled Assumption of the AU Mic System

In Section 4.3, we presented a high-eccentricity two-planet TTV model that does exhibit the super-period (Figure 4), making it seems a potentially good fit for a two-planet system with a stable but unlikely eccentric-aligned configuration with abnormally small masses inconsistent with the RV masses from Cale et al. (2021) and Donati et al. (2023). After having the very small masses replaced with those RV masses, the high-eccentricity two-planet model provides an example of an

unstable eccentric system that does model the TTVs and does persist for 20 Myr, as seen in Figure 6, but it is clearly not dynamically settled given the apparent orbital migrations of AU Mic b and c and the extreme fluctuations in both planets' eccentricities and inclinations in the rebound simulations.

We must caution that rebound does not include tidal circularization and general relativity (GR) precession, which are relevant at these orbital periods, and highly eccentric planets do not exist in older systems. So, in theory, planet d may not exist if the system is currently dynamically unstable. However, future TTVs will clearly rule out this case, and current RVs from Cale et al. (2021) and Donati et al. (2023), while challenging to constrain the eccentricity, do not give any clear indication that this system is highly eccentric. Additionally, the secondary-eclipse model of AU Mic b by K. I. Collins et al. (2023, in preparation) rules out high eccentricity and provides a vastly different argument of periastron ($\sim 90^\circ$ as opposed to our 173° from Table 12). The dynamically unstable case does not account for the fact that the host star was larger in the recent past, so the planets would have likely collided with the star prior to the 20 Myr mark at these high eccentricities. Thus, we likely can rule out this scenario, but more work is needed on dynamical simulations that include GR precession, tidal circularization, and any disk drag from the past for this star.

6.4. Overall Model Comparisons of the AU Mic System

In Section 4.4, we modeled our TTVs and then generated the TTV log-likelihood periodograms to explore a set of planet d's orbital periods that would be in agreement with the observed TTVs of the AU Mic system; it was through the periodograms that we obtained ten possible orbital periods and orbital configurations for AU Mic d. Given that the high-eccentricity two-planet configuration is ruled out for reasons provided in the previous section (Section 6.3), that the low-eccentricity two-planet configuration has both b and c being misaligned despite their well-known transiting status (Plavchan et al. 2020; Martioli et al. 2021; Gilbert et al. 2022; Wittrock et al. 2022), and that both two-planet models are statistically ruled out at high confidence (Tables 16 and 17), this validates AU Mic d, and AU Mic d must possess one of these ten orbital periods and orbital configurations. Based on our TTV model comparison (Tables 16 and 17), our calculated super-period (Figure 3 and Table 9), our RV analysis, and our stability tests (Figure 13), the 12.7 days configuration is the most favored period for AU Mic d.

In the meanwhile, the 5.64 days case is favored by the RVs and the dynamical stability tests but is strongly disfavored by the TTVs. The 5.39, 6.20, and 6.47 days cases are favored by the TTVs and RVs but are disfavored by the stability tests. The 5.08 and 11.9 days cases are favored by the RVs but are disfavored by the TTVs and stability tests. The 12.6 days case is favored by the stability tests but is disfavored by the TTVs and RVs. Lastly, the 5.86 and 14.1 days cases are favored by the TTV but are disfavored by the RVs and stability tests.

Thus, while we do not statistically rule out the other nine potential periods for AU Mic d, they are all statistically disfavored to varying degrees for one reason or another. Since we do not statistically rule out these other potential orbital periods for AU Mic d, we do not consider AU Mic d to be confirmed, only statistically validated. Further, in addition to AU Mic d at its likely orbital period of 12.7 days, it is entirely

plausible there could be an additional fourth planet at 5.39, 6.20, or 6.47 days interior to AU Mic b, which is favored by the TTVs and RVs and is disfavored by the dynamical stability tests. Our existing data are unable to explore this latter possibility.

6.5. Coplanarity of the AU Mic System

One curious feature among the Exo-Striker MCMC models is the inclination of AU Mic d for these ten candidate periods of AU Mic d. AU Mic d is nearly coplanar for the 5.64, 12.6, and 12.7 days cases, with $i_d = 90.08^\circ \pm 4.99^\circ$, $89.63^\circ \pm 1.71^\circ$, and $89.32^\circ \pm 1.16^\circ$, respectively, but the 5.64 days case is strongly disfavored by the TTVs and the 12.6 days case is disfavored by both the TTVs and RVs. For the 6.47 days case, AU Mic d is slightly misaligned at $i_d = 93.57^\circ \pm 3.26^\circ$ but is strongly disfavored from the dynamical stability tests. The remaining candidate periods required considerable misalignment with $i_d = 76.88^\circ \pm 3.05^\circ$, $57.19^\circ \pm 1.83^\circ$, $99.59^\circ \pm 6.78^\circ$, $107.16^\circ \pm 14.19^\circ$, $81.89^\circ \pm 0.64^\circ$, and $74.27^\circ \pm 0.40^\circ$ for the 5.08, 5.39, 5.86, 6.20, 11.9, and 14.1 cases, respectively.

We do not know if the planet formation process could produce such a highly misaligned planet in this young system, especially for the cases between the two coplanar, transiting planets, but we can invoke Occam's razor, which argues that the simplest configurations are the 5.64, 12.6, and 12.74 days cases since these configuration have AU Mic d being coplanar with the system. As discussed in Picogna & Marzari (2014), misalignments are considered to be a particular signature of a past flyby, with a maximum tilting value of 9° , while multiple flybys can lead to higher misalignments. However, after reconstructing the flybys of debris disks using the Gaia Early Data Release 3 data, Bertini et al. (2023) found that AU Mic did not experience any flyby in the last 5 Myr, so any misalignment induced by a recent close flyby appears unlikely. It is also important to point out that if a flyby had occurred, we would expect to see both AU Mic b and c become highly misaligned along with AU Mic d. Moreover, most of the Kepler compact multiplanet systems are well-aligned with a scatter of $\pm 3^\circ$ (Lissauer et al. 2011; Fang & Margot 2012; Fabrycky et al. 2014). These are yet further reasons why we favor the 12.74 days period for the validated AU Mic d, but additional TTV observations will be needed to confirm that this is the case.

6.6. Near-resonant Chains of the AU Mic System

Since AU Mic b exhibits such strong TTVs, one consequence of all ten candidate periods for AU Mic d is that they all place the AU Mic system near a resonant chain of varying order. For the inner d scenario, the near resonances between d and b are 3:5 for 5.08 days, 5:8 for 5.39 days, 2:3 for 5.64 days, 5:7 for 5.86 days, 3:4 for 6.20 days, and 7:9 for 6.47 days. The near-orbital resonance between planets b and c for all inner d cases is 4:9, so the overall near-resonant chains are 12:20:45 for 5.08 days, 5:8:18 for 5.39 days, 8:12:27 for 5.64 days, 20:28:63 for 5.86 days, 3:4:9 for 6.20 days, and 28:36:81 for 6.47 days. For the 11.9 days case, the near resonances between b and d and between d and c are 5:7 and 5:8, or near a 25:35:56 chain overall. Both the 12.6 days and 12.7 days cases have both pairs near a 2:3 chain, or near a 4:6:9 chain overall. The 14.1 days case pairs are near 3:5 and 3:4 chains, or near a 9:15:20 chain overall. These near-MMR chains are the reason why our

Table 20
Masses, Radii, and Time of Conjunctions of AU Mic d

P_d (days)	M_d (M_\oplus)	M_d (M_J)	R_d (R_\oplus)	R_d (R_J)	$T_{C,d}$ (BJD)
5.08	0.737 ± 0.083	0.00232 ± 0.00026	0.926 ± 0.029	0.0826 ± 0.0002	$2458330.88624 \pm 0.16079$
5.39	0.667 ± 0.042	0.00210 ± 0.00013	0.900 ± 0.016	0.0803 ± 0.0001	$2458330.79993 \pm 0.29196$
5.64	0.469 ± 0.045	0.00148 ± 0.00014	0.816 ± 0.022	0.0728 ± 0.0001	$2458331.07793 \pm 0.30077$
5.86	1.049 ± 0.239	0.00330 ± 0.00075	1.021 ± 0.065	0.0911 ± 0.0005	$2458331.46003 \pm 0.28115$
6.20	0.904 ± 0.482	0.00284 ± 0.00152	0.980 ± 0.146	0.0874 ± 0.0011	$2458331.66688 \pm 1.08314$
6.47	0.510 ± 0.112	0.00161 ± 0.00035	0.836 ± 0.051	0.0745 ± 0.0003	$2458331.03312 \pm 0.33184$
11.9	0.392 ± 0.079	0.00123 ± 0.00025	0.776 ± 0.044	0.0693 ± 0.0003	$2458339.91558 \pm 0.08663$
12.6	0.522 ± 0.147	0.00164 ± 0.00046	0.841 ± 0.066	0.0750 ± 0.0004	$2458339.99167 \pm 0.18465$
12.7	1.053 ± 0.511	0.00331 ± 0.00161	1.023 ± 0.139	0.0912 ± 0.0011	$2458340.55781 \pm 0.11641$
14.1	0.628 ± 0.092	0.00198 ± 0.00029	0.885 ± 0.036	0.0790 ± 0.0003	$2458341.05771 \pm 0.20391$

Notes. The masses were calculated using the parameters from *Exo-Striker* MCMC models. AU Mic d is not known to transit, so its measured radius and density are unknown. However, we use the Chen–Kipping mass–radius relation (Chen & Kipping 2017) to estimate the radii of AU Mic d.

TTV periodograms (Figures 10 and 12) yielded ten possible solutions, each with relatively high log-likelihood.

Suggestively, as per the Occam’s razor argument, the near 4:6:9 resonant chain (or near 3:2 pairings between d and b and between c and d) is the simplest one, given that our TTV and RV models, dynamical stability tests, and the calculated TTV super-period favor the 12.7 days case (the 12.6 days case also shares this near-MMR chain but is not as favored). The 3:2 resonance is the most common among the resonant chains; the 4:3, 5:3, 7:5, and 8:5 resonances are not as common, respectively, but are not unprecedented; and the 9:4 and 9:7 resonances do not appear to be very common (Fabrycky et al. 2014). Thus, the distribution of near-MMR chains among the known exoplanets gives stronger credence to the potential near 3:2 resonant chain pairs of the AU Mic system. Regardless of which of the ten candidates periods for AU Mic d is correct, with the 12.7 days case being most favored, all place the AU Mic system near a resonant chain, with the 12.7 days case being the simplest commensurability chain configuration. The presence of near-MMRs in such a young system poses significant constraints for the formation model and evolution of planetary systems. The similarly young V1298 Tau system is also near a similar resonant chain configuration, suggesting this may be a common characteristic of the formation of compact multiplanet systems (David et al. 2019a, 2019b; Tejada Arevalo et al. 2022). This implies that the planetary systems can develop resonant chains very early on, particularly in compact systems, which in turn would quickly establish the stability of the dynamical systems.

6.7. Characterizing AU Mic d

We calculate the mass of AU Mic d (Table 20) using the parameters generated from *Exo-Striker* MCMC models and Equation (3) (Cumming et al. 1999):

$$K = \left(\frac{2\pi G}{P_{\text{orb}}} \right)^{1/3} \frac{M_p \sin i}{(M_* + M_p)^{2/3}} \frac{1}{\sqrt{1 - e^2}}. \quad (3)$$

Since AU Mic d is not known or observed to transit, its radius, and therefore its density, is unknown.

The mass of AU Mic b is poorly constrained from the TTVs due to the limited amount of transit data from AU Mic c and that AU Mic d is not known to be transiting, so we do not derive a TTV mass constraint for AU Mic b. AU Mic c is better constrained given the wealth of transit data from AU Mic b, but since there is no transit data from AU Mic d, we are unable to

meaningfully constrain the mass of AU Mic c either. However, the mass of AU Mic d is the most constrained of the three planets due to the availability of transit data from both planets b and c. The most favored model ($P_d = 12.7$ days) has the mass $M_d = 1.053 \pm 0.511 M_\oplus$, which makes AU Mic d Earth-like in mass. The 5.86 and 6.20 days cases also have AU Mic d’s mass being Earth-like but smaller, while the rest of the cases have AU Mic d being of even smaller mass but still not as small as that of Mars; for comparison, the mass of Mars is $M_{\text{Mars}} = 0.107 M_\oplus$.

These relatively small masses for AU Mic d partially explain the challenges RVs face in characterizing such a planet and the divergent results between TTVs and RVs in which periods for AU Mic d were favored. Additionally, given that the host star AU Mic is highly active, it is incredibly challenging to validate AU Mic d with RVs with current-generation RV measurements and techniques for modeling and mitigating stellar activity. However, given that our RV models indicated the presence of a potential additional planet beyond AU Mic c, it will still be worthwhile to continue to intensely monitor the AU Mic system with RVs.

Given that the 12.7 days case is the most plausible one based on the super-period (Section 4.1), the stability tests (Section 4.6), overall model comparisons (Section 6.4), and the Occam’s razor arguments from Sections 6.5 and 6.6, AU Mic d is most likely to have a roughly Earth-like mass. If confirmed, this would be the first known Earth-mass planet orbiting a young star. If it does transit, it will provide a crucial and valuable opportunity in probing its atmosphere and understanding the evolution of terrestrial planets’ atmosphere.

We use the Chen–Kipping mass–radius relation (Chen & Kipping 2017) to estimate the radius of AU Mic d (Table 20). For $P_d = 12.7$ days, AU Mic d has a radius $1.023 \pm 0.139 R_\oplus$, making its size very Earth-like. The 5.86 and 6.20 days cases still have AU Mic d being Earth-like in size, while the remaining cases have it being smaller, with the smallest size being $0.776 \pm 0.044 R_\oplus$ from the 11.9 days case (for comparison, Mars’ radius is $R_{\text{Mars}} = 0.532 R_\oplus$).

We obtain $T_{C,d} = 2458340.55781 \pm 0.11641$ BJD for $P_d = 12.7$ days (Table 20). Assuming AU Mic d is coplanar with the other planets, we predict the time of transits for AU Mic d and map them onto the raw TESS photometry of AU Mic (Figures 19 and A14).

Any potential transit signatures of AU Mic d are not readily apparent by eye in the TESS photometry. Based on the model of AU Mic c’s depth uncertainty from Gilbert et al. (2022), we

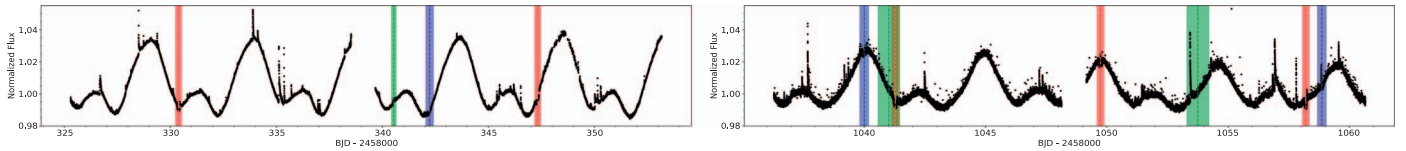


Figure 19. Raw TESS photometry of AU Mic overlaid with observed transits of AU Mic b (red) and c (blue) and range of predicted transit midpoint times for AU Mic d (green). This case is for $P_d = 12.7$ days. Left panel is from Cycle 1, and right panel is from Cycle 3. Owing to AU Mic d’s estimated 160 ppm depth and AU Mic’s intense stellar activity, the potential transit signatures from AU Mic d are not readily apparent by eye, nor detectable at high statistical significance in the TESS light curve given the stellar activity and photon noise.

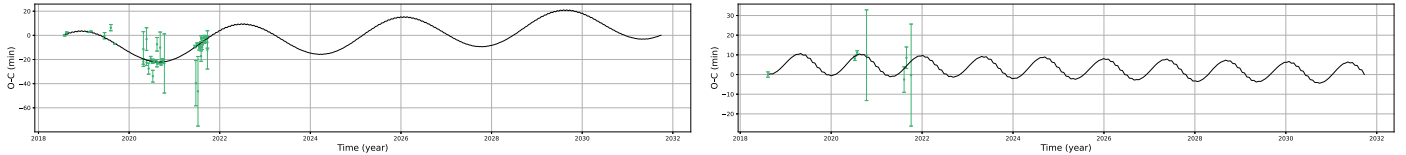


Figure 20. Exo-Striker-generated TTV models of AU Mic b (left) and c (right) projected over 10 yr since last observed transit of the AU Mic system for $P_d = 12.7$ days. The observed TTVs (green) are included with the model (black).

place a 3σ upper limit on AU Mic d’s transit of 460 ppm. With the estimated radius $R_d = 1.023 \pm 0.139 R_{\oplus}$ for AU Mic d (Table 20), its transit depth would be ~ 160 ppm. AU Mic d’s small radius and very shallow transit depth could explain its lack of transit detection, especially in the presence of AU Mic’s intense stellar activity. Future multiwavelength, high-precision, space-based photometry could potentially confirm whether or not AU Mic d transits.

We next estimate whether or not a transit of AU Mic d could be recovered with any statistical significance. The point-to-point white-noise scatter in the AU Mic TESS light-curves model from Gilbert et al. (2022) is 550 ppm for the 20 s cadence data and 370 ppm for the 2 minutes cadence data. There is also the variability in the Gaussian process (GP) component of their model, which has an amplitude of ~ 3000 ppm and is periodic over ~ 0.6 days; this periodic noise is, in principle, removable with the GP process, so the white noise is the remaining limiting factor. As an example, let us consider the 2 minutes cadence data with the 370 ppm scatter. In 1 hr, that noise averages down to ~ 70 ppm. Therefore, to obtain a signal-to-noise ratio ($S/N = 7$) for a 160 ppm transit, we would need approximately 10 hr of transit data. The 20 s cadence data with the 550 ppm scatter is less noisy per unit time; the reason for this is partially unknown, but is believed to be not significantly caused by the cosmic-ray correction (Huber et al. 2022). In that case, the noise averages down to ~ 40 ppm in 1 hr; this would require approximately 4 hr of transit data to achieve a $S/N = 7$ for a 160 ppm transit. Therefore, accounting for the photon noise alone, given the transit duration of AU Mic d would be less than 4 hr if it transited, there is marginal photon noise to recover a statistically significant detection off AU Mic d in the TESS light curves, but this statistical significance would be degraded even further after accounting for the additional uncertainty introduced from the stellar activity. This is best evidenced by the recovered depth uncertainty for AU Mic c of 1.15 ± 0.22 ppt.

Future work is needed to involve a thorough search of all existing space-based light curves of the AU Mic system for any statistically significant 160 ppm signal to confirm AU Mic d’s transiting nature and to tighten the predicted timing of its transits. This will also require additional transit observations of AU Mic to minimize the photon shot noise such that AU Mic d’s transits can potentially be detected. To further constrain this system, and confirm and distinguish the 12.7 days period for

AU Mic d relative to the other nine candidate periods, additional precise space-based transit timing observations of AU Mic b and especially AU Mic c are needed. For this purpose, we modeled the projected TTVs of AU Mic b and c over the course of 10 yr since the last observed transit in the AU Mic system (Figures 20 and A15).

6.8. Inner d Orbits versus Middle d Orbits

Is AU Mic d interior or exterior to AU Mic b? The TTV analysis (Tables 16 and 17) favors AU Mic d to be between AU Mic b and c, and this is additionally supported by the Occam’s razor arguments covered in Sections 6.5 and 6.6, whereas the SPOCK, MEGNO, and Nbody stability analyses (Figure 13) and the RV analysis (Table 19) appear to favor AU Mic d lying interior to AU Mic b. However, this kind of comparison can only be done to a limited extent because the time periods after which the stability is estimated vary. For instance, SPOCK can only predict the stability probability after 10^9 orbits of the innermost planet. If AU Mic d is interior to b, this corresponds to about 14 Myr, but if AU Mic d is between b and d, this corresponds to about 23 Myr. Therefore, the results for the two ranges are not directly comparable, since the longer timescale could naturally make more systems unstable, sorting out more configurations, and making the AU Mic d between b and c case seem less likely than it is. Additionally, the current RV models, including those from Cale et al. (2021) and Zicher et al. (2022), struggle in detecting low-mass planets with $K_d < 1 \text{ m s}^{-1}$ like AU Mic d and most especially in the presence of heightened stellar activity such as that of AU Mic.

The determination of these cases is further complicated by the fact that AU Mic d’s transiting nature is unknown. If AU Mic d does not appear to transit, it can mean that either the planet is misaligned or is so small such that its transit signature is masked by the instrumental and photon noise and/or host star’s activity. Notably, almost all of the TTV orbital configurations that were explored in this paper require d to be misaligned with the other two planets, with the exception of the 5.64, 12.6, and 12.7 days cases. Additionally, AU Mic d in the inner d cases would have a higher probability of transiting AU Mic than in the middle d cases (Table 21). We also found AU Mic d to be relatively small ($\sim 1.02 R_{\oplus}$ assuming d is Earth-like in density), so its transit signature would be very

Table 21

Probabilities of AU Mic d Transiting Based on Its Corresponding Distances from the Host Star Using the Simplest Estimate $P_{tr} = R_s/a_d$ (Borucki & Summers 1984; Sackett 1999)

P_d (days)	a_d (au)	P_{tr}
5.08	0.0462 ± 0.0008	0.0749 ± 0.0027
5.39	0.0481 ± 0.0009	0.0720 ± 0.0026
5.64	0.0495 ± 0.0009	0.0698 ± 0.0025
5.86	0.0508 ± 0.0009	0.0681 ± 0.0024
6.20	0.0528 ± 0.0010	0.0655 ± 0.0024
6.47	0.0543 ± 0.0010	0.0637 ± 0.0023
11.9	0.0813 ± 0.0015	0.0425 ± 0.0015
12.6	0.0849 ± 0.0016	0.0408 ± 0.0015
12.7	0.0853 ± 0.0016	0.0406 ± 0.0015
14.1	0.0912 ± 0.0017	0.0380 ± 0.0014

shallow (~ 160 ppm, which is below the 460 ppm limit for resolving transits in the AU Mic TESS light curves).

Given the complexity of the resonance orders of the different candidate periods, Occam’s razor suggests that the 12.7 days period (and thus the middle d case) is the most likely scenario, given that this configuration has the simplest near-MMR chains and with planet d being coplanar with the system (as discussed in Sections 6.6 and 6.5, respectively). However, we do not rule out the possibility of an interior planet to AU Mic b and the near-2:3 resonance.

With the small mass $M_d = 1.053 \pm 0.511 M_{\oplus}$ of AU Mic d (based on 12.7 days being the most plausible case), this would make d the first Earth-mass planet orbiting a known young star. Thus, d will serve as a very valuable target for a test case study involving the evolution of terrestrial planets and their atmosphere, provided that AU Mic d does transit. Kane et al. (2022) determined that AU Mic d presently does not lie within the habitable zone (HZ). While it is true that AU Mic is still in its pre-main sequence phase and that the HZ regions are predicted to migrate relatively quickly toward the host star until AU Mic becomes a main sequence star (Kane et al. 2022), AU Mic d will still be well interior to the innermost boundary of the HZ. This implies that AU Mic d may become more like Venus instead of Earth. Additionally, the compact near-MMR orbits mentioned in Section 6.6 could lead to significant tidal heating of AU Mic planets’ interior, particularly that of AU Mic d (e.g., Bolmont et al. 2013; Van Laerhoven et al. 2014; Luger et al. 2017), but this effect is not further explored herein.

6.9. Is AU Mic d a Candidate, Validated, or Confirmed Planet?

Throughout this work, we have characterized AU Mic d as a “validated” planet and not as a “confirmed” or “candidate” planet. We consider the threshold for “confirmed” planet to be that it is detected through a second technique that unequivocally rules out false-positive scenarios (even if its orbital period is not well-known), for “validated” planet to be that the planet is statistically verified (even if its orbital period is ambiguous), and for “candidate planet” to be that there is a signal which may be due to a planet but is not statistically verified. Thus, by our definition, “validated” is more conservative than “confirmed.” The main point of “validation” is that we have statistically proven AU Mic d must exist (TTV model comparisons from Tables 16 and 17 and most of RV model comparisons from Table 19 consistently ruled out the two-planet configurations), even if there is some ambiguity in

its orbital period, and Wittrock et al. (2022) ruled out TTVs driven by stellar flares and spot modulations. Moreover, we have come across several published examples where the planets that are “confirmed” via the TTV detection technique apparently may have several possible orbital periods or do not need their orbital period to be well-defined or even known; such examples include Kepler-19 c (Ballard et al. 2011), Kepler-82 f (Freudenthal et al. 2019), Kepler-160 d (Heller et al. 2020), Kepler-448 c (Masuda 2017), Kepler-539 c (Mancini et al. 2016), and KOI-984 c (Sun et al. 2022). We should also point out that many direct-imaging planets are considered confirmed without precise knowledge of their orbital period.

Based on the sensitivity of RVs from Cale et al. (2021) and Zicher et al. (2022), and since $K_d < 1 \text{ m s}^{-1}$, further characterization of AU Mic d is out of reach of the current RV sensitivity, especially in the presence of substantial stellar activity. Furthermore, this planet is too close to the host star for direct imaging at ~ 0.1 au, which is a $\sim 0.01''$ maximum angular separation from the host star at 9.7 pc. Thus, TTVs and transits (if AU Mic d does transit) are the only near-term methods to further characterize this system and confirm planet d. Additional TTVs of AU Mic b and, in particular, AU Mic c are critical for the coming decade with space-based photometry and in further distinguishing the inner d and middle d scenarios. Follow-up with current and future space-based transit observing missions such as the Hubble Space Telescope, CHEOPS, PLANetary Transits and Oscillation of stars (PLATO), and Pandora would be greatly beneficial in searching for and/or ruling out any transit signatures for AU Mic d. To this end, both AU Mic b’s and c’s predicted transit midpoint times in Barycentric Julian Date (BJD), after accounting for the TTVs, over the next three years for each three-planet configuration are listed in Tables A10 through A13.

7. Conclusion

AU Mic is a young, nearby system hosting a dusty disk and two transiting planets. It is known to be very active, which made RV and TTV observations of the AU Mic system quite challenging. However, Wittrock et al. (2022) determined that the impact of spot modulation on observed TTVs are minimal with respect to the observed TTV variability, and the effect of stellar flares had been mitigated in SPIRou (Martiooli et al. 2020), ESPRESSO (Palle et al. 2020), TESS (Gilbert et al. 2022), Spitzer (Wittrock et al. 2022), and CHEOPS (Szabó et al. 2021, 2022) data and masked in the ground data due to larger transit timing uncertainties.

We presented the validation of the new planet, AU Mic d, likely orbiting between planets b and c. We collected 50 data sets from 4 yr of observations on the AU Mic system, including 18 new observations presented herein. We modeled the photometric transits with EXOFASTv2 and the TTVs with Exo-Striker. We determined the super-period of AU Mic b’s TTVs, which we then used to estimate the orbital period of AU Mic d. We modeled the high-eccentricity and low-eccentricity two-planet configurations. The high-eccentricity configurations’ high eccentricities ($e > 0.2$) and very small inferred masses ($K < 0.07 \text{ m s}^{-1}$) are in strong disagreement with those from RV models (Cale et al. 2021; Donati et al. 2023) and a secondary-eclipse search (Kevin et al. 2023, in preparation). While this configuration does model a TTV super-period, it is clearly not dynamically settled given the apparent orbital migrations of AU Mic b and c and the extreme

fluctuations in both planets' eccentricities and inclinations in the rebound simulations. Unlike the high-eccentricity configuration, the low-eccentricity two-planet counterpart does not exhibit the super-period in AU Mic b's TTVs; this configuration also has both b and c being misaligned, which contradicts with transit models and observations (Plavchan et al. 2020; Martioli et al. 2021; Gilbert et al. 2022; Wittrock et al. 2022). Lastly, the TTV model comparisons statistically ruled out both two-planet models. We then developed the TTV log-likelihood periodograms to explore a range of possible orbital periods for AU Mic d; through this technique, we obtained ten possible solutions, which we then follow up with *Exo-Striker*'s Bayesian analysis, stability test packages, including rebound and SPOCK, and RV modeling.

After performing the TTV and RV model comparisons, the stability tests, and the calculation of the super-period of AU Mic b's TTVs, and taking into consideration the Occam's razor arguments regarding the near-MMR chains and coplanarity of the AU Mic system, we determined that the most favored orbital period for AU Mic d is $P_d = 12.73596 \pm 0.00793$ days. The nine other possible periods for AU Mic d are statistically and dynamically disfavored, but not ruled out. The favored solution for AU Mic d places the three planets near the 2:3 orbital resonance pairs, or near 4:6:9 orbital resonance overall. The near-3:2 resonant chain is the most common pairing among the known exoplanets (Fabrycky et al. 2014). Moreover, this particular configuration is statistically the most stable one. The presence of resonant chains among the planets in a very young yet stable system is significant, as it indicates that a compact system can quickly establish resonant chains very early on and which can become dynamically stable within a few hundred short orbital periods. The other disfavored candidate periods for AU Mic d that we modeled would also establish the AU Mic system of three planets in a near-MMR chain, but in higher-order and less commonly occurring resonances. Either way, the scenario with no AU Mic d is statistically ruled out through the TTV and RV model comparisons, and the system must be in a near-resonant chain.

For the period of 12.7 days, our modeling determined the mass $M_d = 1.053 \pm 0.511 M_\oplus$ and the time of conjunction $T_{c,d} = 2458340.55781 \pm 0.11641$ BJD for AU Mic d. This will make AU Mic d the first known Earth-mass planet to orbit a young star and which will serve as a fundamental target for young terrestrial planets' atmospheric characterization and evolution, provided that it does transit. If one assumes AU Mic d is transiting and its density is Earth-like, then its transit depth would be ~ 160 ppm. This very shallow depth in face of AU Mic's heightened stellar activity may explain the apparent lack of transit signals from AU Mic d in the TESS light curves. Future work could involve thoroughly searching all existing space-based light curves of the AU Mic system for any statistically significant 160 ppm signals. The calculated time of conjunction provided in this paper will simplify the search as it provides a constraint on the timing of the expected transits of AU Mic d.

Our RV analysis implies that there may be additional planets beyond AU Mic c, including the 65 days candidate signature from our $\ln \mathcal{L}$ brute-force periodogram. We recommend additional ground- and space-based TTV observations to further characterize AU Mic d and confirm its orbital period, and to search for additional planets beyond AU Mic c. Given the youthfulness of the AU Mic system and the numerous exciting discoveries emerging from it, AU Mic will serve as an excellent

target for HST, CHEOPS, PLATO, Pandora, and future space-based transit observing missions. Acknowledgments

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This research has made use of the Exoplanet Follow-up Observation Program (ExoFOP; DOI: 10.26134/ExoFOP5) website, which is operated by the California Institute of

³⁹ <http://pestobservatory.com>

Technology, under contract with the National Aeronautics and Space Administration under the Exoplanet Exploration Program.

This research has made use of the SIMBAD database, operated at CDS, Strasbourg, France.

This research has made use of NASA's Astrophysics Data System Bibliographic Services.

This research has made use of an online calculator that converts a list of Barycentric Julian Dates in Barycentric Dynamical Time (BJD_TDB) to JD in UT (Eastman et al. 2010)⁴⁰.

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Facilities: ASTEP 400:0.4m (FLI Proline 16800E), Brierfield:0.36m (Moravian G4-16000 KAF-16803), CFHT (SPIRou), CHEOPS, ExoFOP, Exoplanet Archive, IRSA, IRTF (iSHELL), LCOGT (CTIO:1m, SAAO:1m, SSO:1m, and TO:1 m; Sinistro), MAST, MKO CDK700:0.7m (U16), PEST:0.30m (SBIG ST-8XME), Spitzer (IRAC), TESS, VLT:Antu (ESPRESSO)

Software: AstroImageJ (Collins et al. 2017), astropy (Astropy Collaboration et al. 2013, 2018), batman (Kreidberg 2015), celerite (Foreman-Mackey et al. 2017),

celerite2 (Foreman-Mackey et al. 2017; Foreman-Mackey 2018), emcee (Foreman-Mackey et al. 2013), EXOFAST (Eastman et al. 2013), EXOFASTv2 (Eastman et al. 2019), exoplanet (Foreman-Mackey et al. 2021), Exo-Striker (Trifonov 2019), ipython (Perez & Granger 2007), lightkurve (Lightkurve Collaboration et al.), matplotlib (Hunter 2007), MEGNO (Cincotta & Simó 2000; Cincotta et al. 2003), numpy (Harris et al. 2020), rebound (Rein & Liu 2012; Rein & Spiegel 2015), scipy (Virtanen et al. 2020), SPOCK (Tamayo et al. 2020), TAPIR (Jensen 2013)

Appendix Outputs from Alternative Cases

This appendix lists the $O-C$ diagrams (Figures A1–A3), MCMC posteriors (Tables A1–A9), corner plots (Figures A4–A12), rebound plots (Figure A13), predicted transit times (Figure A14 and Tables A10–A13), and future (10 yrs) TTV models (Figure A15) for $P_d = (5.08, 5.39, 5.64, 5.86, 6.20, 6.47, 11.9, 12.6, \text{ and } 14.1)$ days. The 12.7 days case is statistically most favored and is presented in the main text (Sections 4 and 6).

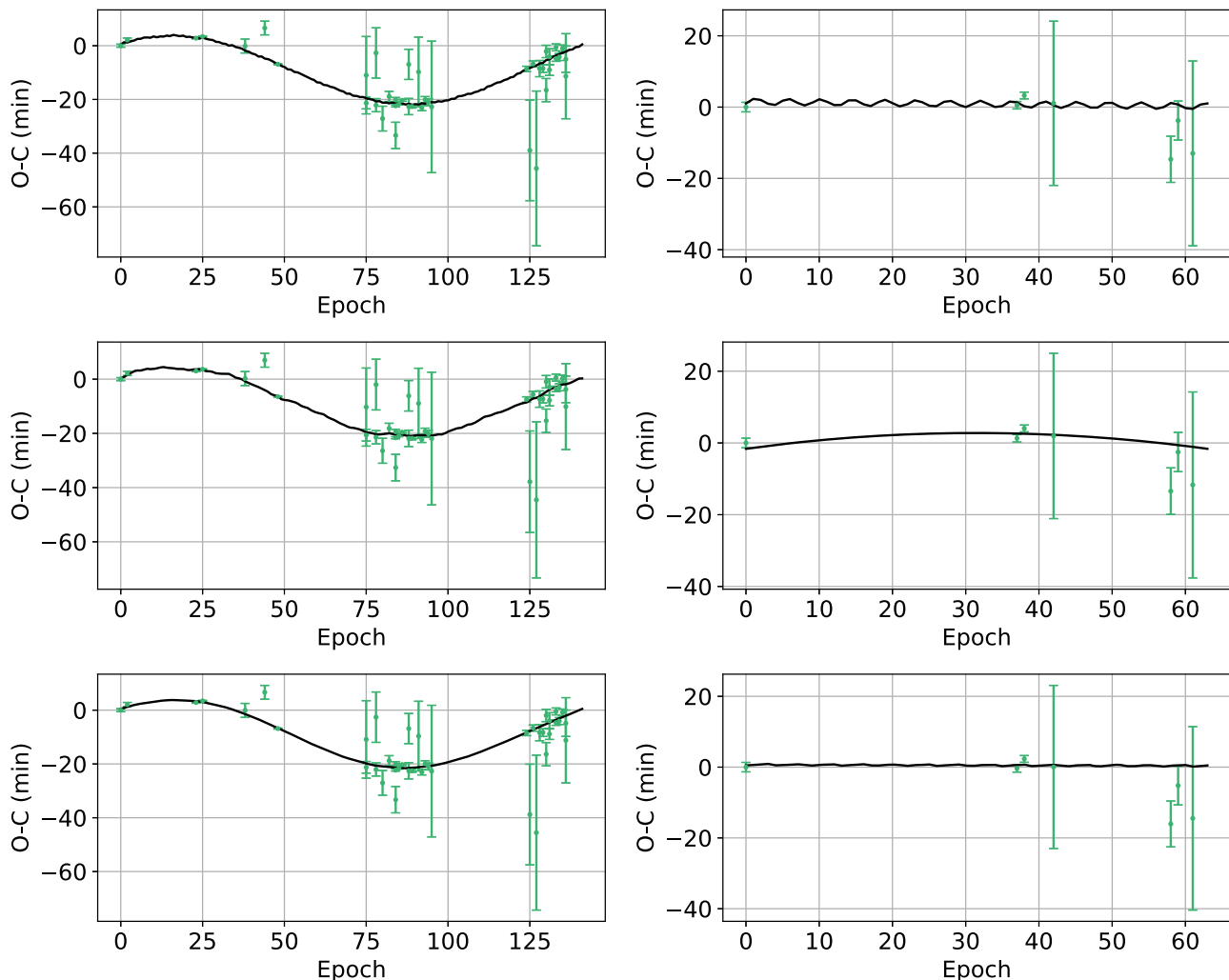


Figure A1. $O-C$ diagrams of AU Mic b (left) and AU Mic c (right), with comparison between TTVs (green) and Exo-Striker-generated MCMC models (black) for $P_d = 5.08$ days (top row), 5.39 days (middle row), and 5.64 days (bottom row).

⁴⁰ <https://astrutils.astronomy.osu.edu/time/bjd2utc.html>

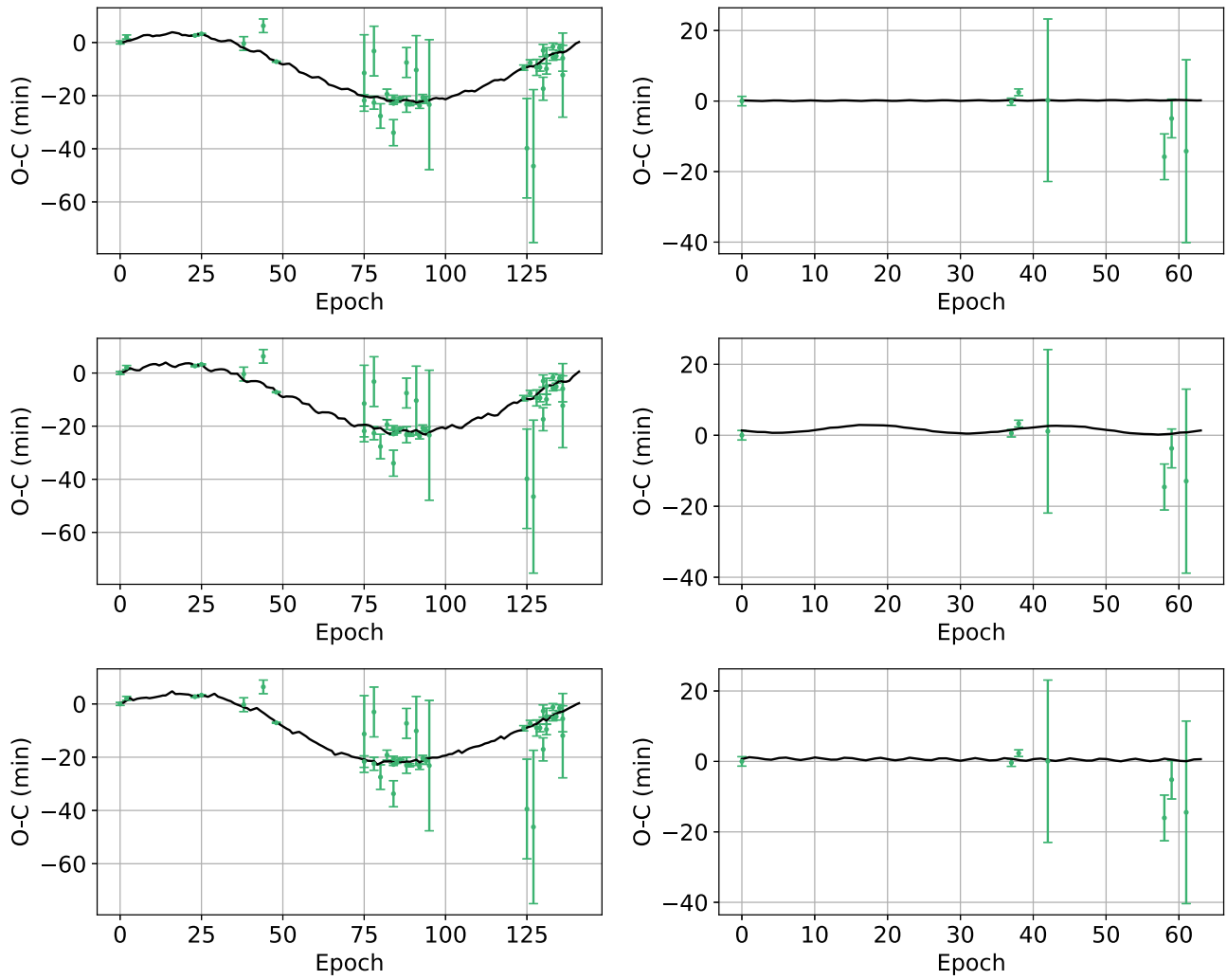


Figure A2. $O-C$ diagrams of AU Mic b (left) and AU Mic c (right), with comparison between TTVs (green) and Exo-Striker-generated MCMC models (black) for $P_d = 5.86$ days (top row), 6.20 days (middle row), and 6.47 days (bottom row).

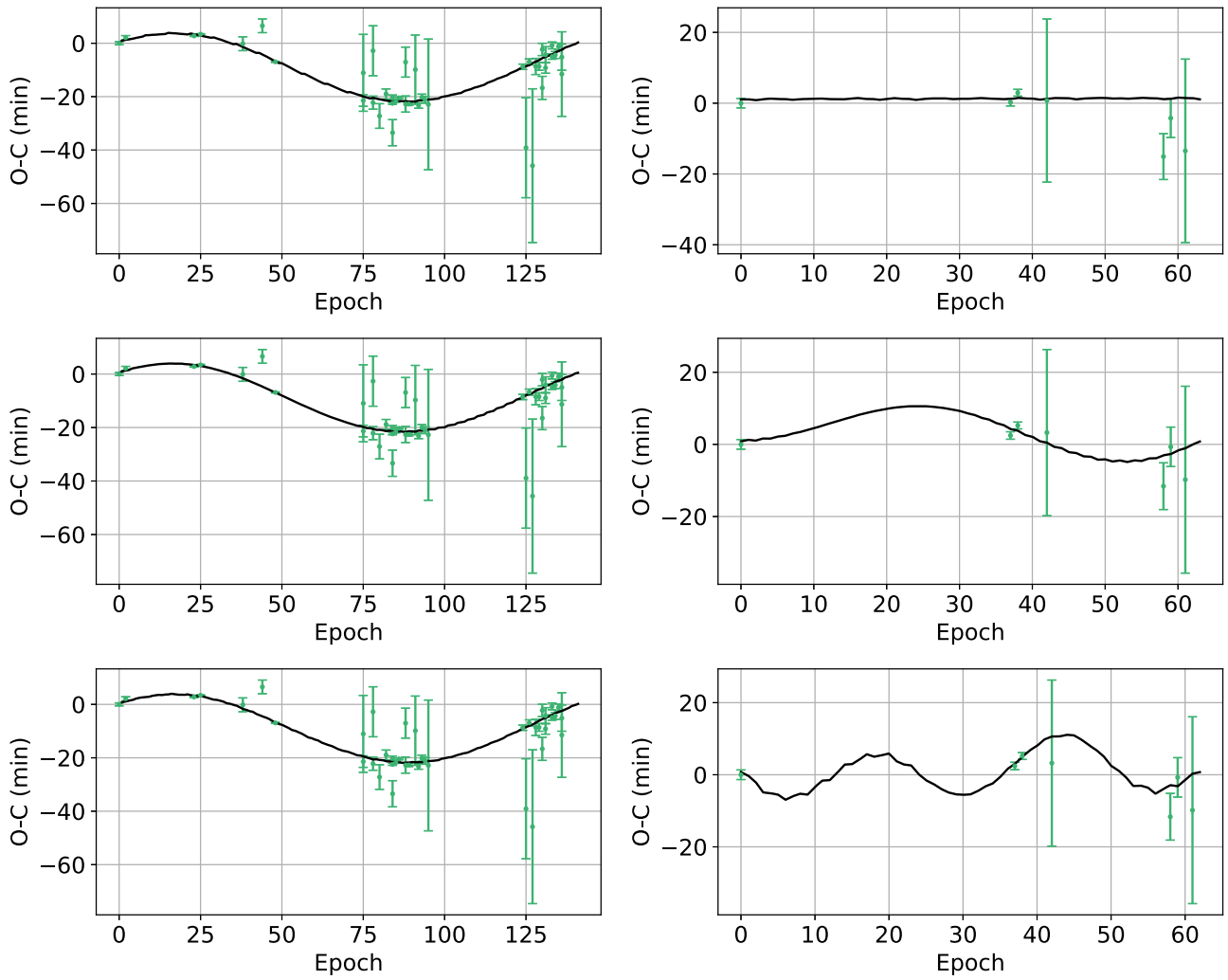


Figure A3. $O-C$ diagrams of AU Mic b (left) and AU Mic c (right), with comparison between TTVs (green) and Exo-Striker-generated MCMC models (black) for 11.9 days (top row), 12.6 days (middle row), and 14.1 days (bottom row).

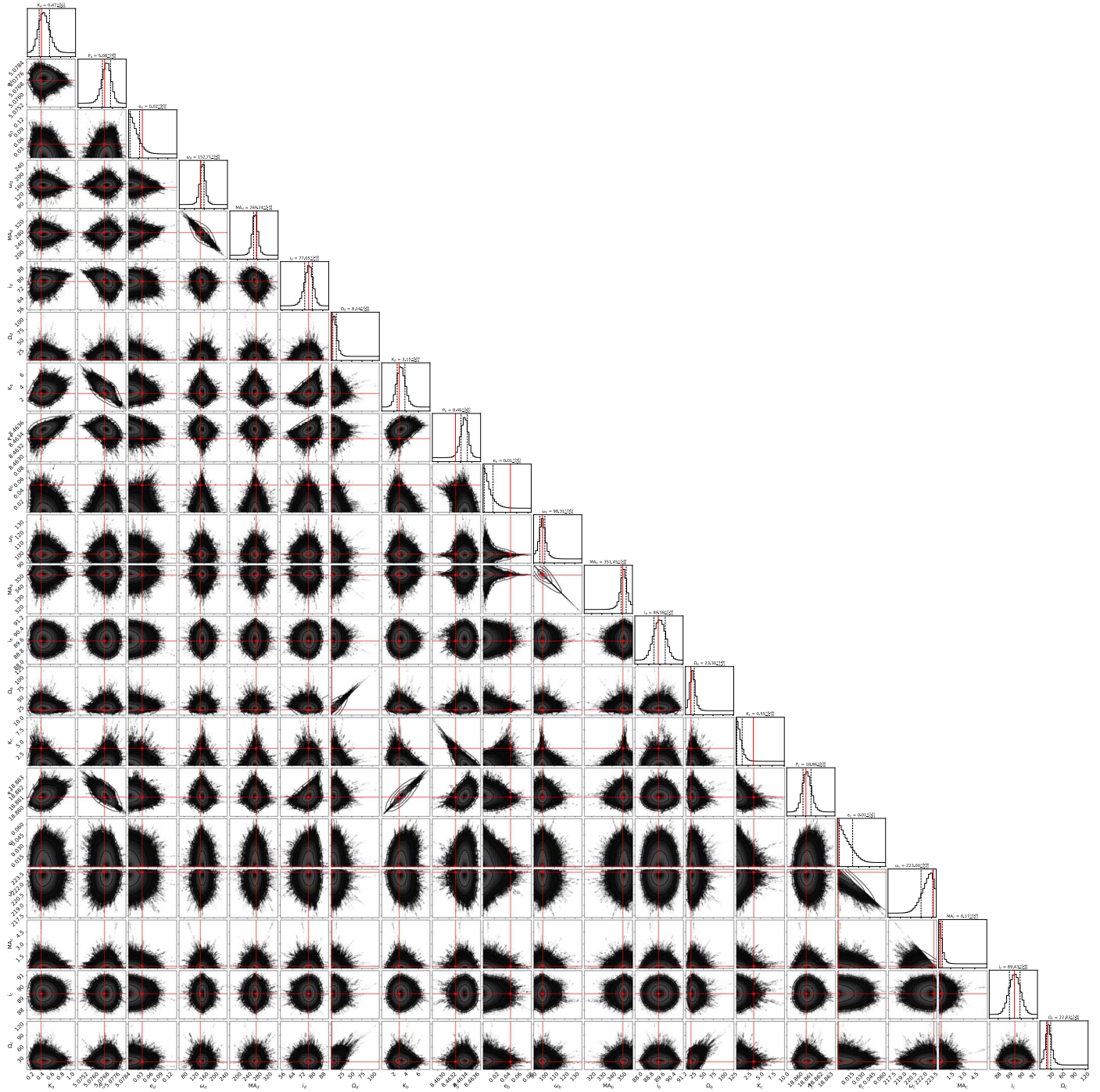


Figure A4. Corner plot of AU Mic b and c's orbital parameters from Exo-Striker MCMC analysis for $P_d = 5.08$ days.

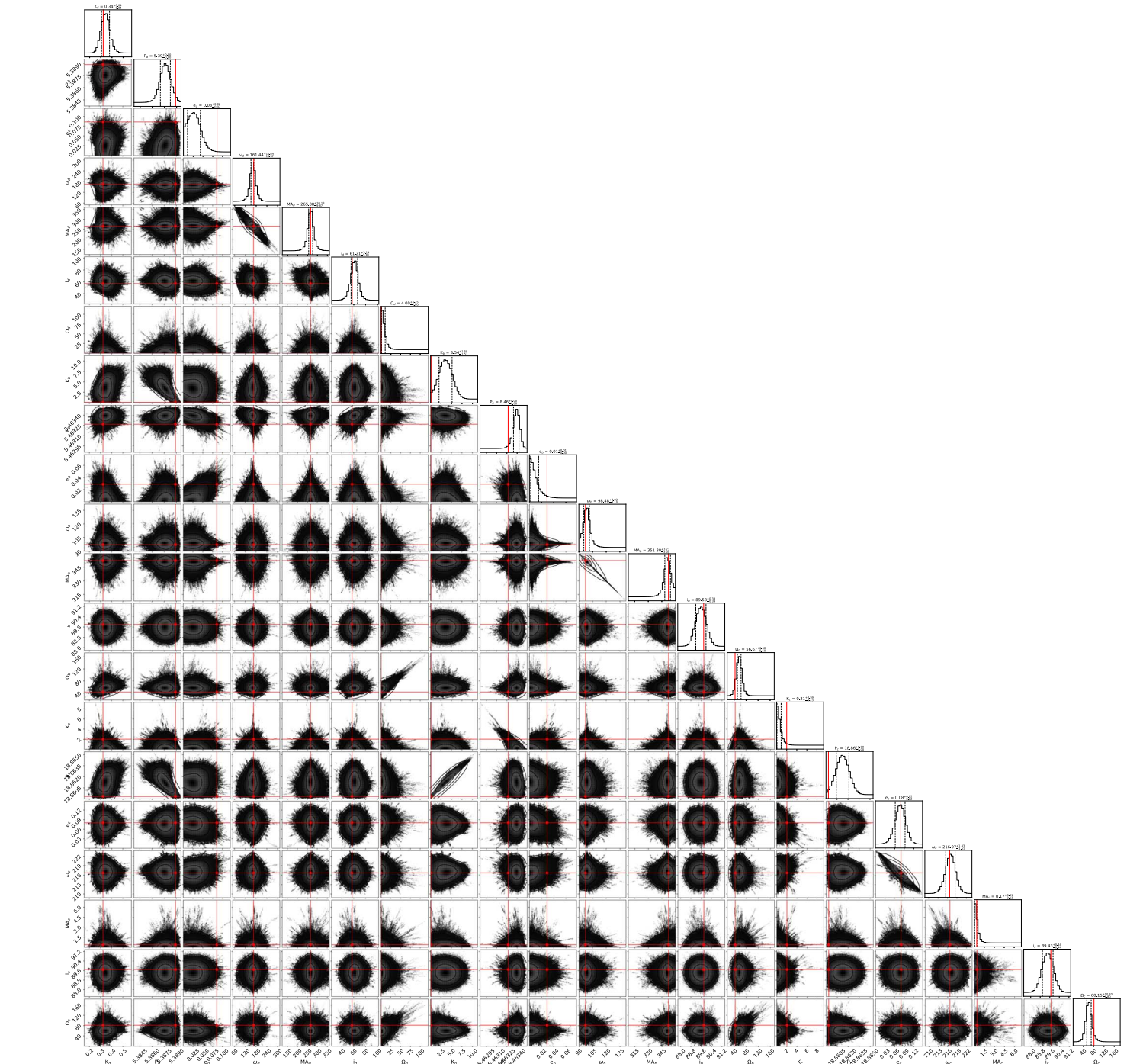


Figure A5. Corner plot of AU Mic b and c's orbital parameters from Exo-Striker MCMC analysis for $P_d = 5.39$ days.

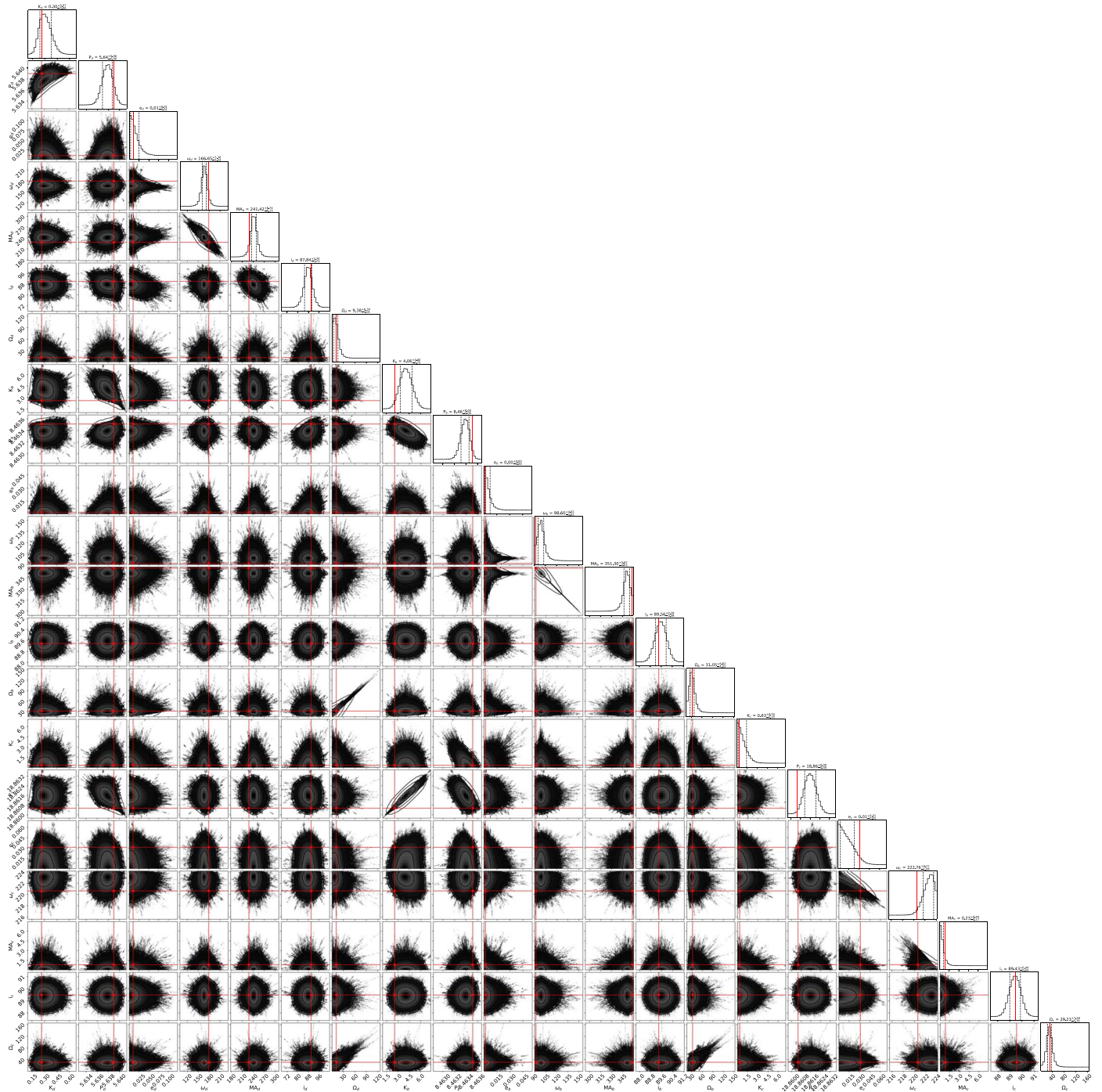


Figure A6. Corner plot of AU Mic b and c's orbital parameters from Exo-Striker MCMC analysis for $P_d = 5.64$ days.

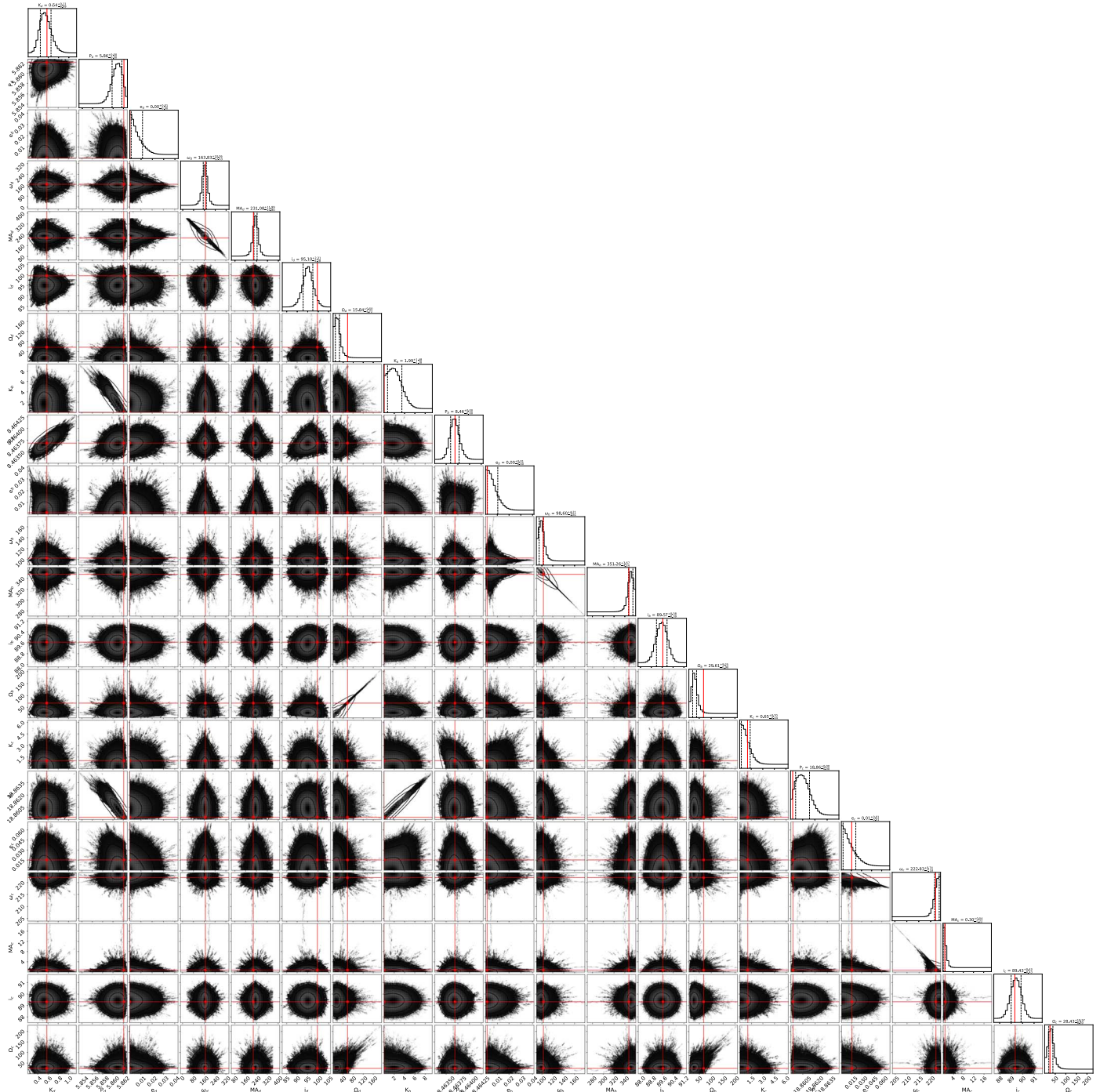


Figure A7. Corner plot of AU Mic b and c's orbital parameters from `EXO-STRIKER` MCMC analysis for $P_d = 5.86$ days.

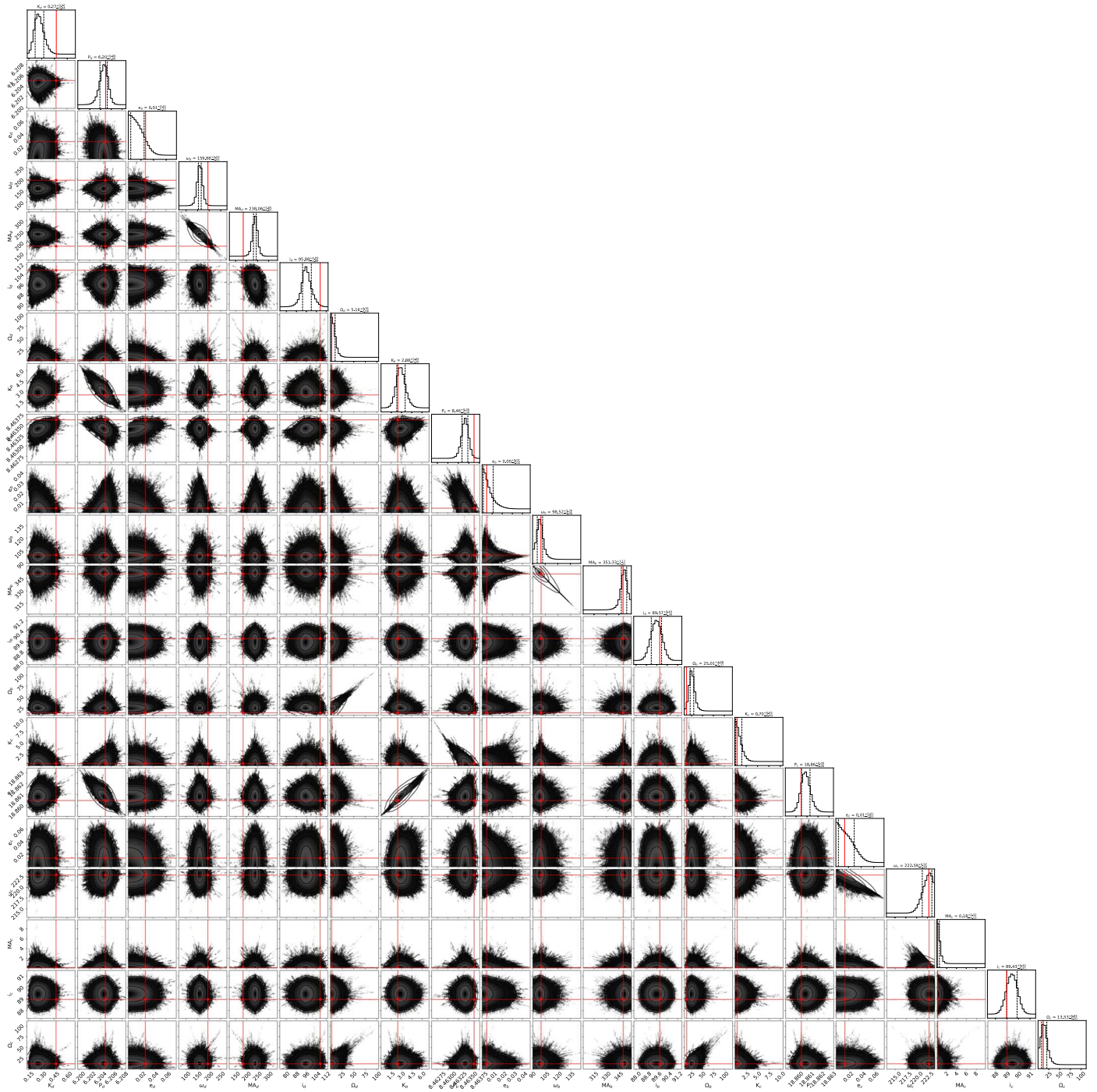


Figure A8. Corner plot of AU Mic b and c's orbital parameters from Exo-Striker MCMC analysis for $P_d = 6.20$ days.

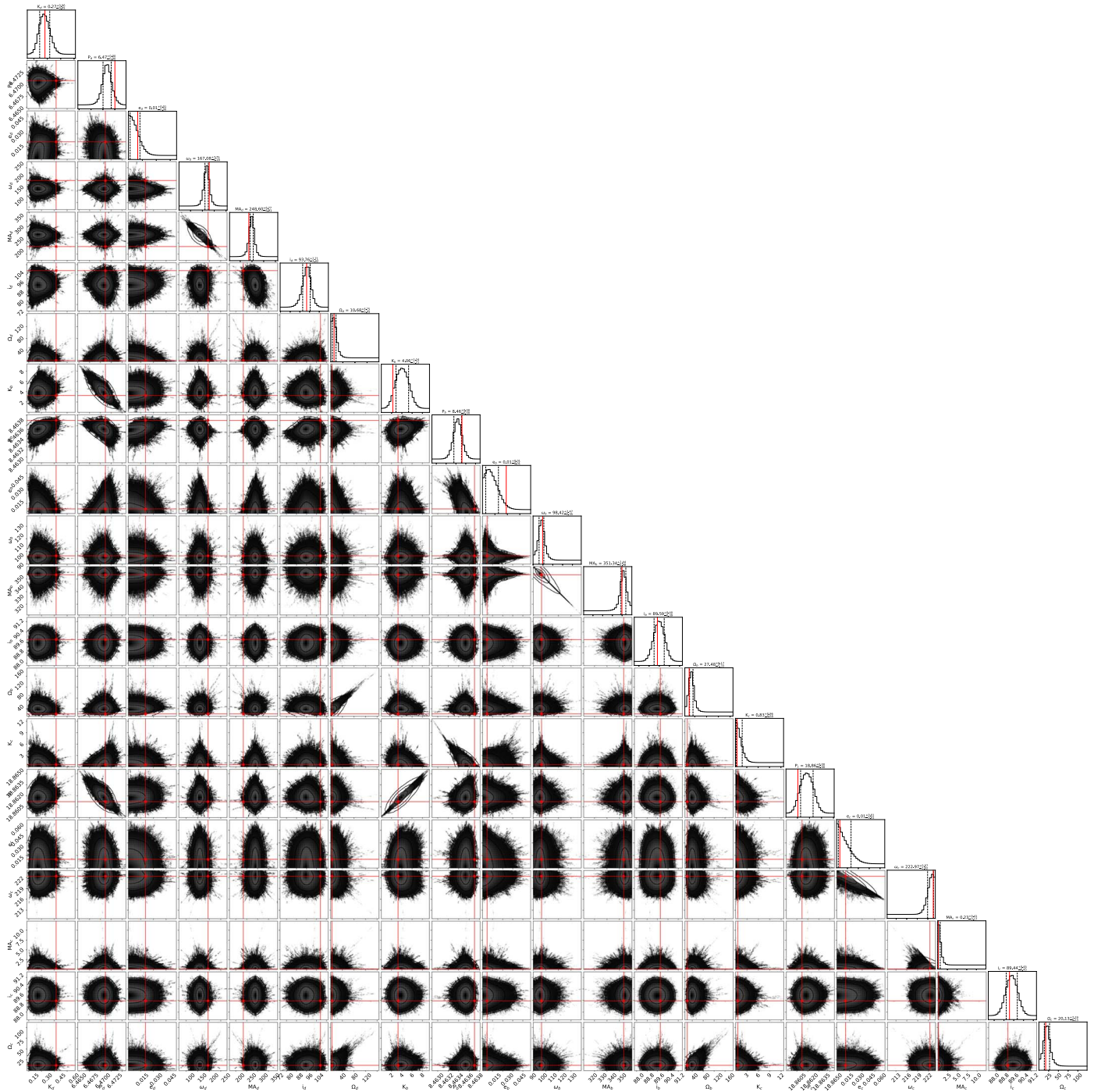


Figure A9. Corner plot of AU Mic b and c's orbital parameters from Exo-Striker MCMC analysis for $P_d = 6.47$ days.

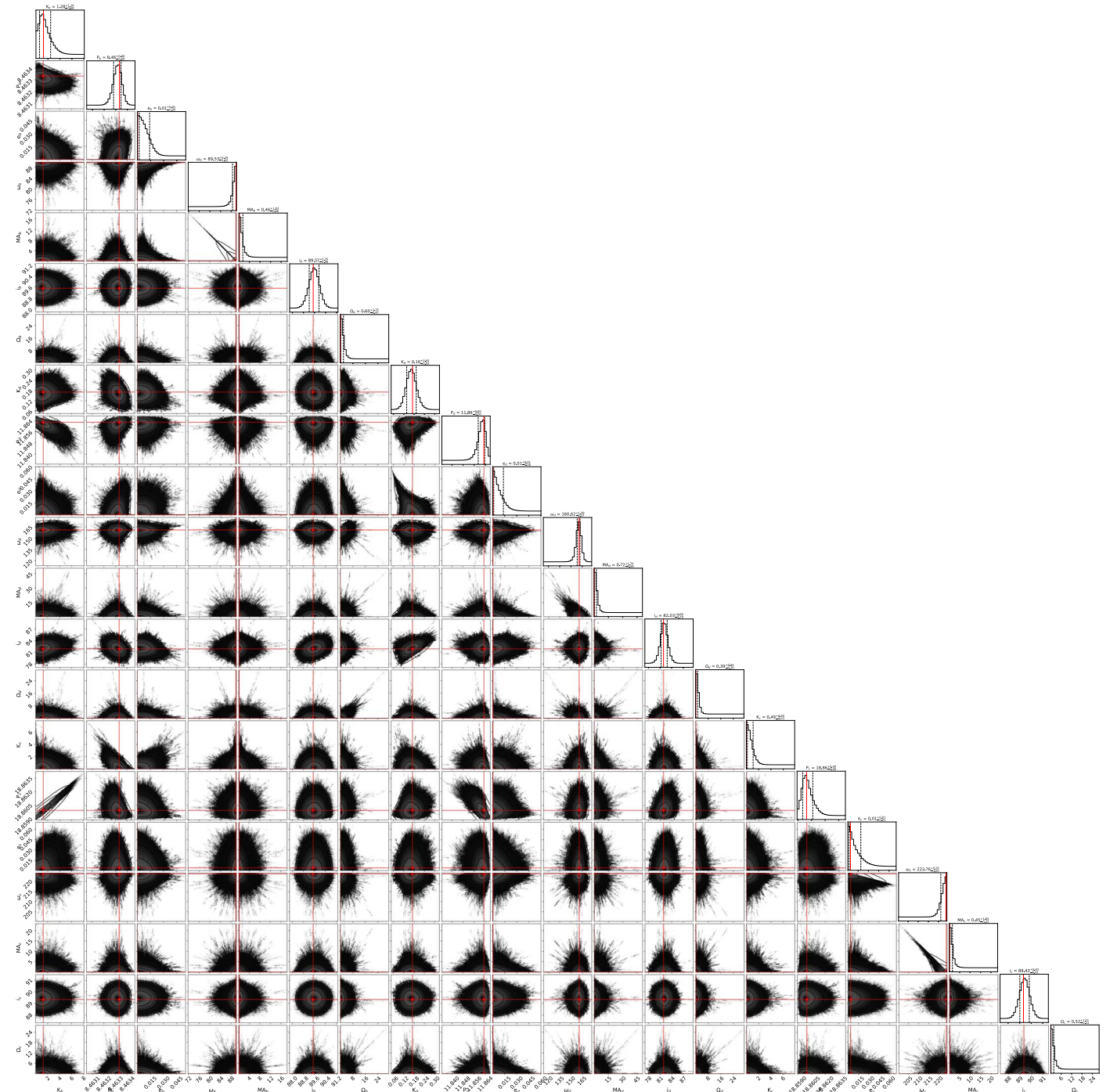


Figure A10. Corner plot of AU Mic b and c's orbital parameters from Exo-Striker MCMC analysis for $P_d = 11.9$ days.

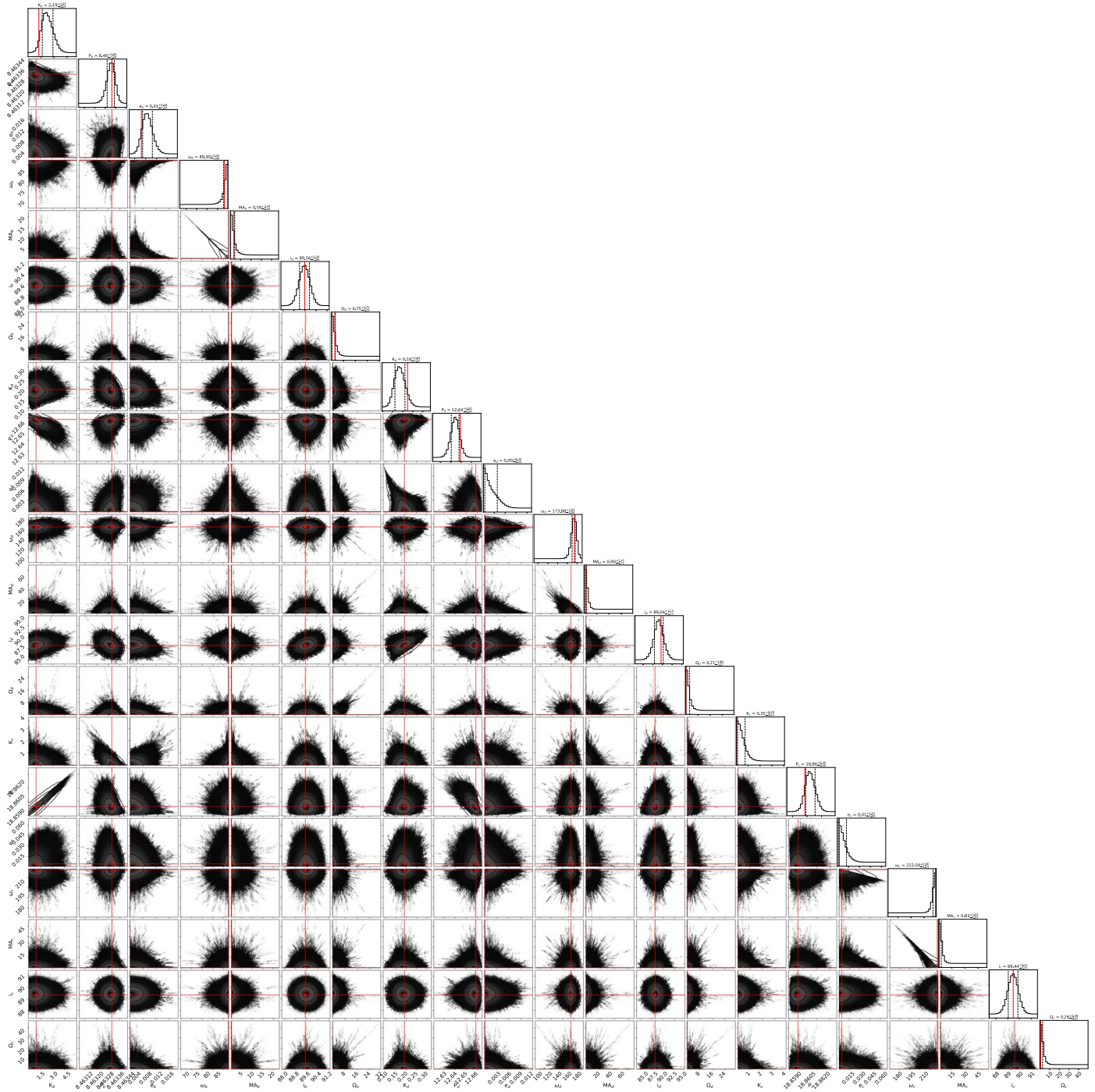


Figure A11. Corner plot of AU Mic b and c's orbital parameters from Exo-Striker MCMC analysis for $P_d = 12.6$ days.

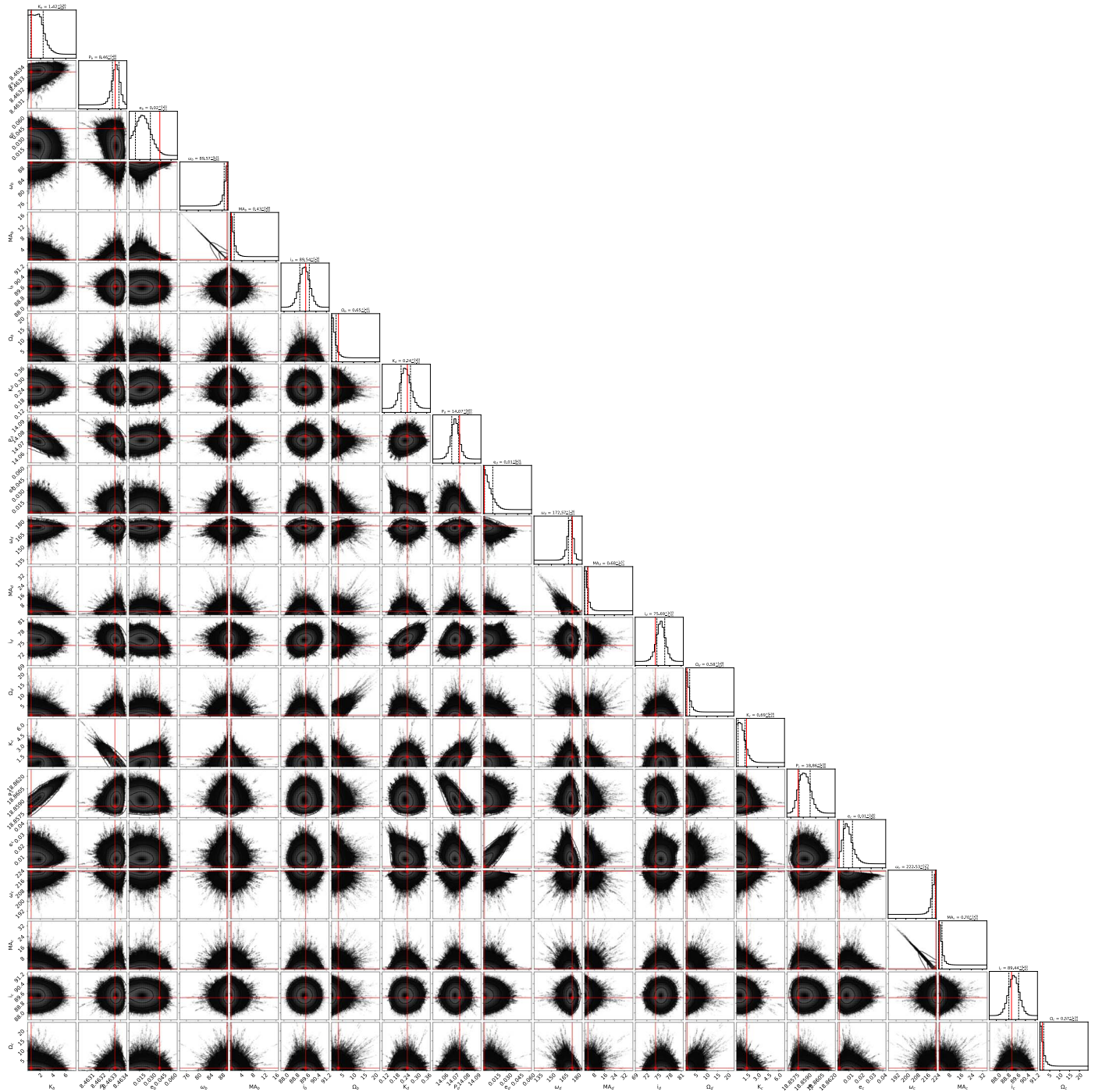


Figure A12. Corner plot of AU Mic b and c's orbital parameters from Exo-Striker MCMC analysis for $P_d = 14.1$ days.

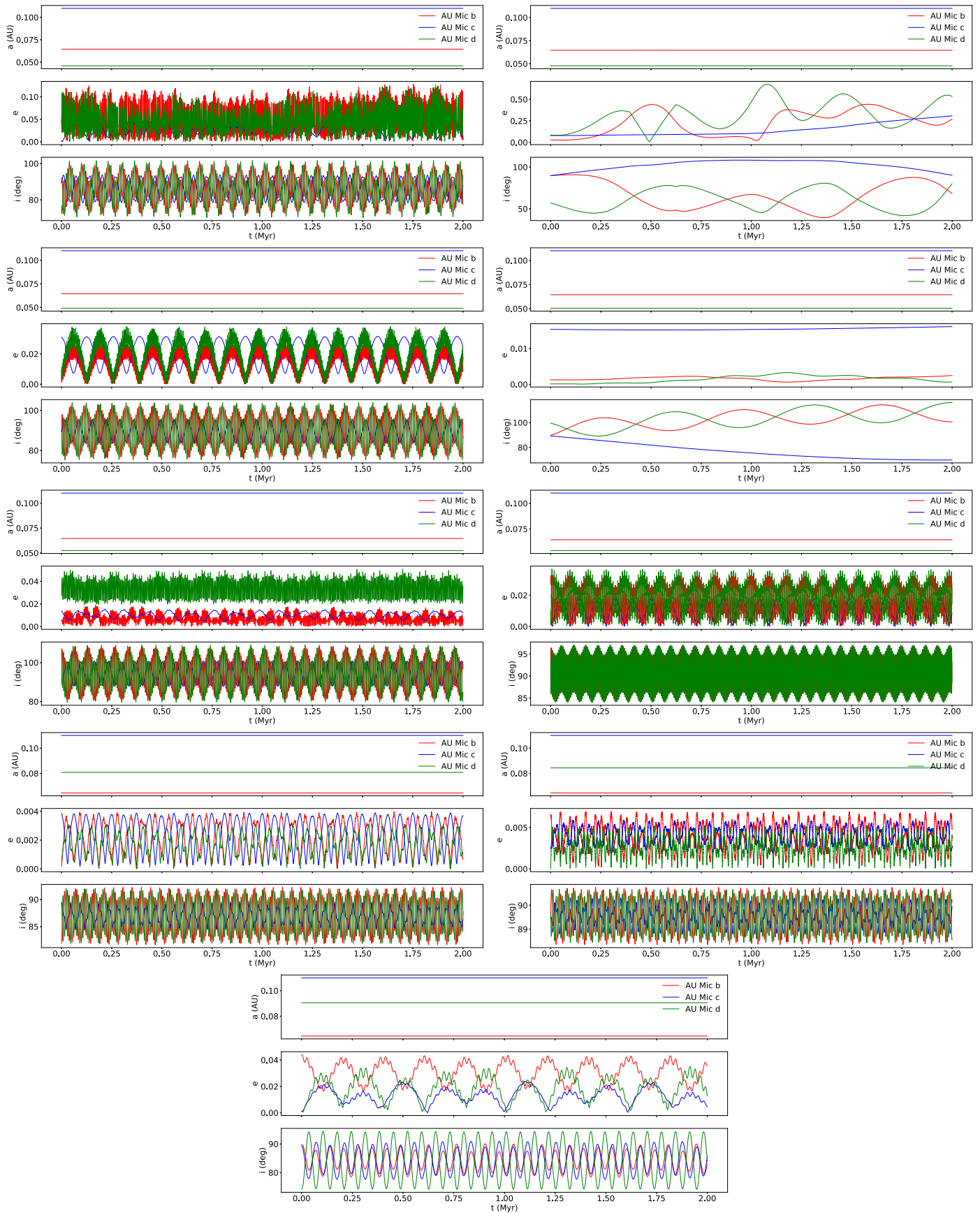


Figure A13. rebound models of the stability of the AU Mic system on a timescale of 2 Myr for $P_d = 5.08$ days (first row left), 5.39 days (first row right), 5.64 days (second row left), 5.86 days (second row right), 6.20 days (third row left), 6.47 days (third row right), 11.9 days (fourth row left), 12.6 days (fourth row right), and 14.1 days (fifth row).

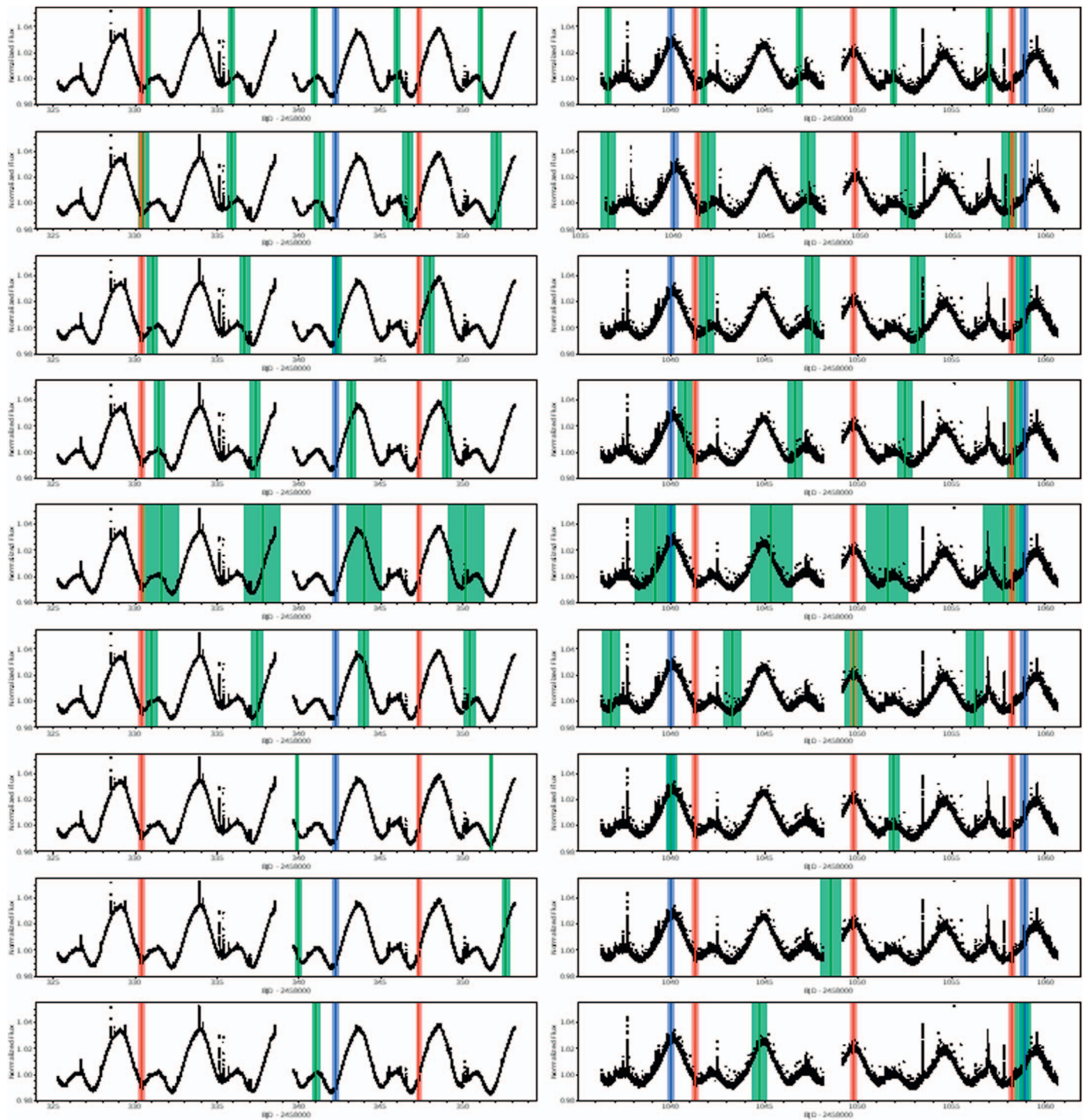


Figure A14. Raw TESS photometry of AU Mic overlaid with observed transits of AU Mic b (red) and c (blue) and a range of predicted transit midpoint times for AU Mic d (green), with $P_d = 5.07$ days (first row), 5.39 days (second row), 5.64 days (third row), 5.86 days (fourth row), 6.20 days (fifth row), 6.47 days (sixth row), 11.9 days (seventh row), 12.6 days (eighth row), and 14.1 days (ninth row). Left column is from Cycle 1, and right column is from Cycle 3. Owing to AU Mic d's estimated 160 ppm depth and AU Mic's intense stellar activity, the potential transit signatures from AU Mic d are not obvious in these plots.

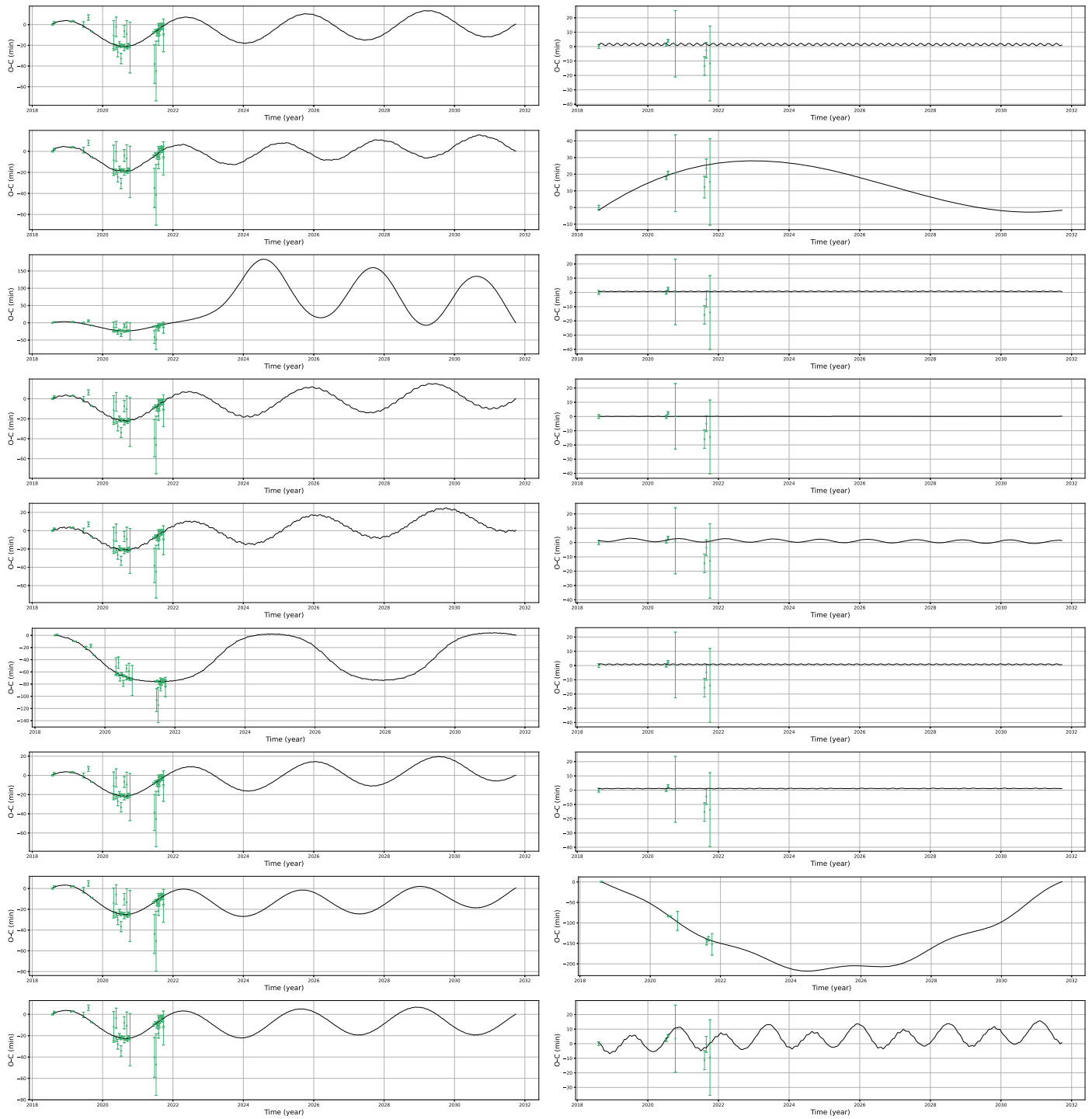


Figure A15. Exo-Striker-generated TTV models of AU Mic b (left) and c (right) projected over 10 yr since last observed transit of the AU Mic system. The observed TTVs (green) are included with the model (black), with $P_d = 5.07$ days (first row), 5.39 days (second row), 5.64 days (third row), 5.86 days (fourth row), 6.20 days (fifth row), 6.47 days (sixth row), 11.9 days (seventh row), 12.6 days (eighth row), and 14.1 days (ninth row).

Table A1
Exo-Striker-Generated Best-fit and MCMC Modeling Parameters for $P_d = 5.08$ Days

Parameter	Unit	Best Fit			MCMC		
		AU Mic b	AU Mic c	AU Mic d	AU Mic b	AU Mic c	AU Mic d
K	m s^{-1}	3.35536	0.22986	0.50720	2.87947 ± 0.34321	3.60029 ± 3.44133	0.41877 ± 0.04442
P_{orb}	day	8.46351	18.86127	5.07706	8.46331 ± 0.00010	18.86102 ± 0.00029	5.07701 ± 0.00017
e	...	0.00000	0.00592	0.00016	0.04872 ± 0.04662	0.00103 ± 0.00159	0.04209 ± 0.03778
ω	deg	98.37057	223.59289	152.78962	98.57816 ± 2.54783	223.75374 ± 1.77297	144.16841 ± 2.43169
M_0	deg	351.61077	0.00025	268.90220	350.54357 ± 1.58108	0.24484 ± 0.20155	274.43129 ± 10.64027
i	deg	89.57924	89.43188	77.33666	89.48110 ± 0.31229	89.39871 ± 0.42666	76.88456 ± 3.04551
Ω	deg	23.10249	22.62173	8.17561	19.11756 ± 0.53911	20.71402 ± 3.01682	2.55408 ± 1.56684
χ^2	...	114.02094	11.26343	...	106.79515	18.02865	...
χ^2_{red}	...	5.22018	31.32109	...	5.20099	31.20595	...
$\ln \mathcal{L}$	209.46399	219.96553	...
BIC	-337.19975	-358.20283	...
AIC _c	-342.70576	-363.70884	...

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding the planets' low K 's generated by Exo-Striker.

Table A2
Exo-Striker-generated Best-fit and MCMC Modeling Parameters for $P_d = 5.39$ Days

Parameter	Unit	Best Fit			MCMC		
		AU Mic b	AU Mic c	AU Mic d	AU Mic b	AU Mic c	AU Mic d
K	m s^{-1}	5.16537	0.50637	0.34978	0.10817 ± 1.90115	2.00680 ± 1.92941	0.32173 ± 0.01505
P_{orb}	day	8.46341	18.86196	5.38693	8.46327 ± 0.00008	18.85933 ± 0.00107	5.38888 ± 0.00188
e	...	0.00001	0.07780	0.00299	0.02897 ± 0.02697	0.07978 ± 0.01753	0.08579 ± 0.07529
ω	deg	98.55512	217.22994	161.43928	97.22742 ± 1.87410	216.68146 ± 1.23336	166.41581 ± 15.66255
M_0	deg	351.43634	0.00001	265.97175	352.33787 ± 4.06365	0.36773 ± 0.32286	265.55255 ± 9.64814
i	deg	89.57209	89.42620	61.86109	89.79654 ± 0.61080	89.68914 ± 0.70596	57.18937 ± 1.827605
Ω	deg	55.99639	58.92122	2.10453	43.45119 ± 7.68065	81.23160 ± 27.93638	6.51204 ± 6.20803
χ^2	...	115.39770	5.10544	...	105.90689	10.01490	...
χ^2_{red}	...	5.02096	30.12578	...	4.83007	28.98045	...
$\ln \mathcal{L}$	211.88028	225.96977	...
BIC	-342.03233	-370.21131	...
AIC _c	-347.53834	-375.71732	...

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding the planets' low K 's generated by Exo-Striker.

Table A3
Exo-Striker-generated Best-fit and MCMC Modeling Parameters for $P_d = 5.64$ Days

Parameter	Unit	Best Fit			MCMC		
		AU Mic b	AU Mic c	AU Mic d	AU Mic b	AU Mic c	AU Mic d
K	m s^{-1}	4.40058	0.46675	0.33788	2.56609 ± 0.74131	0.32794 ± 0.14726	0.26404 ± 0.02325
P_{orb}	day	8.46339	18.86185	5.63768	8.46351 ± 0.00020	18.86055 ± 0.00059	5.63897 ± 0.00207
e	...	0.00000	0.00677	0.00029	0.00179 ± 0.00085	0.03117 ± 0.02806	0.00997 ± 0.00676
ω	deg	98.50502	223.51766	166.51374	91.68412 ± 3.26572	220.60684 ± 1.08879	179.09135 ± 17.81811
M_0	deg	351.47597	0.00094	241.03825	358.29595 ± 11.06134	0.80625 ± 0.74415	228.08583 ± 7.13443
i	deg	89.52517	89.43744	87.60186	89.38133 ± 0.21181	89.48992 ± 0.50234	90.07831 ± 4.98962
Ω	deg	30.10388	28.65682	9.84080	31.92354 ± 5.43454	30.65984 ± 7.23061	11.45968 ± 7.13193
χ^2	...	113.88847	11.22925	...	115.86957	11.71456	...
χ^2_{red}	...	5.21324	31.27943	...	5.31601	31.89603	...
$\ln \mathcal{L}$	209.53001	219.12889	...
BIC	-337.33179	-356.52955	...
AIC _c	-342.83780	-362.03556	...

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding the planets' low K 's generated by Exo-Striker.

Table A4
Exo-Striker-generated Best-fit and MCMC Modeling Parameters for $P_d = 5.86$ Days

Parameter	Unit	Best Fit			MCMC		
		AU Mic b	AU Mic c	AU Mic d	AU Mic b	AU Mic c	AU Mic d
K	m s^{-1}	2.16456	0.23630	0.53136	0.03744 ± 0.66777	1.01987 ± 0.81217	0.57447 ± 0.12867
P_{orb}	day	8.46367	18.86055	5.86086	8.46369 ± 0.00009	18.85928 ± 0.00036	5.86210 ± 0.00231
e	...	0.00000	0.00311	0.00003	0.00129 ± 0.00025	0.01538 ± 0.01280	0.00011 ± 0.00110
ω	deg	98.48083	223.82709	164.24881	102.58694 ± 7.62158	222.06323 ± 0.41503	168.01295 ± 16.83654
M_0	deg	351.49999	0.00131	231.06872	347.37523 ± 0.39120	0.73499 ± 0.65918	216.24219 ± 3.82607
i	deg	89.55965	89.42747	95.02796	89.64773 ± 0.46690	89.32042 ± 0.32416	99.58862 ± 6.78176
Ω	deg	29.19644	28.21852	15.22950	68.15507 ± 45.50567	25.26173 ± 5.59688	57.61649 ± 48.55583
χ^2	...	102.74262	11.75572	...	100.82281	13.43211	...
χ_{red}^2	...	4.77076	28.62458	...	4.76062	28.56373	...
$\ln \mathcal{L}$	214.83902	226.99616	...
BIC	-347.94981	-372.26409	...
AIC _c	-353.45582	-377.77010	...

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding the planets' low K 's generated by Exo-Striker.

Table A5
Exo-Striker-generated Best-fit and MCMC Modeling Parameters for $P_d = 6.20$ Days

Parameter	Unit	Best Fit			MCMC		
		AU Mic b	AU Mic c	AU Mic d	AU Mic b	AU Mic c	AU Mic d
K	m s^{-1}	2.90791	0.46992	0.26161	2.45919 ± 0.14618	0.49589 ± 0.28947	0.47104 ± 0.24820
P_{orb}	day	8.46343	18.86090	6.20441	8.46362 ± 0.00029	18.86047 ± 0.00004	6.20494 ± 0.00105
e	...	0.00000	0.01115	0.00018	0.00452 ± 0.00323	0.01386 ± 0.00986	0.02737 ± 0.02352
ω	deg	98.48979	223.15789	160.36483	99.85587 ± 4.41672	222.82523 ± 1.52204	194.59296 ± 41.32711
M_0	deg	351.49117	0.00036	238.43986	350.03934 ± 2.04421	0.09010 ± 0.04308	184.33273 ± 46.79969
i	deg	89.55720	89.42914	94.74564	89.82232 ± 0.65141	88.96846 ± 0.01337	107.15764 ± 14.18751
Ω	deg	24.51329	12.84297	4.91602	13.49562 ± 8.12320	12.11611 ± 3.83135	2.67773 ± 0.61209
χ^2	...	107.43664	11.84116	...	99.94369	10.06389	...
χ_{red}^2	...	4.96991	29.81945	...	4.58365	27.50189	...
$\ln \mathcal{L}$	212.45061	227.07288	...
BIC	-343.17299	-372.41753	...
AIC _c	-348.67900	-377.92354	...

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding the planets' low K 's generated by Exo-Striker.

Table A6
Exo-Striker-generated Best-fit and MCMC Modeling Parameters for $P_d = 6.47$ Days

Parameter	Unit	Best Fit			MCMC		
		AU Mic b	AU Mic c	AU Mic d	AU Mic b	AU Mic c	AU Mic d
K	m s^{-1}	5.22397	0.51394	0.30268	2.24280 ± 0.60448	0.44272 ± 0.19449	0.27380 ± 0.05928
P_{orb}	day	8.46343	18.86244	6.47074	8.46352 ± 0.00016	18.86056 ± 0.00033	6.47251 ± 0.00249
e	...	0.00000	0.00442	0.00023	0.02862 ± 0.02473	0.00453 ± 0.00215	0.01002 ± 0.00819
ω	deg	98.49871	223.71495	166.62198	99.63829 ± 3.59581	223.56221 ± 1.61921	178.62168 ± 17.77984
M_0	deg	351.48520	0.00062	248.45972	349.79295 ± 0.95313	0.15047 ± 0.08931	236.71532 ± 4.90333
i	deg	89.57471	89.46389	94.91569	89.45294 ± 0.26172	89.25326 ± 0.26737	93.57266 ± 3.26340
Ω	deg	27.67490	19.55833	10.23918	20.65656 ± 1.45446	17.43367 ± 3.48630	11.01673 ± 5.42317
χ^2	...	109.38331	11.23694	...	97.94533	13.80731	...
χ_{red}^2	...	5.02584	30.15506	...	4.65636	27.93816	...
$\ln \mathcal{L}$	211.87437	227.18419	...
BIC	-342.02051	-372.64015	...
AIC _c	-347.52652	-378.14616	...

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding the planets' low K 's generated by Exo-Striker.

Table A7
Exo-Striker-generated Best-fit and MCMC Modeling Parameters for $P_d = 11.9$ Days

Parameter	Unit	Best Fit			MCMC		
		AU Mic b	AU Mic c	AU Mic d	AU Mic b	AU Mic c	AU Mic d
K	m s^{-1}	0.97504	0.38690	0.17182	1.24696 ± 0.65028	0.02093 ± 0.13709	0.17048 ± 0.03388
P_{orb}	day	8.46331	18.85981	11.86306	8.46333 ± 0.00005	18.85996 ± 0.00043	11.86292 ± 0.00406
e	...	0.00000	0.00109	0.00000	0.00018 ± 0.00189	0.00385 ± 0.00196	0.00051 ± 0.00087
ω	deg	89.99086	223.99440	160.83877	89.81844 ± 1.31482	223.69121 ± 2.55320	160.94662 ± 2.61504
M_0	deg	0.00005	0.00014	0.00032	0.17304 ± 0.06999	0.06776 ± 0.02942	0.01696 ± 0.13710
i	deg	89.58985	89.43145	82.02998	89.50347 ± 0.29017	89.33758 ± 0.31993	81.88651 ± 0.63757
Ω	deg	0.00012	0.00000	0.00000	0.00494 ± 0.12589	0.01833 ± 0.09216	0.11655 ± 0.03333
χ^2	...	112.83347	11.27744	...	113.01980	11.41707	...
χ_{red}^2	...	5.17129	31.02773	...	5.18487	31.10922	...
$\ln \mathcal{L}$	210.00233	221.62352	...
BIC	-338.27643	-361.51881	...
AIC _c	-343.78244	-367.02482	...

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding the planets' low K 's generated by Exo-Striker.

Table A8
Exo-Striker-generated Best-fit and MCMC Modeling Parameters for $P_d = 12.6$ Days

Parameter	Unit	Best Fit			MCMC		
		AU Mic b	AU Mic c	AU Mic d	AU Mic b	AU Mic c	AU Mic d
K	m s^{-1}	2.90332	0.48084	0.18469	1.25408 ± 0.39741	0.03573 ± 0.07120	0.22424 ± 0.06278
P_{orb}	day	8.46334	18.86145	12.64081	8.46333 ± 0.00004	18.86027 ± 0.00009	12.64921 ± 0.00874
e	...	0.00849	0.00278	0.00341	0.00646 ± 0.00045	0.00257 ± 0.00086	0.00000 ± 0.00039
ω	deg	89.99407	223.85721	174.65177	88.35089 ± 0.42306	223.25926 ± 2.84483	174.35726 ± 4.76760
M_0	deg	0.00002	0.00048	0.00012	1.65671 ± 1.52820	0.60281 ± 0.41896	2.35570 ± 2.16294
i	deg	89.57125	89.43113	89.06083	89.58284 ± 0.38234	89.45028 ± 0.41961	89.63258 ± 1.70943
Ω	deg	0.00001	0.00000	0.00000	2.17332 ± 2.01326	0.78666 ± 0.62801	0.15814 ± 0.01661
χ^2	...	114.85574	7.30405	...	113.68872	7.65425	...
χ_{red}^2	...	5.08999	30.53995	...	5.05596	30.33574	...
$\ln \mathcal{L}$	211.03081	222.91190	...
BIC	-340.33339	-364.09557	...
AIC _c	-345.83940	-369.60158	...

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding the planets' low K 's generated by Exo-Striker.

Table A9
Exo-Striker-generated Best-fit and MCMC Modeling Parameters for $P_d = 14.1$ Days

Parameter	Unit	Best Fit			MCMC		
		AU Mic b	AU Mic c	AU Mic d	AU Mic b	AU Mic c	AU Mic d
K	m s^{-1}	1.94679	0.79913	0.23459	0.53676 ± 0.09009	1.45679 ± 1.21270	0.25084 ± 0.03547
P_{orb}	day	8.46336	18.85942	14.07157	8.46335 ± 0.00002	18.85837 ± 0.00013	14.07595 ± 0.00724
e	...	0.01119	0.00631	0.00109	0.04387 ± 0.03502	0.00142 ± 0.00394	0.00120 ± 0.00028
ω	deg	89.99234	223.54696	173.15636	89.68870 ± 0.98273	223.47563 ± 2.72212	174.56600 ± 4.52321
M_0	deg	0.00001	0.00034	0.00010	0.33416 ± 0.23087	0.46562 ± 0.31050	2.71969 ± 2.57135
i	deg	89.47507	89.44010	75.59741	89.63164 ± 0.44523	89.25623 ± 0.21991	74.27032 ± 0.40173
Ω	deg	0.00000	0.00000	0.00000	3.15613 ± 3.01037	1.28301 ± 1.15840	0.55842 ± 0.42973
χ^2	...	116.76747	3.09309	...	112.45805	2.94137	...
χ_{red}^2	...	4.99419	29.96514	...	4.80831	28.84985	...
$\ln \mathcal{L}$	212.13172	226.37031	...
BIC	-342.53521	-371.01239	...
AIC _c	-348.04122	-376.51840	...

Note. The K 's listed here are unconstrained, but see Section 6 for discussion regarding the planets' low K 's generated by Exo-Striker.

Table A10
(Continued)

$P_d = 5.08$ days	$P_d = 5.39$ days	$P_d = 5.64$ days	$P_d = 5.86$ days	$P_d = 6.20$ days
2461394.10116 ± 0.03620	2461394.09039 ± 0.02896	2461394.23630 ± 0.07240	2461394.24072 ± 0.03258	2461394.22317 ± 0.10498
2461402.56424 ± 0.03630	2461402.55399 ± 0.02904	2461402.70241 ± 0.07260	2461402.70443 ± 0.03267	2461402.68653 ± 0.10527

Note. The columns are for each three-planet configuration.

Table A11
List of Predicted Midpoint Times in BJD for AU Mic b (After Accounting for TTVs) Over the Next Three Years (Part II)

$P_d = 6.47$ days	$P_d = 11.9$ days	$P_d = 12.6$ days	$P_d = 12.7$ days	$P_d = 14.1$ days
2460310.84935 ± 0.03744	2460310.79751 ± 0.01171	2460310.78970 ± 0.00937	2460310.73989 ± 0.01405	2460310.79791 ± 0.00470
2460319.31324 ± 0.03760	2460319.26066 ± 0.01176	2460319.25322 ± 0.00941	2460319.20283 ± 0.01411	2460319.26120 ± 0.00472
2460327.77704 ± 0.03776	2460327.72416 ± 0.01181	2460327.71651 ± 0.00945	2460327.66589 ± 0.01417	2460327.72462 ± 0.00474
2460336.24126 ± 0.03792	2460336.18739 ± 0.01186	2460336.17972 ± 0.00949	2460336.12873 ± 0.01423	2460336.18794 ± 0.00476
2460344.70557 ± 0.03808	2460344.65064 ± 0.01191	2460344.64333 ± 0.00953	2460344.59174 ± 0.01429	2460344.65158 ± 0.00478
2460353.16872 ± 0.03824	2460353.11392 ± 0.01196	2460353.10664 ± 0.00957	2460353.05484 ± 0.01435	2460353.11501 ± 0.00480
2460361.63262 ± 0.03840	2460361.57714 ± 0.01201	2460361.56992 ± 0.00961	2460361.51776 ± 0.01441	2460361.57852 ± 0.00482
2460370.09642 ± 0.03856	2460370.04046 ± 0.01206	2460370.03356 ± 0.00965	2460369.98073 ± 0.01447	2460370.04188 ± 0.00484
2460378.56013 ± 0.03872	2460378.50369 ± 0.01211	2460378.49690 ± 0.00969	2460378.44389 ± 0.01453	2460378.50539 ± 0.00486
2460387.02382 ± 0.03888	2460386.96730 ± 0.01216	2460386.96022 ± 0.00973	2460386.90681 ± 0.01459	2460386.96899 ± 0.00488
2460395.48733 ± 0.03904	2460395.43065 ± 0.01221	2460395.42389 ± 0.00977	2460395.36982 ± 0.01465	2460395.43250 ± 0.00490
2460403.95106 ± 0.03920	2460403.89401 ± 0.01226	2460403.88729 ± 0.00981	2460403.83303 ± 0.01471	2460403.89609 ± 0.00492
2460412.41477 ± 0.03936	2460412.35744 ± 0.01231	2460412.35063 ± 0.00985	2460412.29602 ± 0.01477	2460412.35952 ± 0.00494
2460420.87841 ± 0.03952	2460420.82077 ± 0.01236	2460420.81439 ± 0.00989	2460420.75912 ± 0.01483	2460420.82320 ± 0.00496
2460429.34214 ± 0.03968	2460429.28420 ± 0.01241	2460429.27781 ± 0.00993	2460429.22237 ± 0.01489	2460429.28685 ± 0.00498
2460437.80578 ± 0.03984	2460437.74755 ± 0.01246	2460437.74121 ± 0.00997	2460437.68546 ± 0.01495	2460437.75058 ± 0.00500
2460446.26979 ± 0.04000	2460446.21127 ± 0.01251	2460446.20502 ± 0.01001	2460446.14854 ± 0.01501	2460446.21409 ± 0.00502
2460454.73380 ± 0.04016	2460454.67471 ± 0.01256	2460454.66846 ± 0.01005	2460454.61185 ± 0.01507	2460454.67774 ± 0.00504
2460463.19677 ± 0.04032	2460463.13818 ± 0.01261	2460463.13191 ± 0.01009	2460463.07496 ± 0.01513	2460463.14134 ± 0.00506
2460471.66050 ± 0.04048	2460471.60173 ± 0.01266	2460471.59572 ± 0.01013	2460471.53807 ± 0.01519	2460471.60507 ± 0.00508
2460480.12424 ± 0.04064	2460480.06515 ± 0.01271	2460480.05920 ± 0.01017	2460480.00142 ± 0.01524	2460480.06882 ± 0.00509
2460488.58781 ± 0.04080	2460488.52869 ± 0.01276	2460488.52268 ± 0.01021	2460488.46458 ± 0.01530	2460488.53238 ± 0.00511
2460497.05144 ± 0.04096	2460496.99216 ± 0.01281	2460496.98653 ± 0.01025	2460496.92778 ± 0.01536	2460496.99611 ± 0.00513
2460505.51486 ± 0.04112	2460505.45592 ± 0.01286	2460505.45005 ± 0.01029	2460505.39118 ± 0.01542	2460505.45977 ± 0.00515
2460513.97852 ± 0.04128	2460513.91947 ± 0.01291	2460513.91355 ± 0.01033	2460513.85445 ± 0.01548	2460513.92369 ± 0.00517
2460522.44213 ± 0.04144	2460522.38303 ± 0.01296	2460522.37746 ± 0.01037	2460522.31766 ± 0.01554	2460522.38736 ± 0.00519
2460530.90564 ± 0.04160	2460530.84670 ± 0.01301	2460530.84100 ± 0.01041	2460530.78110 ± 0.01560	2460530.85110 ± 0.00521
2460539.36925 ± 0.04176	2460539.31022 ± 0.01306	2460539.30455 ± 0.01045	2460539.24439 ± 0.01566	2460539.31473 ± 0.00523
2460547.83269 ± 0.04192	2460547.77383 ± 0.01311	2460547.76846 ± 0.01049	2460547.70760 ± 0.01572	2460547.77847 ± 0.00525
2460556.29656 ± 0.04208	2460556.23735 ± 0.01316	2460556.23199 ± 0.01053	2460556.17106 ± 0.01578	2460556.24231 ± 0.00527
2460564.76038 ± 0.04224	2460564.70117 ± 0.01321	2460564.69558 ± 0.01057	2460564.63441 ± 0.01584	2460564.70601 ± 0.00529
2460573.22328 ± 0.04240	2460573.16480 ± 0.01326	2460573.15948 ± 0.01061	2460573.09770 ± 0.01590	2460573.16978 ± 0.00531
2460581.68696 ± 0.04256	2460581.62842 ± 0.01331	2460581.62306 ± 0.01065	2460581.56119 ± 0.01596	2460581.63341 ± 0.00533
2460590.15066 ± 0.04272	2460590.09221 ± 0.01336	2460590.08666 ± 0.01069	2460590.02465 ± 0.01602	2460590.09729 ± 0.00535
2460598.61418 ± 0.04288	2460598.55578 ± 0.01341	2460598.55057 ± 0.01073	2460598.48796 ± 0.01608	2460598.56108 ± 0.00537
2460607.07774 ± 0.04304	2460607.01945 ± 0.01346	2460607.01417 ± 0.01077	2460606.95148 ± 0.01614	2460607.02492 ± 0.00539
2460615.54107 ± 0.04320	2460615.48303 ± 0.01351	2460615.47778 ± 0.01081	2460615.41496 ± 0.01620	2460615.48855 ± 0.00541
2460624.00461 ± 0.04336	2460623.94690 ± 0.01356	2460623.94171 ± 0.01085	2460623.87825 ± 0.01626	2460623.95228 ± 0.00543
2460632.46811 ± 0.04352	2460632.41056 ± 0.01361	2460632.40529 ± 0.01089	2460632.34177 ± 0.01632	2460632.41601 ± 0.00545
2460640.93147 ± 0.04368	2460640.87424 ± 0.01366	2460640.86892 ± 0.01093	2460640.80527 ± 0.01638	2460640.87979 ± 0.00547
2460649.39501 ± 0.04384	2460649.33808 ± 0.01371	2460649.33282 ± 0.01097	2460649.26860 ± 0.01644	2460649.34360 ± 0.00549
2460657.85836 ± 0.04400	2460657.80168 ± 0.01376	2460657.79640 ± 0.01101	2460657.73213 ± 0.01650	2460657.80720 ± 0.00551
2460666.32220 ± 0.04416	2460666.26539 ± 0.01381	2460666.26006 ± 0.01105	2460666.19574 ± 0.01656	2460666.27097 ± 0.00553
2460674.78595 ± 0.04432	2460674.72902 ± 0.01386	2460674.72393 ± 0.01109	2460674.65909 ± 0.01662	2460674.73467 ± 0.00555
2460683.24876 ± 0.04448	2460683.19288 ± 0.01391	2460683.18753 ± 0.01113	2460683.12264 ± 0.01668	2460683.19857 ± 0.00557
2460691.71237 ± 0.04464	2460691.65657 ± 0.01396	2460691.65116 ± 0.01117	2460691.58628 ± 0.01674	2460691.66221 ± 0.00559
2460700.17595 ± 0.04480	2460700.12027 ± 0.01401	2460700.11502 ± 0.01121	2460700.04960 ± 0.01680	2460700.12591 ± 0.00561
2460708.63936 ± 0.04496	2460708.58414 ± 0.01406	2460708.57860 ± 0.01125	2460708.51311 ± 0.01686	2460708.58950 ± 0.00563
2460717.10277 ± 0.04512	2460717.04777 ± 0.01411	2460717.04223 ± 0.01129	2460716.97674 ± 0.01692	2460717.05318 ± 0.00565
2460725.56600 ± 0.04528	2460725.51149 ± 0.01416	2460725.50607 ± 0.01133	2460725.44004 ± 0.01698	2460725.51699 ± 0.00567
2460734.02943 ± 0.04544	2460733.97511 ± 0.01421	2460733.96961 ± 0.01137	2460733.90355 ± 0.01704	2460733.98056 ± 0.00569

Table A11
(Continued)

$P_d = 6.47$ days	$P_d = 11.9$ days	$P_d = 12.6$ days	$P_d = 12.7$ days	$P_d = 14.1$ days
2461284.12226 ± 0.05584	2461284.09587 ± 0.01745	2461284.07987 ± 0.01397	2461284.01035 ± 0.02094	2461284.09079 ± 0.00699
2461292.58481 ± 0.05600	2461292.55878 ± 0.01750	2461292.54293 ± 0.01401	2461292.47286 ± 0.02100	2461292.55364 ± 0.00701
2461301.04749 ± 0.05616	2461301.02204 ± 0.01755	2461301.00595 ± 0.01405	2461300.93593 ± 0.02106	2461301.01667 ± 0.00703
2461309.51024 ± 0.05632	2461309.48494 ± 0.01760	2461309.46888 ± 0.01409	2461309.39859 ± 0.02112	2461309.47978 ± 0.00705
2461317.97243 ± 0.05648	2461317.94788 ± 0.01765	2461317.93197 ± 0.01413	2461317.86109 ± 0.02118	2461317.94276 ± 0.00707
2461326.43594 ± 0.05664	2461326.41073 ± 0.01770	2461326.39500 ± 0.01417	2461326.32421 ± 0.02124	2461326.40577 ± 0.00709
2461334.89936 ± 0.05680	2461334.87358 ± 0.01775	2461334.85795 ± 0.01421	2461334.78685 ± 0.02130	2461334.86867 ± 0.00711
2461343.36244 ± 0.05696	2461343.33650 ± 0.01780	2461343.32108 ± 0.01425	2461343.24941 ± 0.02136	2461343.33187 ± 0.00713
2461351.82545 ± 0.05712	2461351.79938 ± 0.01785	2461351.78414 ± 0.01429	2461351.71243 ± 0.02142	2461351.79490 ± 0.00715
2461360.28832 ± 0.05728	2461360.26264 ± 0.01790	2461360.24709 ± 0.01433	2461360.17508 ± 0.02148	2461360.25806 ± 0.00717
2461368.75131 ± 0.05744	2461368.72551 ± 0.01795	2461368.71027 ± 0.01437	2461368.63759 ± 0.02154	2461368.72097 ± 0.00719
2461377.21433 ± 0.05760	2461377.18842 ± 0.01800	2461377.17334 ± 0.01441	2461377.10054 ± 0.02160	2461377.18395 ± 0.00721
2461385.67718 ± 0.05776	2461385.65128 ± 0.01805	2461385.63632 ± 0.01445	2461385.56321 ± 0.02166	2461385.64715 ± 0.00723
2461394.14020 ± 0.05792	2461394.11412 ± 0.01810	2461394.09956 ± 0.01449	2461394.02570 ± 0.02172	2461394.11024 ± 0.00725
2461402.60310 ± 0.05808	2461402.57706 ± 0.01815	2461402.56265 ± 0.01453	2461402.48869 ± 0.02178	2461402.57345 ± 0.00727

Note.The columns are for each three-planet configuration.

Table A12
List of Predicted Midpoint Times in BJD for AU Mic c (After Accounting for TTVs) Over the Next Three Years (Part I)

$P_d = 5.08$ days	$P_d = 5.39$ days	$P_d = 5.64$ days	$P_d = 5.86$ days	$P_d = 6.20$ days
2460322.63096 ± 0.03046	2460322.47156 ± 0.11235	2460322.58173 ± 0.06196	2460322.44772 ± 0.03781	2460322.57336 ± 0.00430
2460341.49211 ± 0.03075	2460341.33078 ± 0.11342	2460341.44194 ± 0.06255	2460341.30710 ± 0.03817	2460341.43354 ± 0.00434
2460360.35401 ± 0.03104	2460360.19004 ± 0.11449	2460360.30267 ± 0.06314	2460360.16643 ± 0.03853	2460360.29392 ± 0.00438
2460379.21496 ± 0.03133	2460379.04925 ± 0.11556	2460379.16335 ± 0.06373	2460379.02565 ± 0.03889	2460379.15426 ± 0.00442
2460398.07527 ± 0.03162	2460397.90847 ± 0.11663	2460398.02398 ± 0.06432	2460397.88484 ± 0.03925	2460398.01477 ± 0.00446
2460416.93603 ± 0.03191	2460416.76769 ± 0.11770	2460416.88424 ± 0.06491	2460416.74416 ± 0.03961	2460416.87513 ± 0.00450
2460435.79777 ± 0.03220	2460435.62695 ± 0.11877	2460435.74478 ± 0.06550	2460435.60353 ± 0.03997	2460435.73562 ± 0.00454
2460454.65919 ± 0.03249	2460454.48616 ± 0.11984	2460454.60551 ± 0.06609	2460454.46282 ± 0.04033	2460454.59619 ± 0.00458
2460473.51959 ± 0.03278	2460473.34540 ± 0.12091	2460473.46615 ± 0.06668	2460473.32202 ± 0.04069	2460473.45678 ± 0.00461
2460492.38011 ± 0.03307	2460492.20456 ± 0.12198	2460492.32658 ± 0.06727	2460492.18124 ± 0.04105	2460492.31741 ± 0.00465
2460511.24151 ± 0.03336	2460511.06379 ± 0.12305	2460511.18692 ± 0.06786	2460511.04060 ± 0.04141	2460511.17797 ± 0.00469
2460530.10326 ± 0.03365	2460529.92297 ± 0.12412	2460530.04767 ± 0.06845	2460529.89995 ± 0.04177	2460530.03866 ± 0.00473
2460548.96396 ± 0.03394	2460548.78219 ± 0.12519	2460548.90830 ± 0.06904	2460548.75920 ± 0.04213	2460548.89928 ± 0.00477
2460567.82431 ± 0.03423	2460567.64130 ± 0.12626	2460567.76891 ± 0.06963	2460567.61837 ± 0.04249	2460567.76003 ± 0.00481
2460586.68528 ± 0.03452	2460586.50054 ± 0.12733	2460586.62915 ± 0.07022	2460586.47766 ± 0.04285	2460586.62053 ± 0.00485
2460605.54716 ± 0.03481	2460605.35970 ± 0.12840	2460605.48979 ± 0.07081	2460605.33703 ± 0.04321	2460605.48116 ± 0.00489
2460624.40832 ± 0.03510	2460624.21893 ± 0.12947	2460624.35048 ± 0.07140	2460624.19635 ± 0.04357	2460624.34170 ± 0.00493
2460643.26861 ± 0.03539	2460643.07807 ± 0.13054	2460643.21115 ± 0.07199	2460643.05555 ± 0.04393	2460643.20230 ± 0.00497
2460662.12924 ± 0.03568	2460661.93728 ± 0.13161	2460662.07145 ± 0.07258	2460661.91476 ± 0.04429	2460662.06271 ± 0.00501
2460680.99090 ± 0.03597	2460680.79644 ± 0.13268	2460680.93191 ± 0.07317	2460680.77409 ± 0.04465	2460680.92310 ± 0.00505
2460699.85248 ± 0.03626	2460699.65565 ± 0.13375	2460699.79266 ± 0.07376	2460699.63346 ± 0.04501	2460699.78351 ± 0.00509
2460718.71294 ± 0.03655	2460718.51480 ± 0.13482	2460718.65327 ± 0.07435	2460718.49273 ± 0.04537	2460718.64384 ± 0.00513
2460737.57340 ± 0.03684	2460737.37395 ± 0.13589	2460737.51379 ± 0.07494	2460737.35192 ± 0.04573	2460737.50421 ± 0.00516
2460756.43466 ± 0.03713	2460756.23310 ± 0.13696	2460756.37410 ± 0.07553	2460756.21117 ± 0.04609	2460756.36439 ± 0.00520
2460775.29646 ± 0.03742	2460775.09228 ± 0.13803	2460775.23479 ± 0.07612	2460775.07053 ± 0.04645	2460775.22471 ± 0.00524
2460794.15730 ± 0.03771	2460793.95140 ± 0.13910	2460794.09544 ± 0.07671	2460793.92987 ± 0.04681	2460794.08494 ± 0.00528
2460813.01766 ± 0.03800	2460812.81055 ± 0.14017	2460812.95610 ± 0.07730	2460812.78909 ± 0.04717	2460812.94532 ± 0.00532
2460831.87850 ± 0.03829	2460831.66970 ± 0.14124	2460831.81633 ± 0.07789	2460831.64829 ± 0.04753	2460831.80552 ± 0.00536
2460850.74029 ± 0.03858	2460850.52889 ± 0.14231	2460850.67691 ± 0.07848	2460850.50759 ± 0.04789	2460850.66587 ± 0.00540
2460869.60163 ± 0.03887	2460869.38803 ± 0.14338	2460869.53764 ± 0.07907	2460869.36697 ± 0.04825	2460869.52626 ± 0.00544
2460888.46196 ± 0.03916	2460888.24721 ± 0.14445	2460888.39828 ± 0.07966	2460888.22627 ± 0.04861	2460888.38672 ± 0.00548
2460907.32249 ± 0.03945	2460907.10630 ± 0.14552	2460907.25865 ± 0.08025	2460907.08546 ± 0.04897	2460907.24718 ± 0.00552
2460926.18402 ± 0.03974	2460925.96549 ± 0.14659	2460926.11906 ± 0.08084	2460925.94469 ± 0.04933	2460926.10764 ± 0.00556
2460945.04575 ± 0.04003	2460944.82460 ± 0.14766	2460944.97979 ± 0.08143	2460944.80403 ± 0.04969	2460944.96825 ± 0.00560
2460963.90632 ± 0.04032	2460963.68377 ± 0.14873	2460963.84042 ± 0.08202	2460963.66339 ± 0.05005	2460963.82882 ± 0.00564
2460982.76670 ± 0.04061	2460982.54282 ± 0.14980	2460982.70100 ± 0.08261	2460982.52263 ± 0.05041	2460982.68955 ± 0.00568
2461001.62781 ± 0.04090	2461001.40200 ± 0.15087	2461001.56125 ± 0.08320	2461001.38183 ± 0.05077	2461001.55009 ± 0.00572
2461020.48965 ± 0.04119	2461020.26110 ± 0.15194	2461020.42192 ± 0.08379	2461020.24110 ± 0.05113	2461020.41080 ± 0.00576

Table A12
(Continued)

$P_d = 5.08$ days	$P_d = 5.39$ days	$P_d = 5.64$ days	$P_d = 5.86$ days	$P_d = 6.20$ days
2461039.35065 ± 0.04148	2461039.12026 ± 0.15301	2461039.28260 ± 0.08438	2461039.10047 ± 0.05149	2461039.27144 ± 0.00580
2461058.21099 ± 0.04177	2461057.97934 ± 0.15408	2461058.14325 ± 0.08497	2461057.95978 ± 0.05185	2461058.13218 ± 0.00583
2461077.07172 ± 0.04206	2461076.83851 ± 0.15515	2461077.00352 ± 0.08556	2461076.81900 ± 0.05221	2461076.99274 ± 0.00587
2461095.93343 ± 0.04235	2461095.69763 ± 0.15622	2461095.86406 ± 0.08615	2461095.67821 ± 0.05257	2461095.85333 ± 0.00591
2461114.79490 ± 0.04264	2461114.55680 ± 0.15729	2461114.72476 ± 0.08673	2461114.53753 ± 0.05293	2461114.71392 ± 0.00595
2461133.65530 ± 0.04293	2461133.41590 ± 0.15836	2461133.58542 ± 0.08732	2461133.39690 ± 0.05329	2461133.57444 ± 0.00599
2461152.51578 ± 0.04322	2461152.27502 ± 0.15943	2461152.44587 ± 0.08791	2461152.25617 ± 0.05365	2461152.43496 ± 0.00603
2461171.37716 ± 0.04351	2461171.13412 ± 0.16050	2461171.30617 ± 0.08850	2461171.11536 ± 0.05401	2461171.29529 ± 0.00607
2461190.23897 ± 0.04380	2461189.99327 ± 0.16157	2461190.16692 ± 0.08909	2461189.97461 ± 0.05437	2461190.15573 ± 0.00611
2461209.09968 ± 0.04409	2461208.85235 ± 0.16264	2461209.02757 ± 0.08968	2461208.83397 ± 0.05473	2461209.01603 ± 0.00615
2461227.96003 ± 0.04438	2461227.71145 ± 0.16371	2461227.88819 ± 0.09027	2461227.69331 ± 0.05509	2461227.87644 ± 0.00619
2461246.82097 ± 0.04467	2461246.57056 ± 0.16478	2461246.74842 ± 0.09086	2461246.55254 ± 0.05545	2461246.73660 ± 0.00623
2461265.68281 ± 0.04496	2461265.42972 ± 0.16585	2461265.60905 ± 0.09145	2461265.41173 ± 0.05581	2461265.59689 ± 0.00627
2461284.54399 ± 0.04525	2461284.28882 ± 0.16692	2461284.46973 ± 0.09204	2461284.27104 ± 0.05617	2461284.45715 ± 0.00631
2461303.40433 ± 0.04554	2461303.14796 ± 0.16799	2461303.33042 ± 0.09263	2461303.13040 ± 0.05653	2461303.31748 ± 0.00635
2461322.26496 ± 0.04583	2461322.00705 ± 0.16906	2461322.19073 ± 0.09322	2461321.98970 ± 0.05689	2461322.17774 ± 0.00639
2461341.12657 ± 0.04612	2461340.86621 ± 0.17013	2461341.05116 ± 0.09381	2461340.84890 ± 0.05725	2461341.03802 ± 0.00643
2461359.98817 ± 0.04641	2461359.72530 ± 0.17120	2461359.91190 ± 0.09440	2461359.70813 ± 0.05761	2461359.89843 ± 0.00647
2461378.84865 ± 0.04670	2461378.58445 ± 0.17227	2461378.77255 ± 0.09499	2461378.56748 ± 0.05797	2461378.75881 ± 0.00651
2461397.70909 ± 0.04699	2461397.44349 ± 0.17334	2461397.63308 ± 0.09558	2461397.42683 ± 0.05833	2461397.61934 ± 0.00655

Note. The columns are for each three-planet configuration..

Table A13
List of Predicted Midpoint Times in BJD for AU Mic c (After Accounting for TTVs) Over the Next Three Years (Part II)




























$P_d = 6.47$ days	$P_d = 11.9$ days	$P_d = 12.6$ days	$P_d = 12.7$ days	$P_d = 14.1$ days
2460322.58254 ± 0.03466	2460322.52001 ± 0.04516	2460322.40457 ± 0.00950	2460322.48933 ± 0.00845	2460322.35068 ± 0.01368
2460341.44311 ± 0.03499	2460341.38002 ± 0.04559	2460341.26429 ± 0.00959	2460341.34919 ± 0.00853	2460341.20880 ± 0.01381
2460360.30406 ± 0.03532	2460360.23997 ± 0.04602	2460360.12350 ± 0.00967	2460360.20886 ± 0.00861	2460360.06637 ± 0.01394
2460379.16458 ± 0.03565	2460379.09968 ± 0.04645	2460378.98336 ± 0.00976	2460379.06903 ± 0.00869	2460378.92560 ± 0.01407
2460398.02486 ± 0.03598	2460397.95978 ± 0.04688	2460397.84285 ± 0.00985	2460397.92928 ± 0.00877	2460397.78454 ± 0.01420
2460416.88527 ± 0.03631	2460416.81973 ± 0.04731	2460416.70288 ± 0.00994	2460416.78973 ± 0.00885	2460416.64275 ± 0.01433
2460435.74612 ± 0.03664	2460435.67976 ± 0.04774	2460435.56255 ± 0.01003	2460435.65028 ± 0.00893	2460435.50253 ± 0.01446
2460454.60686 ± 0.03697	2460454.53974 ± 0.04817	2460454.42273 ± 0.01012	2460454.51091 ± 0.00901	2460454.36224 ± 0.01459
2460473.46716 ± 0.03730	2460473.39956 ± 0.04860	2460473.28264 ± 0.01021	2460473.37153 ± 0.00909	2460473.22079 ± 0.01472
2460492.32752 ± 0.03763	2460492.25962 ± 0.04903	2460492.14302 ± 0.01030	2460492.23213 ± 0.00917	2460492.08034 ± 0.01485
2460511.18822 ± 0.03796	2460511.11941 ± 0.04946	2460511.00312 ± 0.01039	2460511.09259 ± 0.00925	2460510.94007 ± 0.01498
2460530.04912 ± 0.03829	2460529.97943 ± 0.04989	2460529.86361 ± 0.01048	2460529.95282 ± 0.00933	2460529.79857 ± 0.01511
2460548.90954 ± 0.03862	2460548.83949 ± 0.05032	2460548.72391 ± 0.01057	2460548.81303 ± 0.00941	2460548.65767 ± 0.01524
2460567.76986 ± 0.03895	2460567.69947 ± 0.05075	2460567.58458 ± 0.01066	2460567.67272 ± 0.00949	2460567.51685 ± 0.01537
2460586.63036 ± 0.03928	2460586.55944 ± 0.05118	2460586.44505 ± 0.01075	2460586.53264 ± 0.00957	2460586.37442 ± 0.01550
2460605.49129 ± 0.03961	2460605.41916 ± 0.05161	2460605.30575 ± 0.01084	2460605.39167 ± 0.00964	2460605.23284 ± 0.01563
2460624.35189 ± 0.03994	2460624.27915 ± 0.05204	2460624.16637 ± 0.01093	2460624.25136 ± 0.00972	2460624.09145 ± 0.01576
2460643.21215 ± 0.04027	2460643.13932 ± 0.05247	2460643.02717 ± 0.01102	2460643.10988 ± 0.00980	2460642.94823 ± 0.01589
2460662.07253 ± 0.04060	2460661.99935 ± 0.05290	2460661.88792 ± 0.01111	2460661.96939 ± 0.00988	2460661.80565 ± 0.01602
2460680.93334 ± 0.04093	2460680.85928 ± 0.05333	2460680.74871 ± 0.01120	2460680.82762 ± 0.00996	2460680.66359 ± 0.01615
2460699.79415 ± 0.04126	2460699.71899 ± 0.05376	2460699.60952 ± 0.01129	2460699.68702 ± 0.01004	2460699.52062 ± 0.01628
2460718.65448 ± 0.04159	2460718.57901 ± 0.05419	2460718.47031 ± 0.01138	2460718.54524 ± 0.01012	2460718.37790 ± 0.01641
2460737.51484 ± 0.04192	2460737.43911 ± 0.05462	2460737.33118 ± 0.01147	2460737.40466 ± 0.01020	2460737.23557 ± 0.01654
2460756.37547 ± 0.04225	2460756.29915 ± 0.05505	2460756.19195 ± 0.01156	2460756.26311 ± 0.01028	2460756.09332 ± 0.01667
2460775.23639 ± 0.04258	2460775.15910 ± 0.05548	2460775.05276 ± 0.01165	2460775.12264 ± 0.01036	2460774.95136 ± 0.01680
2460794.09687 ± 0.04291	2460794.01884 ± 0.05591	2460793.91349 ± 0.01174	2460793.98152 ± 0.01044	2460793.80975 ± 0.01693
2460812.95717 ± 0.04324	2460812.87893 ± 0.05634	2460812.77429 ± 0.01183	2460812.84130 ± 0.01052	2460812.66844 ± 0.01706
2460831.81761 ± 0.04357	2460831.73882 ± 0.05677	2460831.63496 ± 0.01192	2460831.70073 ± 0.01060	2460831.52707 ± 0.01719
2460850.67849 ± 0.04390	2460850.59884 ± 0.05720	2460850.49564 ± 0.01201	2460850.56076 ± 0.01068	2460850.38639 ± 0.01731
2460869.53918 ± 0.04423	2460869.45885 ± 0.05763	2460869.35624 ± 0.01210	2460869.42075 ± 0.01076	2460869.24605 ± 0.01744
2460888.39945 ± 0.04456	2460888.31870 ± 0.05806	2460888.21685 ± 0.01219	2460888.28106 ± 0.01084	2460888.10515 ± 0.01757
2460907.25981 ± 0.04489	2460907.17876 ± 0.05849	2460907.07737 ± 0.01228	2460907.14147 ± 0.01092	2460906.96478 ± 0.01770
2460926.12057 ± 0.04522	2460926.03851 ± 0.05892	2460925.93786 ± 0.01237	2460926.00196 ± 0.01100	2460925.82463 ± 0.01783








Table A13
(Continued)

$P_d = 6.47$ days	$P_d = 11.9$ days	$P_d = 12.6$ days	$P_d = 12.7$ days	$P_d = 14.1$ days
2460944.98145 ± 0.04555	2460944.89852 ± 0.05935	2460944.79827 ± 0.01245	2460944.86252 ± 0.01108	2460944.68405 ± 0.01796
2460963.84182 ± 0.04588	2460963.75860 ± 0.05978	2460963.65866 ± 0.01254	2460963.72308 ± 0.01116	2460963.54371 ± 0.01809
2460982.70216 ± 0.04621	2460982.61863 ± 0.06021	2460982.51900 ± 0.01263	2460982.58358 ± 0.01124	2460982.40300 ± 0.01822
2461001.56271 ± 0.04654	2461001.47858 ± 0.06064	2461001.37932 ± 0.01272	2461001.44397 ± 0.01132	2461001.26188 ± 0.01835
2461020.42363 ± 0.04687	2461020.33830 ± 0.06107	2461020.23952 ± 0.01281	2461020.30426 ± 0.01140	2461020.12112 ± 0.01848
2461039.28417 ± 0.04720	2461039.19829 ± 0.06150	2461039.09978 ± 0.01290	2461039.16423 ± 0.01148	2461038.97974 ± 0.01861
2461058.14446 ± 0.04753	2461058.05844 ± 0.06193	2461057.95990 ± 0.01299	2461058.02429 ± 0.01156	2461057.83773 ± 0.01874
2461077.00487 ± 0.04786	2461076.91849 ± 0.06236	2461076.82014 ± 0.01308	2461076.88370 ± 0.01164	2461076.69570 ± 0.01887
2461095.86571 ± 0.04819	2461095.77842 ± 0.06279	2461095.68014 ± 0.01317	2461095.74351 ± 0.01172	2461095.55354 ± 0.01900
2461114.72647 ± 0.04852	2461114.63813 ± 0.06322	2461114.54039 ± 0.01326	2461114.60236 ± 0.01180	2461114.41089 ± 0.01913
2461133.58677 ± 0.04885	2461133.49817 ± 0.06365	2461133.40031 ± 0.01335	2461133.46200 ± 0.01188	2461133.26774 ± 0.01926
2461152.44712 ± 0.04918	2461152.35820 ± 0.06408	2461152.26063 ± 0.01344	2461152.32040 ± 0.01196	2461152.12497 ± 0.01939
2461171.30781 ± 0.04951	2461171.21824 ± 0.06451	2461171.12049 ± 0.01353	2461171.17995 ± 0.01204	2461170.98195 ± 0.01952
2461190.16874 ± 0.04984	2461190.07822 ± 0.06494	2461189.98086 ± 0.01362	2461190.03811 ± 0.01212	2461189.83846 ± 0.01965
2461209.02915 ± 0.05017	2461208.93796 ± 0.06537	2461208.84070 ± 0.01371	2461208.89761 ± 0.01220	2461208.69618 ± 0.01978
2461227.88946 ± 0.05050	2461227.79808 ± 0.06580	2461227.70118 ± 0.01380	2461227.75589 ± 0.01228	2461227.55347 ± 0.01991
2461246.74995 ± 0.05083	2461246.65789 ± 0.06623	2461246.56109 ± 0.01389	2461246.61538 ± 0.01236	2461246.41023 ± 0.02004
2461265.61085 ± 0.05116	2461265.51790 ± 0.06666	2461265.42162 ± 0.01398	2461265.47400 ± 0.01243	2461265.26893 ± 0.02017
2461284.47147 ± 0.05149	2461284.37793 ± 0.06709	2461284.28161 ± 0.01407	2461284.33360 ± 0.01251	2461284.12733 ± 0.02030
2461303.33175 ± 0.05182	2461303.23788 ± 0.06752	2461303.14227 ± 0.01416	2461303.19274 ± 0.01259	2461302.98495 ± 0.02043
2461322.19214 ± 0.05215	2461322.09790 ± 0.06795	2461322.00242 ± 0.01425	2461322.05256 ± 0.01267	2461321.84420 ± 0.02056
2461341.05294 ± 0.05248	2461340.95763 ± 0.06838	2461340.86320 ± 0.01434	2461340.91223 ± 0.01275	2461340.70353 ± 0.02069
2461359.91376 ± 0.05281	2461359.81762 ± 0.06881	2461359.72348 ± 0.01443	2461359.77234 ± 0.01283	2461359.56210 ± 0.02082
2461378.77410 ± 0.05314	2461378.67771 ± 0.06924	2461378.58442 ± 0.01452	2461378.63253 ± 0.01291	2461378.42180 ± 0.02095
2461397.63445 ± 0.05347	2461397.53782 ± 0.06967	2461397.44494 ± 0.01461	2461397.49293 ± 0.01299	2461397.28144 ± 0.02108

Note. The columns are for each three-planet configuration.

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