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Research Article

Solitary and Rogue Wave Solutions to the Conformable Time Fractional Modified Kawahara Equation in Mathematical Physics

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Utilizing of illustrative scheming programming, the study inspects the careful voyaging wave engagements from the nonlinear time fractional modified Kawahara equation (mKE) by employing the advanced $\exp(-\varphi(\xi))$-expansion policy in terms of trigonometric, hyperbolic, and rational function through some treasured fractional order derivative and free parameters. The undercurrents of nonlinear wave answer are scrutinized and confirmed by MATLAB in 3D and 2D plots, and density plot by specific values of the convoluted parameters is designed. Our preferred advanced $\exp(-\varphi(\xi))$-expansion technique which is parallel to (G'/G) expansion technique is trustworthy dealing for searching significant nonlinear waves that progress a modification of dynamic depictions that ascend in mathematical physics and engineering grounds.

1. Introduction

Nowadays, nonlinear fractional partial differential equations (FPDEs) are lengthily utilized to delimit several prodigies and dynamic procedure in numerous features of mathematical science and scheming, particularly in magnetohydrodynamics, neural material science liquid mechanics, dissemination process, numerical science, plasma material science, geo-optical filaments, strong state material science, and substance energy [1–3].

Frequent investigators arranged through nonlinear evolution equations (NEEs) to form voyaging wave arrangement by executing a few arrangements. The approaches that are engrained in continuing writing are as follows: the double subequation approach [4], multiple exp-function algorithm [5], improved subequation scheme [6, 7], modified simple equation technique [8], tanh-coth scheme [9], sine-cosine strategy [10], first integral approach [11], $(G'/G, 1/G)$-expansion scheme [12], fractional reduced differential transform method [13], extended Kudryashov scheme [14], modified simple equation scheme [15], new extended (G'/G) expansion scheme [16, 17], functional variable method [18], trial solution scheme [19], scheme exp-function approach [20], multiple simplest equation scheme [21], $\exp(-\phi(\xi))$-expansion scheme [22–26], pseudoparabolic model [27–29], sine-Gordon expansion scheme [30], modified extended tanh-function scheme [31], modified auxiliary expansion scheme [32], method of line [33], Bernoulli subequation function technique [34, 35], modified exponential function scheme [36], improved Bernoulli subequation function scheme [37], and the finite difference scheme [38].

The prime goal of this article is to inspect the approached solutions of the nonlinear time fractional modified Kawahara equation in the form

$$\Omega_t^\delta u + u^2 u_x + \Phi u_{xx} + \Psi u_{xxx} = 0, \quad t > 0, x \in \mathbb{R}, \tag{1}$$

where δ denotes a parameter recitation fractional order of the time derivative constant and $0 < \delta \leq 1$. Our preferred modified Kawahara equation (MKE) was previously measured by various investigators; for instance, Atangana et al. [39] planned the exact numerical solutions of time fractional mKE exhausting homotopy decomposition and the Sumudu transform strategy. Guner and Atik determined our stated time fractional MKE employing lengthy $\exp(-\varphi(\xi))$-expansion approach [40]. Kadkhoda and Jafari [41] explained the time fractional MKE by various analytical schemes, namely, fractional exp-function system and tenable particular exact soliton answers. Bhatter et al. [42] solved the time fractional modified nonlinear Kawahara equation using the Mittag-Leffler law and secured some exact soliton solutions that are very important to describe the nonlinear physical phenomena with the sense of Caputo. Recently, Shahen et al. [43] explored the exact solutions of $(2+1)$ dimensional AKNS condition with the virtue of our mentioned advanced $\exp(-\phi(\xi))$-expansion scheme. He received this method as a particular creation of generalized $\exp(-\phi(\xi))$-expansion scheme. The ultimate intension of this study is to smear the advanced $\exp(-\phi(\xi))$-expansion strategy [43] to shape the detailed voyaging wave answers for nonlinear progression environments in scientific substantial science by means of the time fractional nonlinear mKE. The advanced $\exp(-\phi(\xi))$-expansion system is more regimented and steadfast system as compared to G'/G-expansion system. Our favored method is a parallel technique of G'/G-expansion process. The answers enlarged by the specified practice can be articulated in the form of hyperbolic, trigonometric, and rational functions. These events of the clarifications are appropriate for learning certain nonlinear physical dealing. The target of this article is to apply the advanced $\exp(-\phi(\xi))$-expansion strategy [43] to build the precise voyaging wave answers for nonlinear advancement conditions in scientific material science by means of the time fractional nonlinear modified Kawahara equations.

In contrast with the attained solutions [43], to the finest of our knowledge, antibright kink, bright kink, rogue wave, and bright and dark bell solution shapes are new in the case of our advanced $\exp(-\phi(\xi))$-expansion scheme, which are not testified in previously published studies [22–26]. It is important to know that the maximum of the examined solutions in this study has varied structures over the solutions accessible in the fiction in the wave proliferation; the performed approaches are entirely new for this studied mKE equation. Therefore, the developed exact answers may irradiate the authors for advance studies to clarify pragmatic phenomena in the field of shallow water wave and mathematical physics. This article affords evidence that our mentioned MKE equation is suitable in the sense of conformable derivative for obtaining the new traveling soliton structures in any kind of physical system without any obliqueness condition.

The study is set up as follows. In Section 2, the portrayal of the conformable derivative and scheme is deliberated. In Section 3, the advanced $\exp(-\phi(\xi))$-expansion approach has been described. In Section 4, we utilized this plan to the nonlinear modified Kawahara equations. In Section 5, results and discussion are presented. In Section 6, ends are given.

2. Preliminaries and Approaches

2.1. Meaning and Some Topographies of Conformable Derivative. Recently, Khalil et al. [44] showed the basic of conformable derivative with the idea of a limit.

Definition 1. $f:(0,\infty)\longrightarrow\mathbb{R}$; then, the conformable derivative of f order δ is well-defined as

$$\Omega_t^\delta f(t)=\lim_{\varepsilon\longrightarrow}\left(\frac{f\left(t+\varepsilon t^{1-\delta}\right)-f(t)}{\varepsilon}\right),\ \text{for all } t>0,\ 0<\delta\leq 1. \tag{2}$$

Nearly, a famous researcher Abdeljawad [45] has also discovered exponential functions, chain rule, definite and indefinite integration by parts, Gronwall's inequality, Laplace transform, and Taylor power series expansions for conformable derivative in the process of fractional order. The definition of a conformable order derivative can naturally stun the difficulty of exiting the modified Riemann Liouville derivative definition [46] and the Caputo derivative [47].

Theorem 1. *Let $\delta\in(0,1]$ and $f=f(t)$, $g=g(t)$ be δ-conformable differentiable at a point $t>0$, then*

(i) $\Omega_t^\delta(cf+dg)=c\Omega_t^\delta f+d\Omega_t^\delta g$, for all $c,d\in\mathbb{R}$

(ii) $\Omega_t^\delta(t\gamma)=\gamma t^{\gamma-\delta}$, for all $\gamma\in\mathbb{R}$

(iii) $\Omega_t^\delta(fg)=g\Omega_t^\delta(f)+f\Omega_t^\delta(g)$

(iv) $\Omega_t^\delta(f/g)=(g\Omega_t^\delta(f)-f\Omega_t^\delta(g))/g^2$

Furthermore, if f is differentiable, then $\Omega_t^\delta(f(t))=t^{1-\delta}(df/dt)$.

Theorem 2. *Let $f:(0,\delta)\longrightarrow R$ be a real function such as f is differentiable and δ-conformable differentiable. Also, let g be a differentiable function well defined in the range of f. Then,*

$$\Omega_t^\delta(fog)(t)=t^{1-\delta}g(t)^{\delta-1}g'(t)\Omega_t^\delta(f(t))_{t=g(t)}, \tag{3}$$

where prime means the conventional derivatives with respect to t.

In this research, we have mainly taken the preferred equation with the sense of conformable derivative. In condition of general theory of calculus, there are numerous functions that do not have Taylor power arrangement representations on particular point whereas in the theory of conformable derivative they do have. The conformable derivative does well in the chain rule and product rule while involved plans appear in case of usual fractional calculus. The conformable derivative of a constant function is correspondent to zero where it is not the issue for Riemann fractional calculus.

3. Enlargement of Advanced $\exp(-\phi(\xi))$-Expansion Method

In this part, we have deliberated our mentioned advanced $\exp(-\phi(\xi))$-expansion scheme stepwise in details. Assume a nonlinear time-fractional NPD equation in the following form:

$$\mathfrak{R}\left(\Pi, \Pi_x, T_t^{\theta}\Pi, \Pi_{xx}, T_{tt}^{2\theta}\Pi, \Pi_{xxx}, \cdots\cdots\right) = 0, \tag{4}$$

where $\Pi = \Pi(x, t)$ is an anonymous function and $\mathfrak{R}$ is the polynomial function of Π, it is a distinct kind of partial derivatives, in which the nonlinear terms and the highest order of derivatives are intricate.

Step 1. Now, we assume a wave transformation variable with a view to nondimensionality. We transform all self-governing variable into one variable, as follows

$$\Pi(x, t) = u(\xi), \quad \xi = k\frac{x^{\eta}}{\eta} \pm V\frac{t^{\theta}}{\theta}. \tag{5}$$

By utilizing this variable, Equation (5) permits us reducing Equation (4) in an ODE for $\Pi(x, t) = u(\xi)$ into the form

$$P\left(\cdots u'', u', u,\right) = 0. \tag{6}$$

Step 2. Let us assume that a polynomial can start the solution of O.D. Equation (6) in $\exp(-\phi(\xi))$ as

$$u = \sum_{i=0}^{N} A_i \exp(-\phi(\xi))^i, \quad A_N \neq 0, \tag{7}$$

where N is the positive integer, which can be acquired by harmonizing the uppermost order of derivatives to the uppermost order nonlinear terms, seen in Equation (6).

And the derivative of $\phi(\xi)$ gratifies the ODE in the subsequent form

$$\phi'(\xi) - \lambda \exp(\phi(\xi)) - \mu \exp(-\phi(\xi)) = 0. \tag{8}$$

Then, the solutions of O.D. Equation (6) are as follows.

Case 1. Hyperbolic type of function solutions (when $\lambda\mu < 0$):

$$\varphi(\xi) = \ln\left(\sqrt{\frac{\lambda}{-\mu}}\tanh\left(\sqrt{-\lambda\mu}(\xi + C)\right)\right),$$
$$\varphi(\xi) = \ln\left(\sqrt{\frac{\lambda}{-\mu}}\coth\left(\sqrt{-\lambda\mu}(\xi + C)\right)\right). \tag{9}$$

Case 2. Trigonometric function solution (when $\lambda\mu > 0$):

$$\varphi(\xi) = \ln\left(\sqrt{\frac{\lambda}{\mu}}\tan\left(\sqrt{\lambda\mu}(\xi + C)\right)\right),$$
$$\varphi(\xi) = \ln\left(-\sqrt{\frac{\lambda}{\mu}}\cot\left(\sqrt{\lambda\mu}(\xi + C)\right)\right). \tag{10}$$

Case 3. When $\mu > 0$ and $\lambda = 0$:

$$\varphi(\xi) = \ln\left(\frac{1}{-\mu(\xi + C)}\right). \tag{11}$$

Case 4. When $\mu = 0$ and $\lambda \in \mathfrak{R}$:

$$\varphi(\xi) = \ln(\lambda(\xi + C)), \tag{12}$$

where C is a constant $\lambda\mu < 0$or$\lambda\mu > 0$ is contingent on the sign of μ.

Step 3. By plugging Equation (7) in Equation (6) and consuming Equation (4), gathering all like the order of $\exp(-m\phi(\xi))$, $m = 0, \pm1, \pm2, \pm3, \cdots$ organized, then we accomplish a polynomial form $\exp(-m\phi(\xi))$, and associating each coefficient of this obtained polynomial equivalent to zero yields a set of a system of algebraic equations (SAE).

Step 4. Let the constants' determination be obtained as one or more solutions by deciding the mathematical circumstances in phase 3. Plugging the constant calculations along with the arrangements for Equation (5), from the nonlinear evaluation eq., we can obtain modern and far-reaching detailed moving wave preparations (4).

4. Solicitation of the Preferred Method

In this subclass, we imposed our proposed advanced $\exp(-\phi(\xi))$-expansion technique in Equation (1) and henceforth used the following transformation:

$$u(x, t) = u(\xi), \quad \xi = kx - \frac{\Lambda t^{\delta}}{\delta}. \tag{13}$$

where k and Λ are nonzero constants. We find the ODE from Equation (1)

$$-\Lambda u' + ku^2u' + \Phi k^2u'' + \Psi k^3u''' = 0. \tag{14}$$

Now, we integrate Equation (14) with respect to ξ and we get

$$-\Lambda u + \frac{k}{3}u^3 + \Phi k^2u' + \Psi k^3u'' = 0, \tag{15}$$

where prime signifies the derivative with regard to ξ.

Now, we calculate the equilibrium number of Equation (15) between the linear term u'' and the nonlinear term u^3 which is m equal to 1 so the solution Equation (15) takes the form

$$u(\xi) = A_0 + A_1 \exp(-\varphi(\xi)). \quad (16)$$

Differential Equation (16) with respect to ξ and substituting the value of u, u', u'' into Equation (15) and connecting the coefficients of $e^{i\varphi(\xi)}$ correspondent to zero, where $i = 0, \pm1, \pm2, \cdots$.

Resolving those SAE, we achieve one set of solutions as follows.

Set 1.

$$\begin{aligned} k &= \pm \frac{\sqrt{-(1/36\lambda\mu)}\Phi}{\Psi}, \\ \Lambda &= \mp \frac{2}{9} \frac{\sqrt{-(1/36\lambda\mu)}\Phi^3}{\Psi^2}, \\ A_0 &= \mp \Phi \sqrt{-\frac{1}{6\Psi}}, \\ A_1 &= \pm \frac{1}{6} \frac{\Phi\sqrt{-(1/6\Psi)}}{\sqrt{-(1/36\lambda\mu)}\mu}. \end{aligned} \quad (17)$$

Case 1: when $\lambda\mu < 0$, we get following solutions of hyperbolic type.

Segment 1

$$\begin{aligned} u_{(1,2)}(x,t) &= \frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \pm \frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \frac{1}{\sqrt{-(1/\lambda\mu)}\mu\sqrt{(-\lambda/\mu)} \tanh\left(\sqrt{-\lambda\mu}(\xi + C)\right)}, \\ u_{(3,4)}(x,t) &= \frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \pm \frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \frac{1}{\sqrt{-(1/\lambda\mu)}\mu\sqrt{(-\lambda/\mu)} \coth\left(\sqrt{-\lambda\mu}(\xi + C)\right)}. \end{aligned} \quad (18)$$

Segment 2

$$\begin{aligned} u_{(5,6)}(x,t) &= -\frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \mp \frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \frac{1}{\sqrt{-(1/\lambda\mu)}\mu\sqrt{(-\lambda/\mu)} \tanh\left(\sqrt{-\lambda\mu}(\xi + C)\right)}, \\ u_{(7,8)}(x,t) &= -\frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \mp \frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \frac{1}{\sqrt{-(1/\lambda\mu)}\mu\sqrt{(-\lambda/\mu)} \coth\left(\sqrt{-\lambda\mu}(\xi + C)\right)}, \end{aligned} \quad (19)$$

where $k = \pm(\sqrt{-(1/36\lambda\mu)}\Phi)/\Psi$, $\Lambda = \mp 2/9(\sqrt{-(1/36\lambda\mu)}\Phi^3/\Psi^2)$, and $\xi = \pm((\sqrt{-(1/36\lambda\mu)}\Phi)/\Psi)\, x \pm ((2/9)(\sqrt{-(1/36\lambda\mu)}\Phi^3/\Psi^2)t^\delta/\delta)$.

Case 2: when $\lambda\mu > 0$, we get the following solutions of trigonometric type.

Segment 3

$$\begin{aligned} u_{(9,10)}(x,t) &= \frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \pm \frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \frac{1}{\sqrt{-(1/\lambda\mu)}\mu\sqrt{(\lambda/\mu)} \tan\left(\sqrt{\lambda\mu}(\xi + C)\right)}, \\ u_{(11,12)}(x,t) &= \frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \mp \frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \frac{1}{\sqrt{-(1/\lambda\mu)}\mu\sqrt{(\lambda/\mu)} \cot\left(\sqrt{\lambda\mu}(\xi + C)\right)}. \end{aligned} \quad (20)$$

Segment 4

$$\begin{aligned} u_{(13,14)}(x,t) &= -\frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \mp \frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \frac{1}{\sqrt{-(1/\lambda\mu)}\mu\sqrt{(\lambda/\mu)} \tan\left(\sqrt{\lambda\mu}(\xi + C)\right)}, \\ u_{(15,16)}(x,t) &= -\frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \pm \frac{\Phi}{6}\sqrt{-\frac{6}{\Psi}} \frac{1}{\sqrt{-(1/\lambda\mu)}\mu\sqrt{(\lambda/\mu)} \cot\left(\sqrt{-\lambda\mu}(\xi + C)\right)}, \end{aligned} \quad (21)$$

where $k = \pm\sqrt{-(1/36\lambda\mu)}\Phi/\Psi$, $\Lambda = \mp(2/9)((\sqrt{-(1/36\lambda\mu)}\Phi^3)/\Psi^2)$, and $\xi = \pm((\sqrt{-(1/36\lambda\mu)}\Phi)/\Psi)\, x \pm ((2/9)(\sqrt{-(1/36\lambda\mu)}\Phi^3/\Psi^2)t^\delta/\delta)$.

Case 3 and Case 4: when $\lambda = 0$, the implementing value of A_1 is unspecified. So the solution cannot be obtained. For this reason, this case can be excluded.

Likewise, when $\mu = 0$, the executing value of A_1 is undefined. So the solution cannot be obtained. So this case can also be excluded.

5. Results and Discussions

In this segment, we have discussed the theoretical discussion and its dynamical variations on obtained wave solutions. For our convenience, we have only showed the significant wave solutions. Figure 1 represents the soliton solutions of $u_7(x,t)$ for the parameters $\Phi = 2.5, \Psi = 1, \lambda = 3, C = 1, \mu = -1$ within $-10 \le x \le 10$ and $-10 \le t \le 10$. It is noteworthy that we have chosen the fractional order derivative $\delta = 0.5$, 0.75, and 1 respectively. In every set of figures, a, b, c illustrate the 3D chart; a_1, b_1, c_1 illustrate the 2D chart; and the right side vertical figure illustrates the density plot for the value of $\delta = 0.5$, 0.75, and 1 respectively. Figure 2 illustrates the antibright kink solutions of $u_7(x,t)$ for $\Phi = -2.5, \Psi = 1, \lambda = 3, C = 1, \mu = -1$ within $-10 \le x \le 10$ and $-10 \le t \le 10$. Figure 3 illustrates the bright kink shape of $u_{13}(x,t)$ for $\Phi = 2.5, \Psi = 2, \lambda = 4, C = 1, \mu = 2$ within $-10 \le x \le 10$ and $-10 \le t \le 10$. Figure 4 illustrates the solitonic shape solution of $u_{13}(x,t)$ for $\Phi = -2.5, \Psi = 2, \lambda = 4, C = 1, \mu = 2$ within $-10 \le x \le 10$ and $-10 \le t \le 10$. Figure 5 shows the kink solution shape of $u_5(x,t)$ for $\Phi = 2.9, \Psi = 0.9, \lambda = 3, C = 1, \mu = -1$ within $-10 \le x \le 10$ and $-10 \le t \le 10$. Figure 6 shows the rogue wave solution shape of u_{14} for $\Phi = 2.5, \Psi = 2, C = 1, \lambda = 4, \mu = 2$ within $-10 \le x \le 10$ and $-10 \le t \le 10$.

5.1. Graphical Explanation. This section signifies the graphical depiction of the time fractional MKE. By utilizing mathematical software tool MATLAB density plot, 3D and 2D plots of some attained wave solutions have been exposed in Figures 1–6 to envision the important tool of the main equations. In the concept of mathematical physics, a soliton or solitary wave is defined as a self-reinforcing wave packet that upholds its shape while it propagates at a constant amplitude and velocity. Solitons are the solutions of a widespread class of weakly nonlinear dispersive partial differential equations describing physical systems.

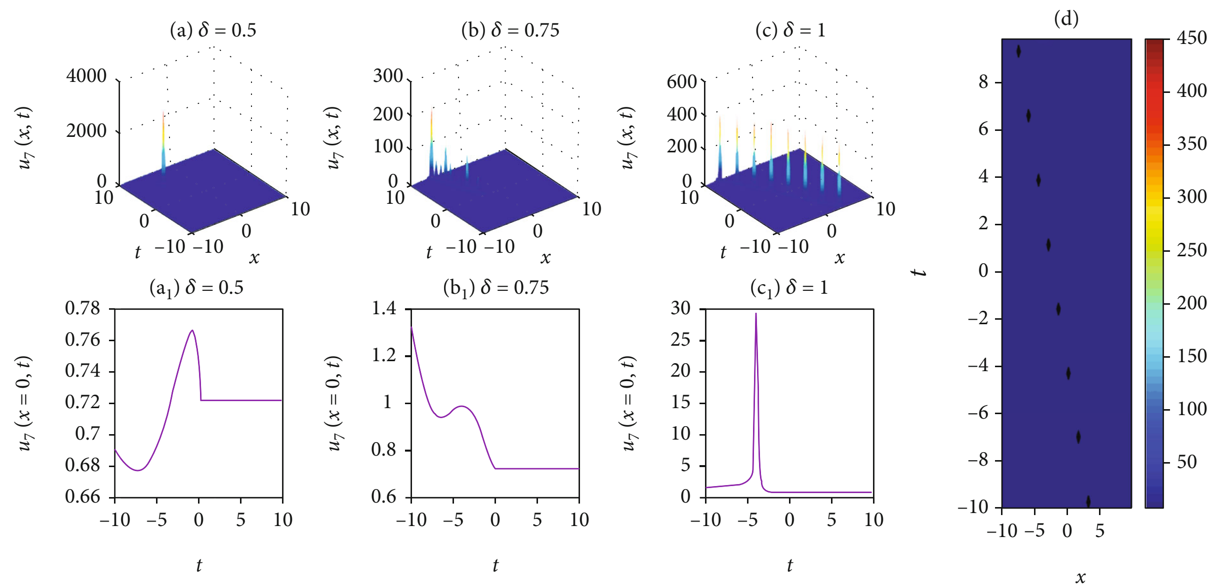

FIGURE 1: Graph of soliton solutions of $u_7(x, t)$. (a–c) 3D plots. (a_1–c_1) 2D plots. The right side vertical figure shows the density plot for the value of $\delta = 0.5$, 0.75, and 1, respectively.

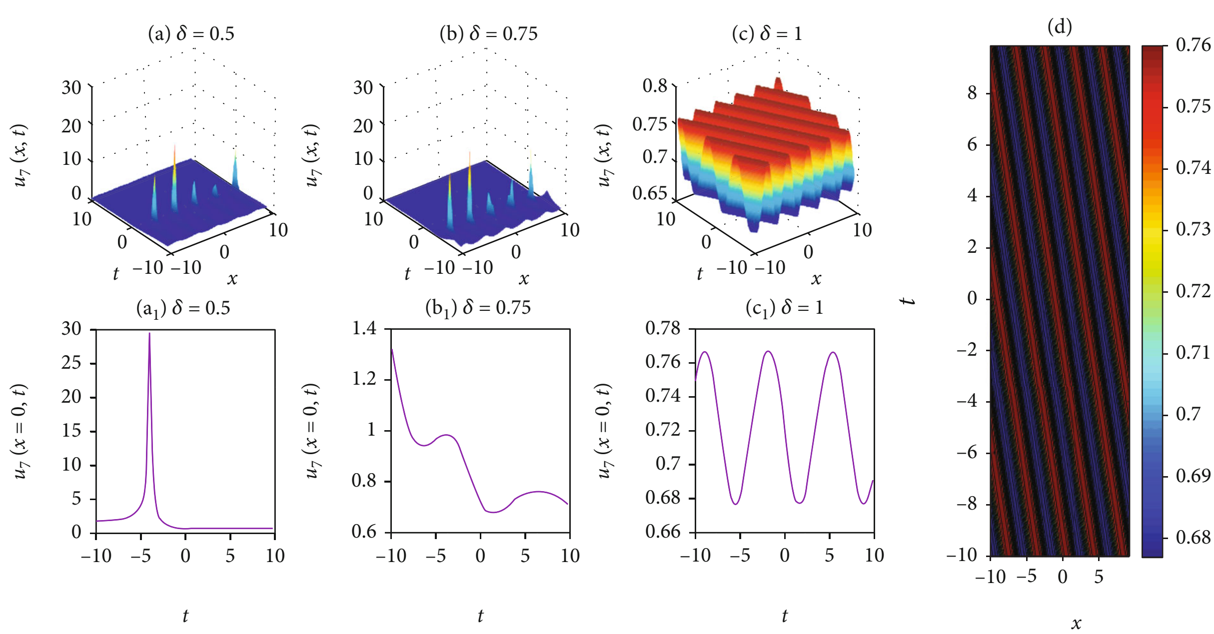

FIGURE 2: Graph of antibright kink solutions of $u_7(x, t)$. (a–c) 3D plots. (a_1–c_1) 2D plots. The right side vertical figure shows the density plot for the value of $\delta = 0.5$, 0.75, and 1, respectively.

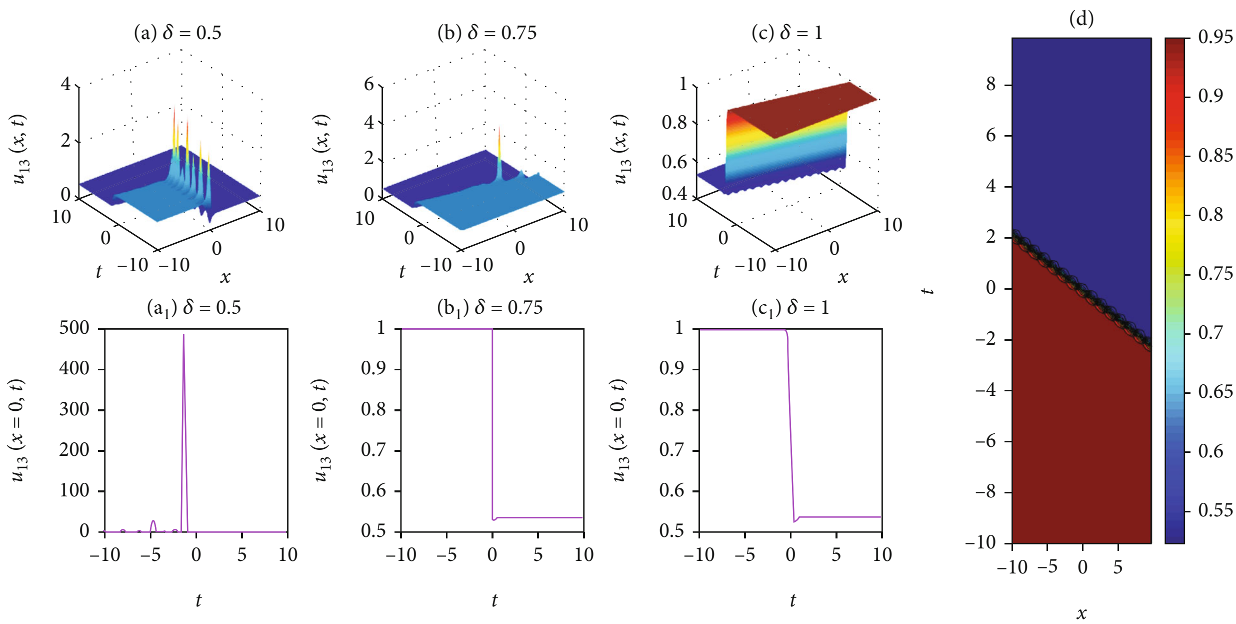

FIGURE 3: Graph of bright kink shape of $u_{13}(x, t)$. (a–c) 3D plots. (a_1–c_1) 2D plots. The right side vertical figure shows the density plot for the value of $\delta = 0.5$, 0.75, and 1, respectively.

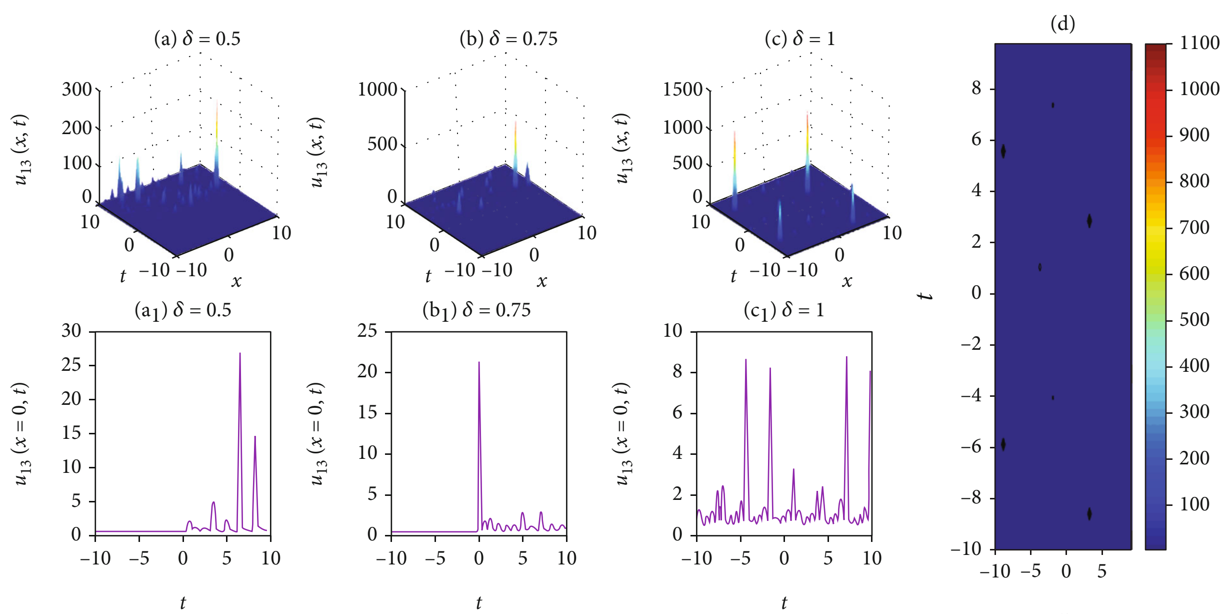

FIGURE 4: Graph of solitonic shape solution of $u_{13}(x, t)$. (a-c) 3D plots. (a_1–c_1) 2D plots. The right side vertical figure shows the density plot for the value of $\delta = 0.5$, 0.75, and 1, respectively.

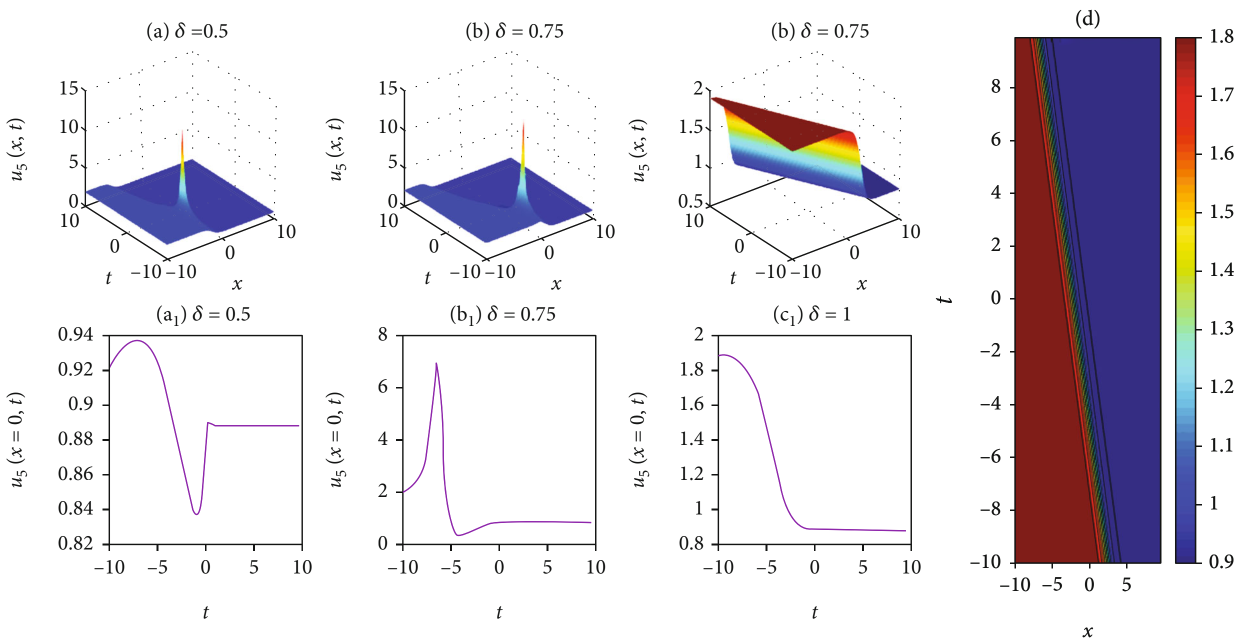

FIGURE 5: Graph of kink solution shape of $u_5(x, t)$. (a–c) 3D plots. (a_1–c_1) 2D plots. The right side vertical figure shows the density plot for the value of $\delta = 0.5$, 0.75, and 1, respectively.

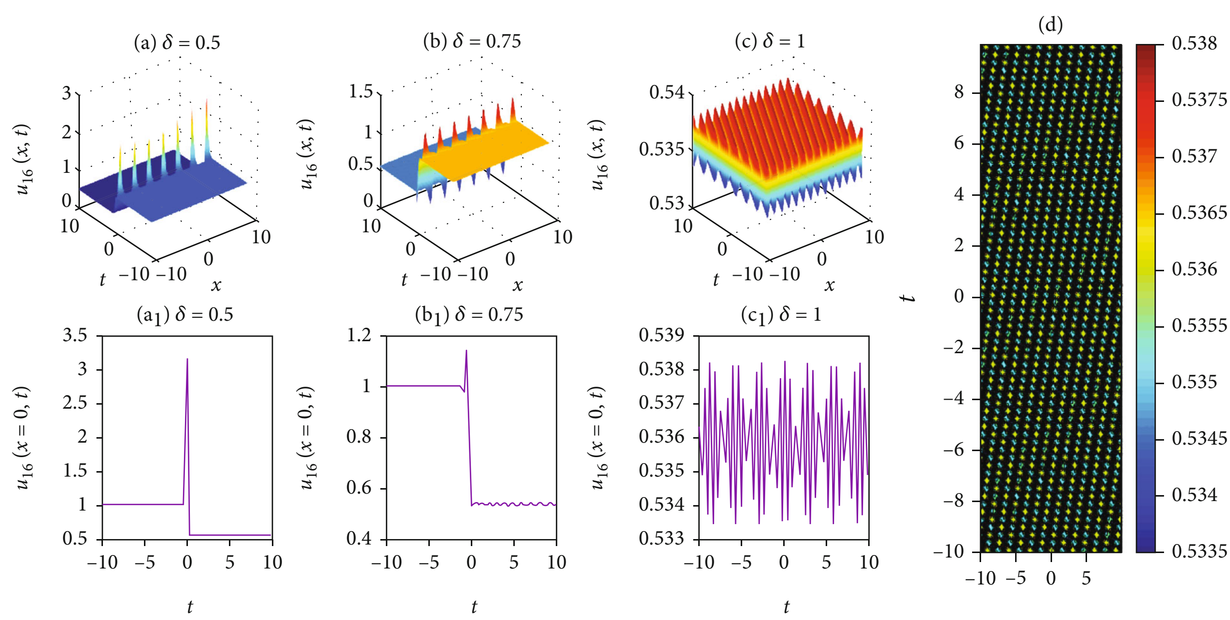

FIGURE 6: Graph of rogue wave solution shape of $u_{14}(x, t)$. (a–c) 3D plots. (a_1–c_1) 2D plots. The right side vertical figure shows the density plot for the value of $\delta = 0.5$, 0.75, and 1, respectively.

6. Conclusions

In this section, we have experiential learning that double wandering wave preparations as far as trigonometric, hyperbolic, and measurements for the time fractional mKE are efficiently imposed by applying the advanced $\exp(-\phi(\xi))$-expansion technique. From our obtained outcomes from this article, the advanced $\exp(-\phi(\xi))$-expansion technique approach is straight, incredible, helpful, and powerful. The demonstration of this system is trustworthy and simple and provides frequent new measures. As an outcome, the advanced extension method illustrates an important technique to determine novel voyaging wave arrangements. The obtained arrangements in this paper uncover that the technique is a powerful and effective material of defining more definite voyaging wave arrangements than other strategies for the nonlinear advancement conditions emerging in numerical physical science.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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