



Analysis of Optimal Control Strategies for Preventing Computer Virus Infection and Reduce Program Files Damage with Other Symptoms

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JSRR/2021/v27i730408

Editor(s):

(1) Shunpu Zhang, University of Nebraska – Lincoln, USA.

Reviewers:

(1) Rajeshwari Hegde, India.

(2) Annapoorani Venkatesan, Anna University, India.

Complete Peer review History: <https://www.sdiarticle4.com/review-history/71819>

Original Research Article

Received 20 May 2021
Accepted 26 July 2021
Published 03 August 2021

ABSTRACT

Program files damage and other computer virus symptoms has become a very threatening issue to computer performance. This paper considered an $SEID_pR$ model with incidence of infected and program files damaged computers and saturated incidence of vaccination and treatment function. Two control functions have been used; one for vaccinating the susceptible computer population and the other for the treatment of the program files damaged computer population. The Pontryagin's Maximum Principle has been used to characterize the optimal control whose numerical results show the positive impact of the two controls used for controlling the infection dynamics of computer virus. Actually the intention of this study is to minimize the number of infected and program files damaged computer systems and at the same time minimize the cost associated to the controls. Efficiency analysis is also studied to determine the best control strategy among vaccination and treatment. Numerical simulations were carried out in this model to demonstrate the analytical results and it was revealed that combination of vaccination and treatment is the most successful way to minimize the incidence of program files damage.

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Keywords: Computer virus; program files damage; vaccination and treatment; optimal control; pontryagin's maximum principle; efficiency analysis.

1. INTRODUCTION

With the constant upgrading and rapid development of the information technology, the efficiency and speed of network information transmission has been improved a lot to a certain degree. Information stability and security following has been paid attention to by the network experts and people gradually [1]. However, as is known to all, the highly developed internet brings people all sorts of convenient life while at the same time with many problems following which one of them is the computer virus infection and its spread.

A computer virus is a man-made destructive computer program or code. One of the fundamental characteristics of a virus is that it replicates its code to other programs or computers. This process of replication is called infection. A typical virus generally (although not in all the cases) adds itself to the end of a program and in that case the size of the host program increases because of the addition of this extra (viral) code. In order to execute this viral code, the virus overwrites the first bytes of the file with a "jump" instruction which makes the execution jump to the viral code. After the viral code is executed, the virus repairs the first few bytes overwritten by the virus in order to return control to the original file. Thus a typical virus (not in case of a worm or Trojan) has to depend on a host program to survive and operate [2].

During the process of infection the contact of the host file (or the file that is infected) has to change in order to include the virus code. As there are different types of the viruses, their methods of infection are also different. Some viruses infect the boot sector and partition table (boot sector viruses). Some viruses remain in memory all the time (memory resident viruses) since the computer is switched on. Some viruses remain in the body of executable files (file viruses), or document files (macro viruses) or other types of files. Some viruses infect email (email viruses) and travel through computer networks. Some viruses are complete programs and work independently without depending on other files (worms, Trojans). Some viruses take over actions of operating systems (rootkits) thereby causing the system to malfunction [2].

Every infection does not cause same level of damage. Some infections cause very minor

disturbance while other infections cause moderate to high level of damage. The dangerous viruses may cause serious damage like formatting the hard disk or destroying the data etc thereby making the computer unusable. Almost all viruses are attached to an executable file, which means the virus may exist on your computer but it cannot attack your computer unless if run or open the malicious program. Therefore, it is important to note that a virus cannot be spread without a human action (such as running an infected program) to keep it going [3].

A biological virus like HIV or HBV cannot reproduce on its own, rather there must be a favorable viral – host interaction before it can replicate [4,5,6]. Similarly, malware is not a standalone program rather a code snippet that insert itself into other applications. When that application executes the malware code unknowingly, the results range from irritation to disastrous. As the infected application executes (usually at the request of the user), and the malware is loaded into the CPU memory before any of the legitimate executes. Computer Malware is not keen to alert their presence in a computer system. Just as biological virus wants to keep its host alive so that it will continue to use it as a vehicle to reproduce and spread, so too does computer malware attempt to do its damage in the background while the computer still limps along [7].

Under appropriate conditions, computer virus spreads to uninfected computers from the infected computer through many kinds of ways. It enters the computer and gets executed, thereby searching for other programs or storage media in line with the conditions of their infection and target to insert the code. This is to enable it to achieve the purpose of replication. Once the computer is infected and not promptly treated, the virus will spread speedily on the computer and maybe a large number of executable files will be infected. These infected files become a new source of infection and when data are exchanged with other computers that are not vaccinated over the network, they will be infected. Computer malware can enter any computer through different means such as; an email attachment, file downloads from the internet, connection to a website, mobile hard disk. Since the network has no permanent

immunity to the computer malware, they are prone to be infected [8].

To prevent computer viruses from causing program file damage and other computer virus symptoms, like slowing down of operating system, computer applications and internet speed; crashing; hard drive malfunction; running out of storage space; etc, an anti-virus software program is installed and constantly updated. An anti-virus software program is a computer program that can be used to scan files to identify and eliminate computer viruses and other malicious software (malware) [9]. Anti-virus software does up to three tasks which include:

- ❖ Detecting whether or not a code is a virus or not (detection).
- ❖ Once a virus is detected, the identification process distinct from detection by classifying which virus it is.
- ❖ The computer is disinfected. Disinfection is the process of removing detected viruses, and is sometime called cleaning.

There are different performance indicators identified for measuring the performance of antivirus software which are as follows: Virus Definition Update, Antivirus Upgrades, On-Access Scanner, On-Demand Scan, Scheduled Scanning, Auto Clean infected File Scanning, Scanning Compressed Files, Email Shield, Web Shield and Antivirus Technical Support.

Although an antivirus aims at preventing any virus attack there are chances that some files are infected earlier to antivirus installation or during a period when the virus signatures are not updated. In that case, the antivirus must disinfect the infected files. This is one of the difficult and most important functions of any antivirus program. The antivirus has to apply various methods in order to remove the virus code from the infected file and restore the file in its original form. An antivirus first tries to detect the presence of viruses using different detection methods. Signature detection being the most popular method may be applied first. If no signature is found then the antivirus applies other methods like heuristic scanning. If no suspicion is raised on a file, then the file is deemed to be uninfected.

The first attempt of any antivirus product is to repair the damaged files or sector of the disk.

However, if the antivirus does not know the method of repairing the infection, then it isolates the infected file to quarantine for a possible repairing in future. If a virus is found to be too dangerous or the file is severely damaged then the antivirus may decide to delete the infected file. Thus, the actions are generally configured in a sequential order, such as, repairing the file (if it cannot be repaired) and deleting the file (least preferred). Our aim in this paper is to highlight the seriousness of computer virus infection and prevent program files damage with other symptoms using vaccination and treatment.

2. RELATED LITERATURES

Associating computer virus with biological virus we formulate a compartmental epidemic model for the optimal control of viruses in a computer network with vaccination and quarantine being of great significant.

Mishra and Singh [10] formulated an SEIQR (Susceptible, Exposed, Infectious Quarantined, and Recovered) models for the transmission of malicious objects with simple mass action incidence and standard incidence rate in computer network. Threshold, equilibrium and their stability are discussed for the simple mass action incidence and standard incidence rate. They showed the global stability and asymptotic stability of endemic equilibrium for simple mass action incidence.

Zhang et al, [11] proposed and analyzed a computer virus spread model concerning impulsive control strategy. They proved that there exists globally attractive infection – free periodic solution when the vaccination rate is larger than θ_0 . Moreover, they showed that the system is uniformly persistent if vaccination rate is less than θ_1 .

Dong et al, [12] in their paper stated that the development of antiviral software always lags behind the emergence of a new virus and the point-to-group information propagation mode, a new computer virus model with point to group and discontinuous antivirus strategy is presented. From their model, they found that in the case that the equilibrium is asymptotically stable, the convergence to the equilibrium can actually be achieved in finite time, and the time can be estimated in terms of the model parameters, the initial number of the uninfected computer and latent computer and the initial antivirus strength, which means the virus in the

network can be controlled or eliminated in finite time by increasing the antivirus strength.

Kazeem et al, [13] in their study formulated a deterministic computer virus model, incorporating removal devices. They studied the basic properties of the model, calculated the reproduction number and the steady state which was found to be stable. Time optimal control was included and Pontryagin's Maximum Principle was used to characterize all the necessary conditions for controlling the spread of computer virus. They showed that the most effective strategy for controlling computer virus is through the combination of the three control, that is, intensive public education on the use of removable devices, public campaign on computer virus free and treatment of infected computers.

Nwokoye et al, [14] motivated by the epidemic theory proposed the Q-SEIR and Q-SEIRV models to present the dynamics of the pre-quarantining of nodes in wireless sensor networks. They established that the disease free equilibrium is asymptotically stable. Runge Kutta –Fehlberg Method of order 4 and 5 was used to solve and simulate the proposed systems of equation. They showed that the impact of pre-quarantine compartment in the proposed model is very strong on the recovery nodes.

Shahrear [15] proposed a compartmental model SAEIQRS (susceptible – Antidotal – Exposed – Infected –Quarantined – Recovered – Susceptible), of virus transmission in a computer network. They applied the differential transmission method (DTM) to obtain an approved solution of each compartment. An accuracy of order $O(h^6)$ was achieved the result of DTM was validated with fourth order Runge Kutta (RK4) method. The local stability of their model was analyzed based on the basic reproduction number for virus free and endemic equilibrium. Lyapunov Function was used to demonstrate the global stability of virus free equilibria.

On the optimal control of a malware propagation model, Guillen et al [16], stated that the important way to control malware epidemic process is to take into account the security measures that are associated to the systems of ordinary differential equations that govern the dynamic of such systems. Here they considered two types of control measures, that is the analysis of the basic reproduction number and the study of control measure functions. They used the theory of

optimal control that is associated to the systems of ordinary equations in order to find a new function to control malware epidemic through time.

Al – Tuwairqi and Bahashwan [17], build a mathematical model to study the impact of external removable devices on a network with weakly and strongly protected computers. Their model, describes the dynamics between weak, strong, infected computers and susceptible, infected removable media. Analytical investigations of the model produce two equilibrium points: virus free and endemic equilibria. They also investigated the local and global stability conditions of the equilibrium points depending primarily on the basic reproduction (R_0) of the model. Their observation was that user awareness plays an essential role in limiting the spread of viruses.

Chinebu et al [8] considered the problem which computer malware cause to personal computers with its control by proposing a compartmental model SVEIRS (Susceptible Vaccinated-Exposed-infected-Recovered-Susceptible) for malware transmission in computer network using nonlinear ordinary differential equation. Through the analysis of the model, the basic reproduction number R_0 were obtained, and the malware free equilibrium was proved to be locally asymptotical stable if R_0 is less than unity and globally asymptotically stable if R_0 is less than some threshold using a Lyapunov function. Also, the unique endemic equilibrium exists under certain conditions and the model underwent backward bifurcation phenomenon. Their results showed that vaccination and treatment is very essential for malware control.

A large number of mathematical models have been developed to simulate, analyse and understand computer virus in a related work, none have considered vaccination and treatment in preventing program files damage. In this research, we proposed an optimal control model in computer network considering effective vaccination and treatment on the virus infection. The proposed model consists of five compartments of susceptible, exposed, infected, program files damaged and recovered computers.

3. MODEL FORMULATION

Mathematical modeling is very significant in providing quantitative insights into the

mechanism of various diseases which lead to designing better prediction, management and control policies. In analyzing and controlling computer viruses to reduce program files damage and other symptoms progression and complications, we develop a mathematical model and introduce two control variables in it. Here, we construct a model which considers the feature of program files damage and other computer virus infection symptoms due to virus transmission and then apply optimal control theory to minimize the progression or complications of the computer virus.

Let the total population of computers (nodes) at time t be denoted by $N(t)$. The computers that are not affected by virus attack, but are prone to become affected by this virus infection are denoted by $S(t)$. The virus transmission progression is very important in the dynamics of program files damage and other symptoms, since viruses are not standalone program but a code snippet that insert itself into other applications and develop over some time progression. Observe that once computer viruses are not diagnosed initially in a computer system, it infects other programs in the system during its incubation (latent) period and is denoted by $E(t)$. In this case, $E(t)$ is the number of infected computers that are not actually infectious at time t . There are also some computers $I(t)$ that are affected by virus and can transmit the infection at any time. When these infectious computers remain undiagnosed for long time, they become horrible, then, each time a user starts a computer that is already infectious by a virus, they may be

launching a series of programs that can damage program files and exhibit other computer virus symptoms. Some infected computers (nodes) remain in the infected class while they are infectious and then move to the recovered class after the run of antivirus software. Since long time progression of these viruses leads to program files damage and exhibition of other computer virus symptoms, most infected computers that were not recovered, whose program files has been damaged or were exhibiting other computer virus symptoms are quarantined while they are infectious and then moved to the recovered class after their treatment. We now consider these set of computers as $D_p(t)$. The computers that have recovered with temporal immunity are denoted by $R(t)$. We represent the flow chart of the compartmental model of preventing computer virus infection and reducing program files damage with other symptoms in Image 1 below.

Let

$$N(t) = S(t) + E(t) + I(t) + D_p(t) + R(t) \quad (1)$$

Let the force of infection associated with computer virus be denoted by λ and is given by

$$\lambda = \beta(I(t) + \eta Q(t)) \quad (2)$$

where β is the effective contact rate for computer virus attack through nodes and other means, and the parameter η is the modification factor for the program files damage with other symptoms.

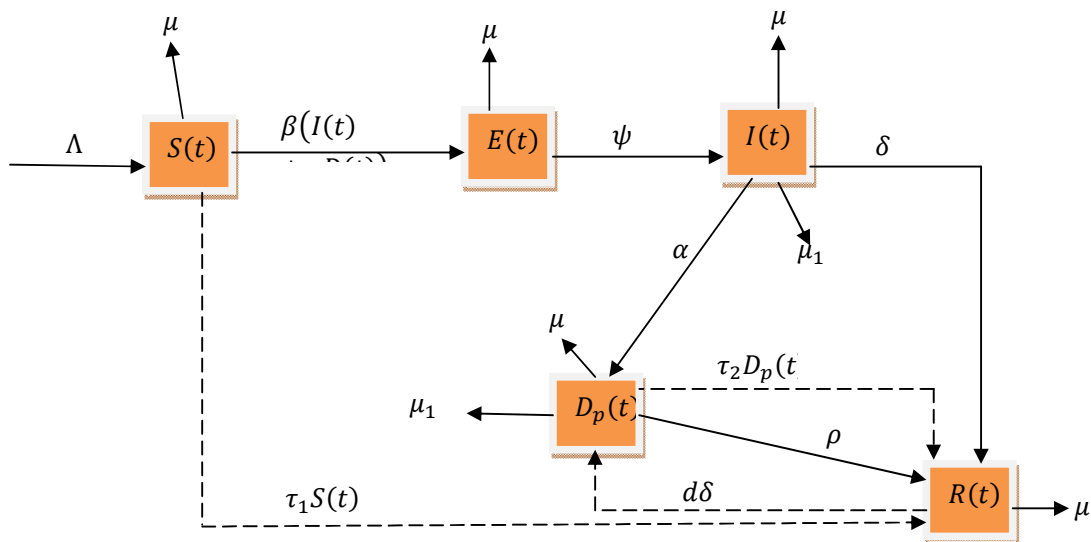


Image 1. The flow diagram of the model

To formulate the optimal control model, we made the following assumptions

- i. All the newly connected computers are virus free and susceptible.
- ii. Both old and newly connected computers are vaccinated (installed with antivirus software).
- iii. Vaccinated computers become exposed to computer virus due to waning and none updating of the antivirus software.
- iv. Recovered computers may move to $D_p(t)$ due to relapse (waning) and lack of antivirus update.

The above formulations, assumptions and flow diagram, leads to the formulation of the following set of non linear differential equations

$$\begin{aligned} \frac{dS(t)}{dt} &= \Lambda - \mu S(t) - \beta (I(t) + \eta D_p(t)) S(t) - \tau_1 S(t) \\ \frac{dE(t)}{dt} &= \beta (I(t) + \eta D_p(t)) S(t) - (\mu + \psi) E(t) \\ \frac{dI(t)}{dt} &= \psi E(t) - (\mu + \mu_1) I(t) - (\alpha + \delta) I(t) \quad (3) \\ \frac{dD_p(t)}{dt} &= \alpha I(t) + d\delta I(t) - (\mu + \mu_1 + \rho) D_p(t) - \tau_2 D_p(t) \\ \frac{dR(t)}{dt} &= \rho D(t) + \delta I(t) + \tau_1 S(t) + \tau_2 D_p(t) - \mu R(t) - d\delta I(t) \end{aligned}$$

Where,

$$\mathbb{T} = \{(\tau_1(t), \tau_2(t)): 0 \leq u_i \leq \tau_i(t) \leq v_i \leq 1, \quad i = 1,2\} \quad \forall \quad t \in [0, t_f]$$

Considering these two control variables, the performance index is given by

$$\text{Min } J(\tau_1, \tau_2) = \int_0^{t_f} \left(I(t) + D_p(t)(t) + \frac{A_1}{2} \tau_1^2 + \frac{A_2}{2} \tau_2^2 \right) dt \quad (4)$$

We can reformulate model system (3) as an optimal control problem with the performance index (4) as

$$(Q_c) \begin{cases} \text{minimize } J(y, \tau) = \int_0^{t_f} D(t, y(t), \tau(t)) dt \\ \text{subject to} \\ \dot{y}(t) = f(y(t)) + g(y(t)\tau(t)), \forall t \in [0, t_f] \\ \tau(t) \in \mathbb{T}(t), \forall t \in [0, t_f] \\ y(0) = y_0 \end{cases} \quad (5)$$

With the initial conditions $S(0) > 0, E(0) \geq 0, I(0) \geq 0, D_p(0) \geq 0, R(0) \geq 0$.

From model system (3), we have assumed that all newly connected computers at the rate Λ are virus free and susceptible. Death rate other than virus attack is constant and denoted by μ , while μ_1 is the virus attack induced death rate. β is the transmission rate and η is the infectiousness of the infectious computers relative to program files damage and other symptoms. Exposed computers become infectious at a rate ψ . Infections computers are cured (treated) at a rate δ or can progress to program files damage and other symptoms at a rate α . ρ is the recovery rate of computers who's program files has been damage or who exhibits' other computer virus symptoms. Portion of the recovered program files may be damaged or may show other symptoms of computer virus infection again at a rate $d\delta$.

Here, we considered two control variables (τ_1, τ_2) : (i) before infection antivirus is used to prevent the infection, that is vaccination; after infection, (ii) treatment is based on repairing the infected files, restoring original files from a backup or putting into quarantine. So that $\tau_1(t)$ and $\tau_2(t)$ denotes the vaccination and the treatment control respectively.

Now model system (3) as an optimal control model and the set of control variable $(\tau_1(t), \tau_2(t)) \in \Gamma$ is a Lebesgue measurable,

Where,

$$y(t) = \begin{pmatrix} S(t) \\ E(t) \\ I(t) \\ D_p(t) \\ R(t) \end{pmatrix}, g(y) = \begin{pmatrix} -S & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -D_p \\ S & D_p \end{pmatrix}, f(y) = \begin{pmatrix} \Lambda - \beta(I + \eta D)S - \mu S \\ \beta(I + \eta D)S - (\mu + \psi)E \\ \psi E - (\mu + \mu_1)I - (\alpha + \delta)I \\ \alpha I + d\delta I - (\mu + \mu_1 + \rho)D_p \\ \rho D - \mu R + (1 - d)\delta I \end{pmatrix}, \tau(t) = \begin{pmatrix} \tau_1(t) \\ \tau_2(t) \end{pmatrix}$$

are the integrand of the performance index and is denoted by

$$L(y, \tau) = I(t) + D_p(t) + \frac{A_1}{2} \tau_1^2 + \frac{A_2}{2} \tau_2^2 \tag{6}$$

3.1 Existence of the Optimal Control

To prove the existence of the optimal control, we have to show the existence of the state variables and the existence of the objective functional. A state variable is one of the variables used to describe the state of the dynamic system. The objective functional is a mathematical equation that describes the control output target that corresponds to the minimization of the progression or complication of the computer virus infection with respect to control.

3.2 Existence of the State Variables

Theorem 1: The biologically feasible region $\Omega = \left\{ (S(t) + E(t) + I(t) + D_p(t) + R(t)) \in \mathbb{R}_+^5 : N \leq \frac{\Lambda}{\mu} \right\}$ is positively invariant and attracts all the solutions in \mathbb{R}_+^5 .

Proof: The state equation (3) with the initial condition can be written in the following form as

$$\begin{aligned} \frac{dS(t)}{dt} &= \Lambda - (\beta(I(t) + \eta D_p(t)) - \mu)S(t) + (0)E(t) + (0)I(t) + (0)D_p(t) + (0)R(t) \\ \frac{dE(t)}{dt} &= \beta(I(t) + \eta D_p(t))S(t) - (\mu + \psi)E(t) + (0)I(t) + (0)D_p(t) + (0)R(t) \\ \frac{dI(t)}{dt} &= \psi E(t) - (\mu + \mu_1 + \delta + \alpha)I(t) + (0)S(t) + (0)D_p(t) + (0)R(t) \tag{7} \\ \frac{dD_p(t)}{dt} &= \alpha I(t) - (\mu + \mu_1 + \rho)D_p(t) + d\delta I(t) + (0)S(t) + (0)E(t) + (0)R(t) \\ \frac{dR(t)}{dt} &= \rho D_p(t) - \mu R(t) + \delta(1 - d)I(t) + (0)S(t) + (0)E(t) \end{aligned}$$

Since system (1) is written as $N(t) = S(t) + E(t) + I(t) + D_p(t) + R(t)$, then system (7) can be represented thus

$$\frac{dN(t)}{dt} = \frac{dS(t)}{dt} + \frac{dE(t)}{dt} + \frac{dI(t)}{dt} + \frac{dD_p(t)}{dt} + \frac{dR(t)}{dt} \tag{8}$$

Therefore, if we substitute the right hand side of system (7) into system (8), we have

$$\begin{aligned} \frac{dN(t)}{dt} &= \Lambda - (\beta(I(t) + \eta D_p(t)) - \mu)S(t) + \beta(I(t) + \eta D_p(t))S(t) - (\mu + \psi)E(t) + \psi E(t) \\ &\quad - (\mu + \mu_1 + \delta + \alpha)I(t) + \alpha I(t) - (\mu + \mu_1 + \rho)D_p(t) + d\delta I(t) + \rho D_p(t) - \mu R(t) \\ &\quad + \delta(1 - d)I(t) \end{aligned}$$

$$N'(t) = \Lambda - \mu (S(t) + E(t) + I(t) + D_p(t) + R(t)) - \mu_1 (I(t) + D(t))$$

So that we have

$$N'(t) \leq \Lambda - \mu N(t)$$

$$N(t) \leq N(0)e^{-\mu t} + \frac{\Lambda}{\mu}(1 - e^{-\mu t}) = P_0 \in \mathbb{R}_+^5$$

Since at disease free equilibrium, $\mu_1 = 0$ implies $\mu_1(I(t) + D_p(t)) = 0$. we now have that

$$N(t) \leq \frac{\Lambda}{\mu} + Ce^{-\mu t}$$

And

$$\lim_{t \rightarrow \infty} \text{Sup } N(t) \leq P_0$$

which conclude

Using a standard comparison theory by Lakshmikanthan et al,[18], Zhang [19], it can be

$$S(t), E(t), I(t), D_p(t), R(t) \leq P_0 \text{ as } t \rightarrow \infty.$$

Shown that

Then, we can rewrite system (7) in the following

$$\phi_t = V\phi + F(\phi) \tag{9}$$

Where

$$\phi = \begin{bmatrix} S(t) \\ E(t) \\ I(t) \\ D_p(t) \\ R(t) \end{bmatrix}, \phi_t = \begin{bmatrix} \frac{dS(t)}{dt} \\ \frac{dE(t)}{dt} \\ \frac{dI(t)}{dt} \\ \frac{dD_p(t)}{dt} \\ \frac{dR(t)}{dt} \end{bmatrix}, F(\phi) = \begin{bmatrix} -\beta(I(t) + \eta D_p(t))S(t) \\ \beta(I(t) + \eta D_p(t))S(t) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

And

$$V := \begin{bmatrix} -\mu & 0 & 0 & 0 & 0 \\ 0 & -(\mu + \psi) & 0 & 0 & 0 \\ 0 & \psi & -(\mu + \mu_1 + \delta + \alpha) & 0 & 0 \\ 0 & 0 & \alpha & -(\mu + \mu_1 + \rho) & 0 \\ 0 & 0 & \delta(1 + d) & \rho & -\mu \end{bmatrix}$$

$$F(\phi_1) - F(\phi_2) = \begin{bmatrix} -\beta(I_1 + \eta D_{p1})S_1 \\ \beta(I_1 + \eta D_{p1})S_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -\beta(I_2 + \eta D_{p2})S_2 \\ \beta(I_2 + \eta D_{p2})S_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

Equation (9) is non linear system with a bounded coefficient. We have

$$\mathcal{H}(\phi) = \phi_t = V\phi + F(\phi)$$

For the existence of optimal control and optimality system, the boundedness of solution of the system for finite time is needed and we assume for $\tau \in T$ there exists a bounded solution.

$$\begin{aligned} F(\phi_1) - F(\phi_2) &= | -\beta(I_1 + \eta D_{p1})S_1 + \beta(I_2 + \eta D_{p2})S_2 | + | \beta(I_1 + \eta D_{p1})S_1 - \beta(I_2 + \eta D_{p2})S_2 | \\ &\leq 2\beta(|S_1| |I_1 - I_2| + |S_1 - S_2| |I_2 + \eta D_{p1}| + |\eta S_2| |D_{p1} - D_{p2}|) \\ &\leq 2\beta(|I_1 S_1 + \eta D_{p1} S_1| - |I_2 S_2 + \eta D_{p2} S_2|) \\ &\leq B | \phi_1 + \phi_2 | \end{aligned}$$

Where $B = 2gP_0$.

Also, we get

$$\begin{aligned} |\mathcal{H}(\phi_1) + \mathcal{H}(\phi_2)| &\leq ||V|| |\phi_1 + \phi_2| + B|\phi_1 + \phi_2| \leq Q|\phi_1 + \phi_2| \\ Q &= \text{Max}(B, ||V||) \end{aligned}$$

Thus, it follows that the function \mathcal{H} is uniformly Lipschitz continuous. From the definition of the control $T(t)$ and the resection on $S(t), E(t), I(t), D_p(t)$ and $R(t) \geq 0$, we see that a solution of the system (9) exists.

3.3 Existence of the Objective Functional

To prove the existence of the objective functional, we use the following theorem Fleming and Rishel [20].

Theorem 2: Let

$$\hat{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_n(t) \end{bmatrix}$$

be a system of n state variables, and let $\tau(t)$ be a control variable with a set of admissible control T , that satisfy the following differential equation $y_i'(t) = g(t, y_i(t), \tau(t))$ for $i = 1, 2, \dots, n$. With associated objective functional

$$J(\tau) = \int f(t, \hat{y}(t), \tau(t)) dt$$

There exists an optimal control which minimizes $J(\tau)$ if the following conditions are satisfied:

1. F is non empty.
2. The control set T must be closed and convex.
3. The right hand side of the state system is continuous, is bounded above by a linear combination of the control and state and can be written as a linear function of τ with coefficients defined by the time and the state.
4. The integrand of the objective functional is convex on T and is bounded below by $-L_2 + L_1(\tau)^\gamma$ with $L_1 > 0$ and $\gamma > 1$.

We define F as a class of $(S_0, E_0, I_0, D_0, R_0, \tau)$ such that τ is a piecewise function on $[0, t_f]$ with values in T . In order to prove that F is nonempty, we use a simplified version of an existence result in Boyce and Di Prima [21], which is stated below.

Theorem 3: Let each of the functions $F_1, F_2, F_3, \dots, F_n$ and the partial derivatives $\frac{dF_1}{dy_1}, \dots, \frac{dF_1}{dy_n}, \dots, \frac{dF_n}{dy_1}, \dots, \frac{dF_n}{dy_n}$ be continuous in a region \mathbb{R} of $t, y_1, y_2, y_3, \dots, y_n$ space defined by $\alpha < t < \beta, \alpha_1 < y_1 < \beta_1, \alpha_2 < y_2 < \beta_2, \alpha_3 < y_3 < \beta_3, \dots, \alpha_n < y_n < \beta_n$ and let the point $(t, y_1^0, y_2^0, y_3^0, \dots, y_n^0)$ be in \mathbb{R} . Then there is an interval $[t = t_0] < h$ in which there exist a unique solution $y_1 = \phi_1(t), y_2 = \phi_2(t), y_3 = \phi_3(t) \dots, y_n = \phi_n(t)$ of the system of differential equations.

$$\begin{cases} y_1' = F_1(t, y_1, y_2, y_3, \dots, y_n) \\ y_2' = F_2(t, y_1, y_2, y_3, \dots, y_n) \\ y_3' = F_3(t, y_1, y_2, y_3, \dots, y_n) \\ \vdots \\ y_n' = F_n(t, y_1, y_2, y_3, \dots, y_n) \end{cases} \quad (11)$$

That also satisfies the initial conditions

$$\begin{aligned} y_1(t_0) = y_1^0, y_2(t_0) = y_2^0, y_3(t_0) = y_3^0, \dots, y_n(t_0) \\ = y_n^0 \end{aligned} \quad (12)$$

Theorem 4: Let $y_i = F_i(t, y_1, y_2, y_3, \dots, y_n)$ for $i \in [1, n]$ be a system of n differential equations with initial conditions $y_i(t_0) = y_i^0$ for $i \in [1, n]$. if each of the functions $F_1, F_2, F_3, \dots, F_n$ and the partial derivatives $\frac{dF_1}{dy_1}, \dots, \frac{dF_1}{dy_n}, \dots, \frac{dF_n}{dy_1}, \dots, \frac{dF_n}{dy_n}$ are continuous in \mathbb{R}^{n+1} space, then there exists a unique solution $y_1 = \eta_1(t), y_2 = \eta_2(t), y_3 = \eta_3(t), \dots, y_n = \eta_n(t)$, that satisfies the initial conditions.

With the help of the above two theorems (theorem 3 and theorem 4) we try to prove the existence of the objective functional. We show that there exists an optimal control τ^* that minimizes $J(\tau)$ over the control set T .

Proof of condition 1: we consider

$$\begin{aligned} \frac{dS(t)}{dt} &= F_1(t, S, E, I, D_p, R) \\ \frac{dE(t)}{dt} &= F_2(t, S, E, I, D_p, R) \\ \frac{dI(t)}{dt} &= F_3(t, S, E, I, D_p, R) \\ \frac{dD_p(t)}{dt} &= F_4(t, S, E, I, D_p, R) \\ \frac{dR(t)}{dt} &= F_5(t, S, E, I, D_p, R) \end{aligned} \quad (13)$$

Where F_1, F_2, F_3, F_4 and F_5 build up the right hand side of the equation (3). Let $\tau(t) = L$ for some constants L . F_1, F_2, F_3, F_4 and F_5 must be linear and they are also continuous everywhere. Moreover the partial derivatives of F_1, F_2, F_3, F_4 and F_5 with respect to all states are constants and they are also continuous everywhere, so by the above theorem 4, there exists a unique solution $S = \eta_1(t), E = \eta_2(t), I = \eta_3(t), D_p = \eta_4(t)$ and $R = \eta_5(t)$ which satisfies the initial conditions. Therefore, the set of controls and corresponding state variables is non empty. Hence the condition 1 is satisfied.

Proof of condition 2: By definition, T is closed. We take two controls $(\tau_1, \tau_2) \in T$ and $\varphi \in [0, 1]$ such that $0 \leq \varphi\tau_1 + (1 - \varphi)\tau_2$. We also observe that $\varphi\tau_1 \leq \varphi$ and $(1 - \varphi)\tau_2 \leq (1 - \varphi)$. Then,

$$\varphi\tau_1 + (1 - \varphi)\tau_2 \leq \varphi + (1 - \varphi) = 1$$

Hence,

$$0 \leq \varphi\tau_1 + (1 - \varphi)\tau_2 \leq 1 \text{ for all } (\tau_1, \tau_2) \in T \text{ and } \varphi \in [0, 1].$$

So, T is convex and therefore, condition 2 is satisfied.

$$|\hat{F}(t, \hat{Y}, \tau)| \leq |\hat{m}| |\hat{Y}| + |\hat{S}| |\tau_1(t)| + |\hat{D}_p| |\tau_2(t)| \leq Q(|\hat{Y}| + |T(t)|)$$

Since \hat{S} and \hat{D}_p are bounded and Q includes the upper bound of the constant matrix. Hence, we say that the right hand side is bounded by a sum of the state and the control. Therefore, condition 3 is satisfied.

Proof of condition 4: Let us consider that the integrand of the objective functional be

$$f(\tau = I(t) + D_p(t) + \tau^2)$$

Proof of condition 3: if we consider,

$$\begin{aligned} F_1 &\leq \Lambda - \tau_1 S \\ F_2 &\leq C_1 E \\ F_3 &\leq \psi E - C_2 I \\ F_4 &\leq \alpha I - K_1 D_p - \tau_2 D_p \\ F_5 &\leq \rho D_p + K_2 I + \tau_1 S + \tau_2 D_p \end{aligned}$$

Then system (13) can be written as

$$\hat{F}(t, \hat{Y}, \tau) \leq \hat{m} \left(t, \begin{bmatrix} S \\ E \\ I \\ D_p \\ R \end{bmatrix} \right) \hat{Y}(t) + \hat{n} \left(t, \begin{bmatrix} S \\ E \\ I \\ D_p \\ R \end{bmatrix} \right) \tau(t)$$

Where

$$\hat{m} \left(t, \begin{bmatrix} S \\ E \\ I \\ D_p \\ R \end{bmatrix} \right) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 & 0 \\ 0 & \psi & -C_2 & 0 & 0 \\ 0 & 0 & \alpha & -K_1 & 0 \\ 0 & 0 & K_2 & \rho & 0 \end{pmatrix} \quad (14)$$

And

$$\hat{n} \left(t, \begin{bmatrix} S \\ E \\ I \\ D_p \\ R \end{bmatrix} \right) = \begin{pmatrix} -S & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -D_p \\ S & D_p \end{pmatrix} \quad (15)$$

Which gives the linear function of the control τ defined by time and state variables. Then we can find out the bound of the right hand side. It is noted that all parameters are constant and greater than or equal to zero. Therefore we can write,

Where

$$\frac{A_1}{2}\tau_1^2 + \frac{A_2}{2}\tau_2^2 = \tau^2$$

We take two controls $(\tau_1, \tau_2) \in T$ and $0 < \theta < 1$, then we can write,

$$\begin{aligned} \tau_1^2 - 2\tau_1\tau_2 + \tau_2^2 &= (\tau_1 - \tau_2)^2 \geq 0 \\ \Rightarrow \theta(1-\theta)\tau_1^2 + \theta(1-\theta)\tau_2^2 &\geq \theta(1-\theta)2\tau_1\tau_2 \\ \Rightarrow (\theta - \theta^2)\tau_1^2 + [(1-\theta) - (1-\theta^2)]\tau_2^2 &\geq 2\theta(1-\theta)\tau_1\tau_2 \\ \Rightarrow \theta\tau_1^2 + (1-\theta)\tau_2^2 &\geq \theta^2\tau_1^2 + (1-\theta^2)\tau_2^2 + 2\theta(1-\theta)\tau_1\tau_2 \\ \Rightarrow \theta\tau_1^2 + (1-\theta)\tau_2^2 &\geq [\theta\tau_1 + (1-\theta)\tau_2]^2 \\ \Rightarrow I(t) + D_p(t) + \theta\tau_1^2 + (1-\theta)\tau_2^2 &\geq I(t) + D_p(t) + [\theta\tau_1 + (1-\theta)\tau_2]^2 \\ \Rightarrow [I(t) + D_p(t)][\theta + (1-\theta)] + \theta\tau_1^2 + (1-\theta)\tau_2^2 &\geq [I(t) + D_p(t)] + [\theta\tau_1 + (1-\theta)\tau_2]^2 \\ \Rightarrow \theta[I(t) + D_p(t)] + (1-\theta)[I(t) + D_p(t)] + \theta\tau_1^2 + (1-\theta)\tau_2^2 &\geq [I(t) + D_p(t)] + [\theta\tau_1 + (1-\theta)\tau_2]^2 \\ \Rightarrow \theta f(\tau_1) + (1-\theta)f(\tau_2) &\geq f(\theta\tau_1 + (1-\theta)\tau_2) \end{aligned}$$

Which implies that $f(\tau)$ is convex on T .

Now we can show that

$$J(\tau) \geq -L_2 + L_1\tau^\gamma$$

with $\gamma > 1, L_1 \geq 0$. Here,

$$\begin{aligned} J(\tau) &= I(t) + D_p(t) + \frac{A_1}{2}\tau_1^2 + \frac{A_2}{2}\tau_2^2 \\ &= I(t) + D_p(t) + \tau^2 \\ \text{Since } \frac{A_1}{2}\tau_1^2 + \frac{A_2}{2}\tau_2^2 &= \tau^2 \\ J(\tau) &\geq -[I(t) + D_p(t)] + \tau^2 = -L_2 + L_1\tau^2 \end{aligned}$$

where $L_2 > 0$ which depends on upper bounds of $I(t), D_p(t)$. We can also see that $\gamma = 2 > 1, L_1 > 0$. Therefore, condition 4 is also satisfied. From the above discussion the existence of the objective functional has been established.

3.4 Characterization of the Optimal Control

By applying Pontryagin's Maximum Principle (PMP) to the Hamiltonian (H) we can derive the necessary conditions for the optimal control. Therefore, using PMP, to find the optimal vaccination and treatment term the standard Hamiltonian function (H) with respect to (τ_1, τ_2) can be defined as follows:

$$H(t, y(t), \tau(t), q(t), \lambda(t)) = (q(t), f(y(t)) + g(y(t))\tau(t)) - \lambda L(y(t), \tau(t)), \lambda \in \mathbb{R}$$

Where $q = (q_S, q_E, q_I, q_{D_p}, q_R) \in \mathbb{R}^5$ denotes the adjoint variables. Suppose that the pair (y^*, τ^*) is the optimal solution of the above optimal control problem. Then, the maximum principle asserts the existence of a scalar $\lambda_0 = 0$, an absolutely continuous function $q(t)$, such that the following conditions are satisfied:

1. $\max\{|q(t)| : t \in [0, t_f]\} + \lambda_0 > 0$;
2. $\dot{q}(t) = \lambda L_y[t] - (q[t], f_y[t] + g_y[t]\tau^*(t))$;
3. $q(t) = (0, 0)$;

$$4. H(y^*(t), \tau^*(t), q(t)) = \max_{\tau} \{H(y^*(t), q(t), \tau(t))\} \text{ where } u_1 \leq \tau_1 \leq v_1, u_2 \leq \tau_2 \leq v_2$$

where time argument [t] denotes the evaluation along with the optimal solution. Then, from condition 2, adjoint equations in normal form (i.e., $\lambda = 1$) are explicitly given by

$$\begin{aligned} q_S' &= -\frac{\partial H}{\partial S} = -q_S\{-\beta(I + \eta D_p) - \mu - \tau_1\} - q_E\{\beta(I + \eta D_p)\} - \tau_1 q_R \\ &= \beta(I + \eta D_p)(q_S - q_E) + q_S(\mu + \tau_1) - \tau_1 q_R \\ q_E' &= -\frac{\partial H}{\partial E} = q_E(\mu + \psi) - \psi q_I = \mu q_E + \psi(q_E - q_I) \\ q_I' &= -\frac{\partial H}{\partial I} = 1 + \beta q_S S - \beta q_E S + (\mu + \mu_1)q_I + (\alpha + \delta)q_I - (\alpha + d\delta)q_{D_p} - (1 - d)\delta q_R \\ &= 1 + \beta S(q_S - q_E) + (\mu + \mu_1 + \alpha + \delta)q_I - (\alpha + d\delta)q_{D_p} - (1 - d)\delta q_R \\ q_{D_p}' &= -\frac{\partial H}{\partial D_p} = 1 + \beta \eta q_S S - \beta \eta q_E S + (\mu + \mu_1 + \rho)q_{D_p} + \tau_2 q_{D_p} - (\rho + \tau_2)q_R \\ &= 1 + \beta \eta S(q_S - q_E) + (\mu + \mu_1 + \rho + \tau_2)q_{D_p} - (\rho + \tau_2)q_R \\ q_R' &= -\frac{\partial H}{\partial R} = \mu q_R \end{aligned}$$

With transversality condition $q_i(t_f) = 0, i = 1, 2, 3, 4$ and 5.

Now by applying Pontryagin's Maximum Principle Lenhart and Workman [22] we have the following theorem and proving theorem 5, we show the existence of controls.

Theorem 5: There exist optimal control (τ_1^*, τ_2^*) that minimizes the objective functional J over T given by

$$\tau_1^* = \max_{[0, t_f]} \left\{ 0, \min \left(1, \frac{(q_S - q_R)S^*}{A_1} \right) \right\} \text{ and } \tau_2^* = \max_{[0, t_f]} \left\{ 0, \min \left(1, \frac{(q_{D_p} - q_R)D_p^*}{A_2} \right) \right\}$$

$$\frac{\partial H}{\partial \tau_1} = A_1 \tau_1 - q_S S + q_R S = 0 \Rightarrow \tau_1 = \frac{(q_S - q_R)S}{A_1} = \hat{\tau}_1;$$

$$\frac{\partial H}{\partial \tau_2} = A_2 \tau_2 + q_R D_p - q_{D_p} D_p = 0 \Rightarrow \tau_2 = \frac{(q_{D_p} - q_R)D_p}{A_2} = \hat{\tau}_2$$

According to the property of T , the two controls (τ_1^*, τ_2^*) are bounded with upper bound 1 and lower bound 0. Therefore,

$$\tau_1^*(t) = \begin{cases} 0 & \text{if } \hat{\tau}_1 \leq 0 \\ \frac{(q_S - q_R)S}{A_1} & \text{if } 0 < \hat{\tau}_1 < 1 \\ 1 & \text{if } \hat{\tau}_1 \geq 1 \end{cases}$$

This can be written in compact form as

$$\tau_1^* = \max_{[0, t_f]} \left\{ 0, \min \left(1, \frac{(q_S - q_R)S}{A_1} \right) \right\}$$

Similarly

$$\tau_2^*(t) = \begin{cases} 0 & \text{if } \hat{\tau}_2 \leq 0 \\ \frac{(q_{D_p} - q_R)D_p}{A_2} & \text{if } 0 < \hat{\tau}_2 < 1 \\ 1 & \text{if } \hat{\tau}_2 \geq 1 \end{cases}$$

In the same way, this can be written in compact form as

$$\tau_2^* = \max_{[0, t_f]} \left\{ 0, \min \left(1, \frac{(q_{D_p} - q_R) D_p}{A_2} \right) \right\}$$

Thus we get optimal solutions as

$$(\tau_1^*, \tau_2^*) = \left(\max_{[0, t_f]} \left\{ 0, \min \left(1, \frac{(q_S - q_R) S}{A_1} \right) \right\}, \max_{[0, t_f]} \left\{ 0, \min \left(1, \frac{(q_{D_p} - q_R) D_p}{A_2} \right) \right\} \right).$$

And this completes the proof.

Table 1. Parameter description and value

Parameter	Description	Value
Λ	Recruitment rate into susceptible class	0.30
μ	Natural death rate	0.10
μ_1	Computer virus induced death rate	0.65
β	Contact rate	0.30
η	Modified factor for program files damage	0.10
ψ	Progression rate from exposed class to infected class	0.45
δ	Treatment rate of infected computers	0.0021 – 1.80
α	Program files damage rate	0.02
ρ	Recovered rate of program files damage	0.0021 – 0.80
$d\delta$	Rate at which recovered damaged program files show virus infection again	0.0025 – 0.025

4. NUMERICAL SOLUTIONS AND EFFICIENCY ANALYSIS

To justify the impact of optimality control strategies, we conducted numerical simulation to confirm the theoretical predictions discussed in the previous sections using Runge Kutta Method of order four (RK4) written in MATLAB programming. We used a set of logical parameter values and the initial values for the susceptible, exposed, infected, program files damage and recovered were $(0) = 65, E(0) = 10, I(0) = 0, D_p(0) = 0, R(0) = 5$. We also considered the initial values $S(0) = 65, E(0) = 10, I(0) = 6, D_p(0) = 2, R(0) = 5$ as in [23]. All the parameters with their values are shown in Table 1.

The simulations are performed with time 12 months. Firstly, we numerically simulated the optimality system when no control is applied (i.e., vaccination control $\tau_1 = 0$ and treatment control $\tau_2 = 0$). The result is presented in Fig. 1. We also conducted numerical simulation of the optimality system when either of the controls is applied (i.e., $\tau_1 \neq 0$ and $\tau_2 = 0$, (Fig. 2); $\tau_1 = 0$ and $\tau_2 \neq$

0, (Fig. 3). Further we simulated the optimality systems where both control strategies are applied (i.e., $\tau_1 \neq 0$ and $\tau_2 \neq 0$); Fig. 4.

We also solved the optimality systems for each of the compartments using either of the control strategy and both to compare their effect on the compartments to when there is no control. So we take the control variable $\tau_1 \neq 0$ (i.e., treatment control $\tau_2 = 0$). The simulation results in the presence of vaccination only are shown in Figs. 5–9. Again, we run the program of the optimality system when only treatment control is employed and we take $\tau_2 \neq 0$ (i.e., vaccination control $\tau_1 = 0$). We presented the simulation results in Figs. 10 – 14. Finally, numerical simulations of the optimality systems are performed considering both control strategies i.e., vaccination control ($\tau_1 \neq 0$) and treatment control ($\tau_2 \neq 0$) and the results are presented in Figs. 15–19. Here, vaccination and treatment control was considered at a greater extent, that is, $\tau_1 = 1$ and $\tau_2 = 1$.

Since in this paper, we have considered two controls, in which one is vaccination control τ_1 and the other is treatment control τ_2 . If one of

these controls is to be used, it will be necessary to determine which of them is more efficient to reduce program files damage in computer system. Based on this we performed an efficiency analysis [24,25,26], which will allow us to determine the best control strategy. Here, we distinguish two control strategies STR-1 and STR-2 where STR-1 is the strategy where $\tau_1 \neq 0, \tau_2 = 0$ and STR-2 is the strategy where $\tau_1 = 0, \tau_2 \neq 0$. To determine the best control strategy among these two, we have to calculate the efficiency index $(E.I) = \left(1 - \frac{A^c}{A^o}\right) \times 100$, where A^c and A^o are the cumulated number of program files damaged computers with and without control, respectively. The best strategy

will be the one whom efficiency index will be bigger [24,25]. It can be noted that the cumulated number of program files damage computers during the time interval $[0, 12]$ is defined by $A = \int_0^{12} D_p(t) dt$. We evaluated the value of integration and we have $A^o = 0.025425$. The value of A^c and the efficiency index (E.I) for STR-1 and STR-2 are given in Table 2.

It can be seen from Table 2 that STR-2 is the best strategy among STR-1 and STR-2, and this permits the reduction of the number of incident cases. Therefore, treatment is more effective than vaccination to minimize program files damage in computer system.

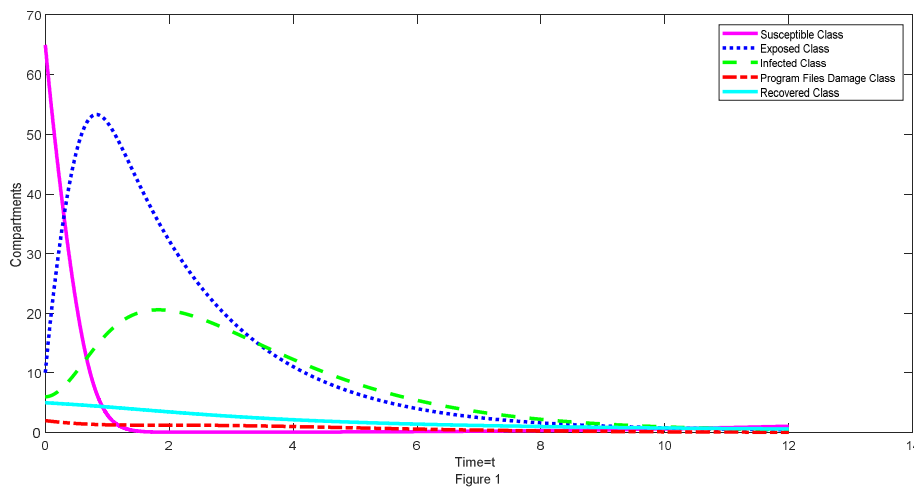


Fig. 1. Simulation of the dynamics of the compartments, when no control measure is applied

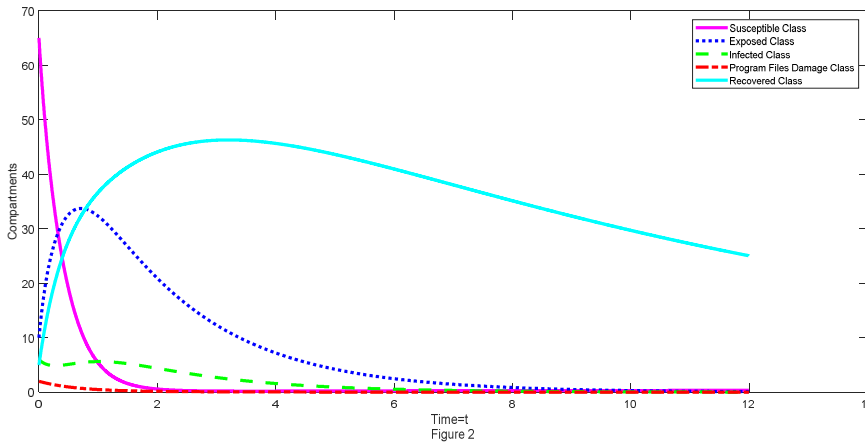


Fig. 2. Simulation of the dynamics of the compartments, when only vaccination control (τ_1) is applied as optimal control

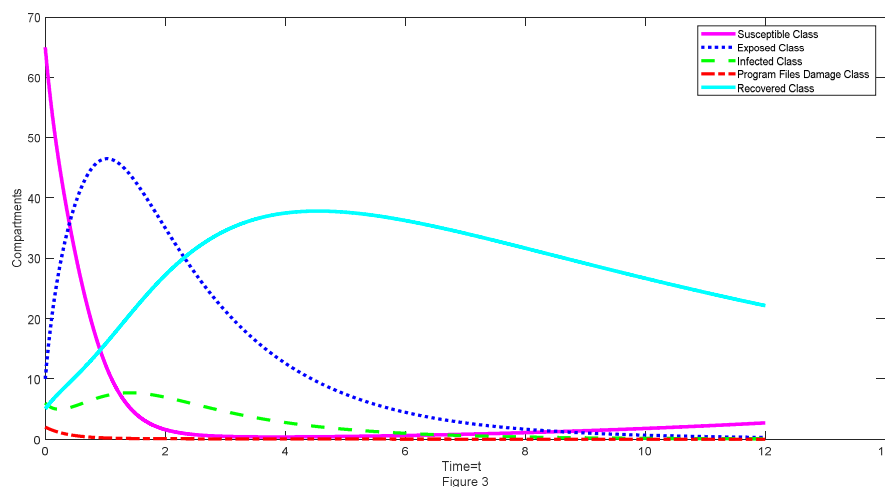


Fig. 3. Simulation of the dynamics of the compartments, when only treatment control (τ_2) is applied as optimal control

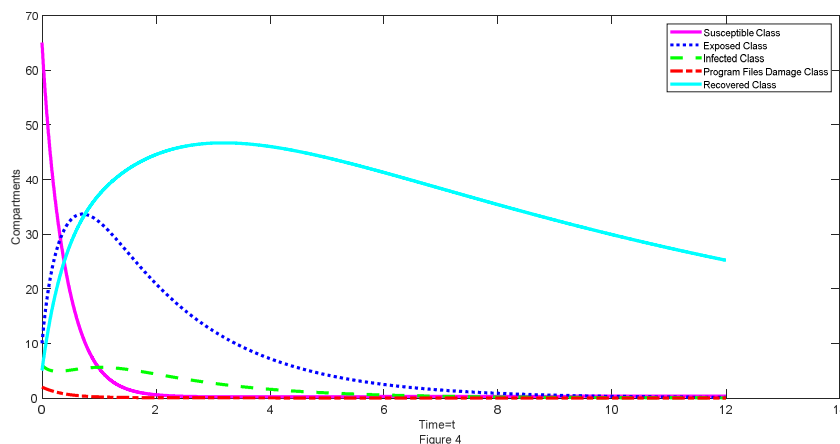


Fig. 4. Simulation of the dynamics of compartments, when vaccination control (τ_1) and treatment control (τ_2) are applied as optimal control

The effect of vaccination as a control measure on the susceptible, exposed, infected, program files damaged and recovered computer for 12 months timeline is represented in Figs. 5-9. We observed that the control measure to a little extent influenced the susceptible computers, but appreciably control the exposed, infected, program files damage and recovered computers. As anticipated, both the infected and program files damaged computers have increased in the absence of vaccination than the computers that are having the control measure. Conversely, the number of recovered computer systems increases at the time vaccination control is

applied when compared to the computer systems without optimal control.

Figs. 10 -14 represents the effects of treatment as a control measure on the susceptible, exposed, infected, program files damaged and recovered computer systems over the period of 12 months. It has been noticed that the control actually influenced the susceptible, exposed, infected, program files damaged and recovered computer systems to a greater extent. We also saw that infected and program files damaged computers decreases significantly in the presence of treatment control than computers without control measure.

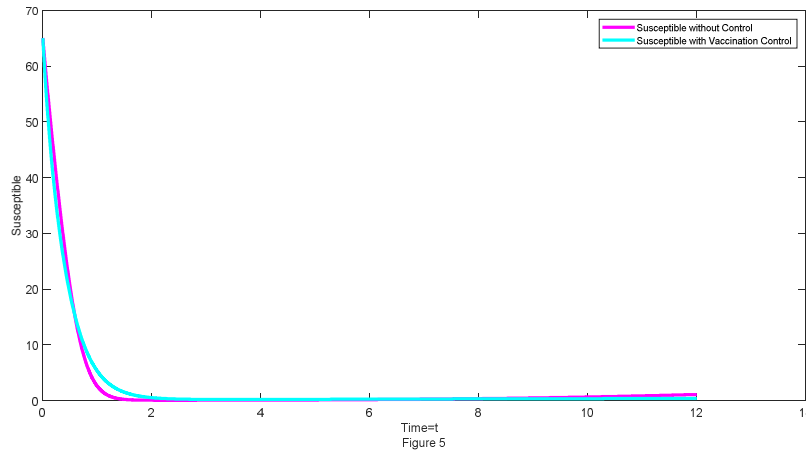


Fig. 5. Simulation of the dynamics of susceptible computers, when only vaccination control (τ_1) is applied as optimal control

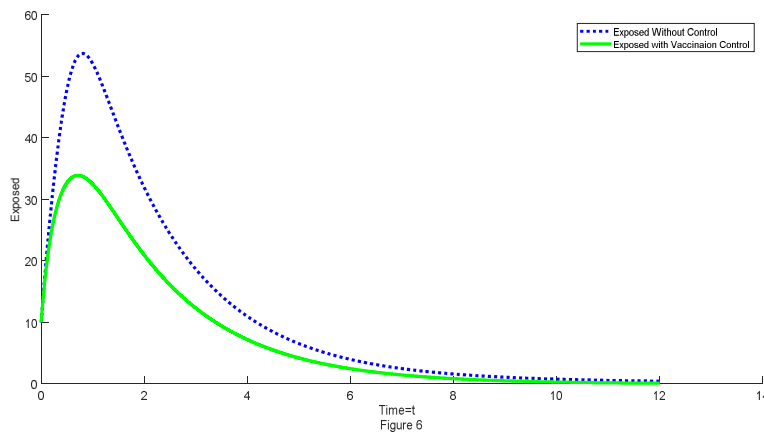


Fig. 6. Simulation of the dynamics of exposed computers, when vaccination control (τ_1) is applied as optimal control

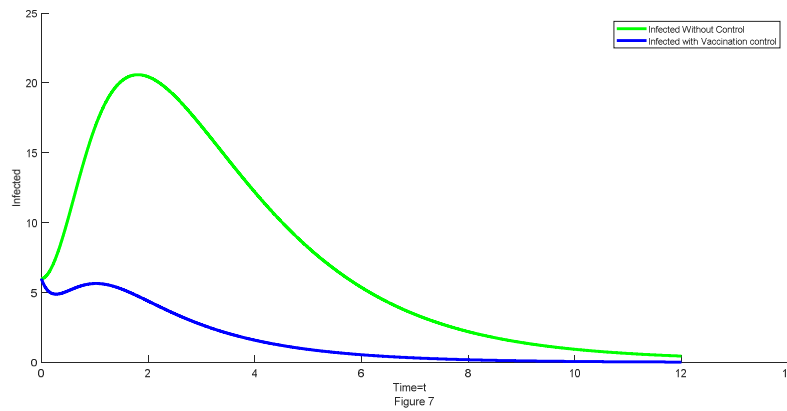


Fig. 7. Simulation of the dynamics of infected computers, when only vaccination control (τ_1) is applied as optimal control

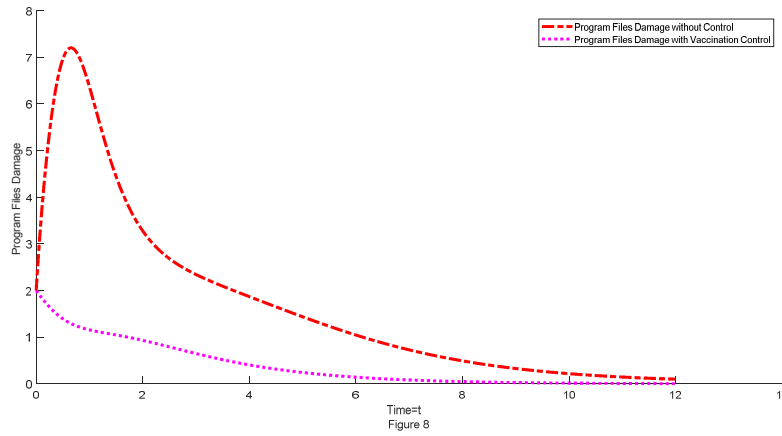


Fig. 8. Simulation of the dynamics of program files damaged computers, when only vaccination control (τ_1) is applied as optimal control

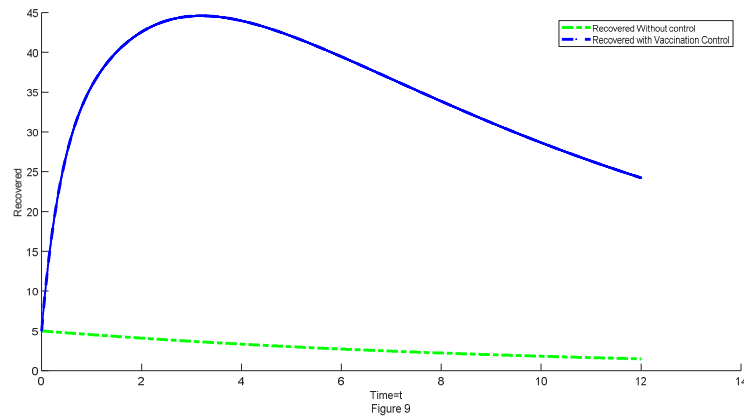


Fig. 9. Simulation of the dynamics of recovered computers, when only vaccination control (τ_1) is applied as optimal control

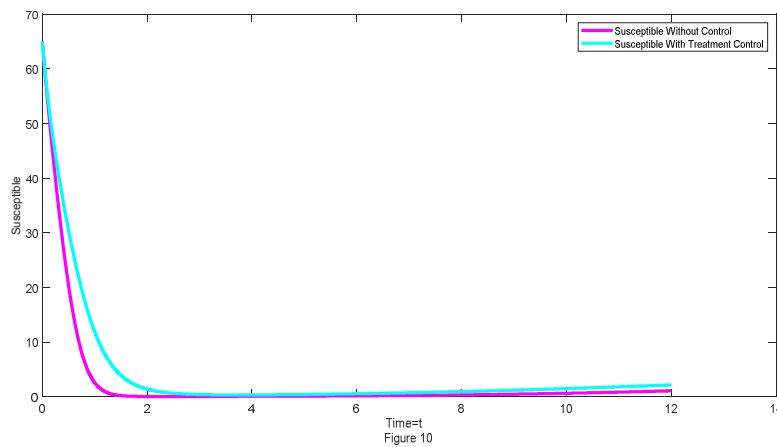


Fig. 10. Simulation of the dynamics of susceptible computers, when only treatment control (τ_2) is applied as optimal control

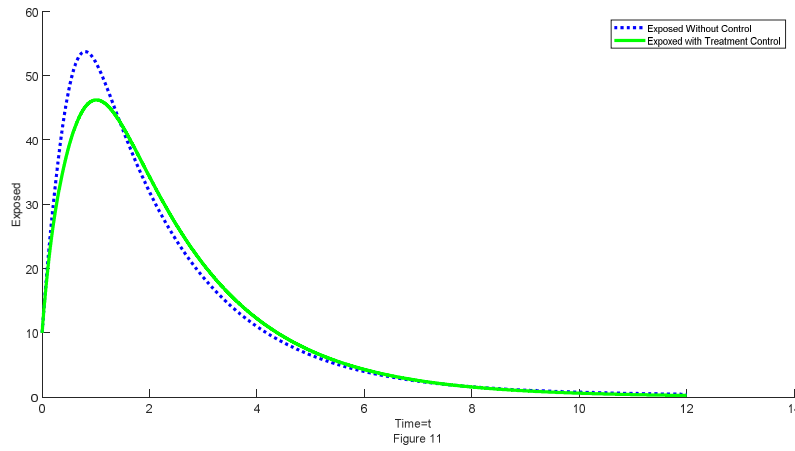


Fig. 11. Simulation of the dynamics of exposed computers, when only treatment control (τ_2) is applied as optimal control

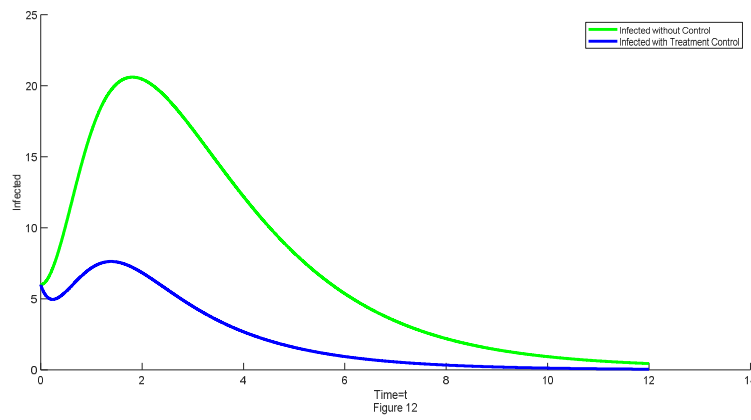


Fig. 12. Simulation of the dynamics of infected computers, when only treatment control (τ_2) is applied as optimal control

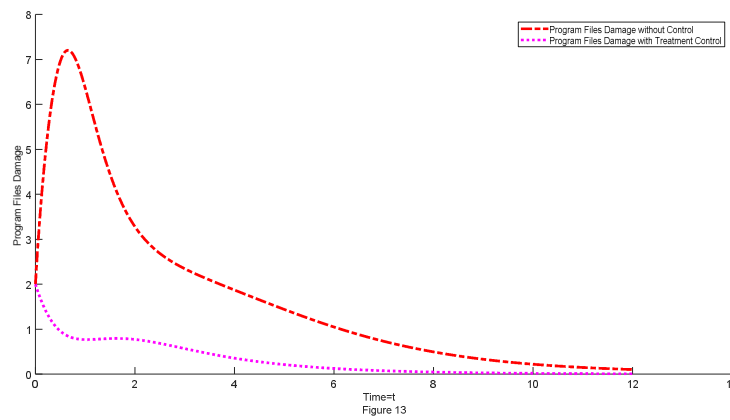


Fig. 13. Simulation of the dynamics of program files damaged computers, when only treatment control (τ_2) is applied as optimal control

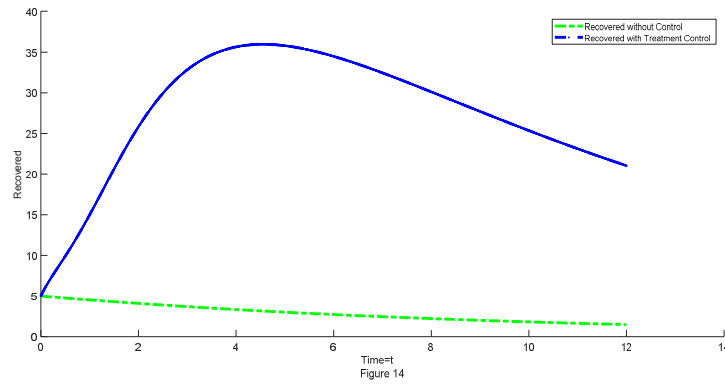


Fig. 14. Simulation of the dynamics of program files damaged computers, when only treatment control (τ_2) is applied as optimal control

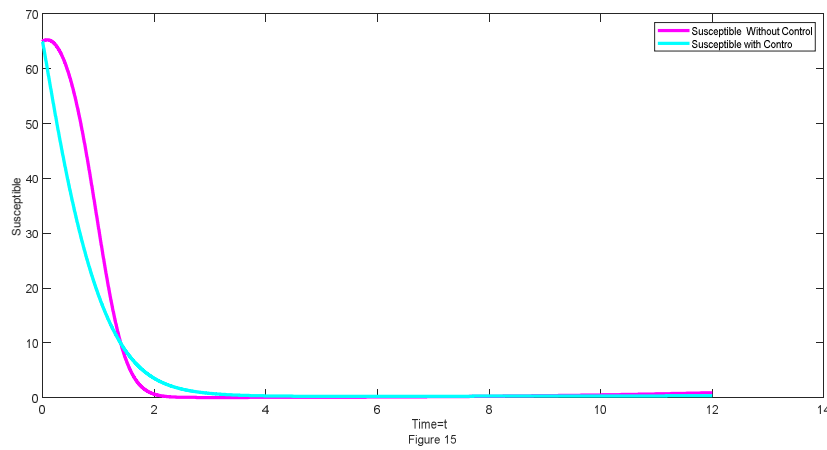


Fig. 15. Simulation of the dynamics of susceptible, when vaccination control (τ_1) and treatment control (τ_2) are applied as optimal control

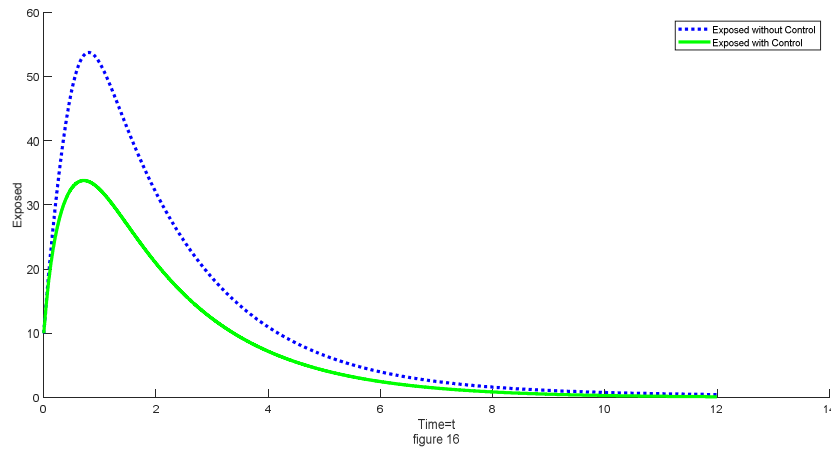


Fig. 16. Simulation of the dynamics of exposed computers, when vaccination control (τ_1) and treatment control (τ_2) are applied as optimal control

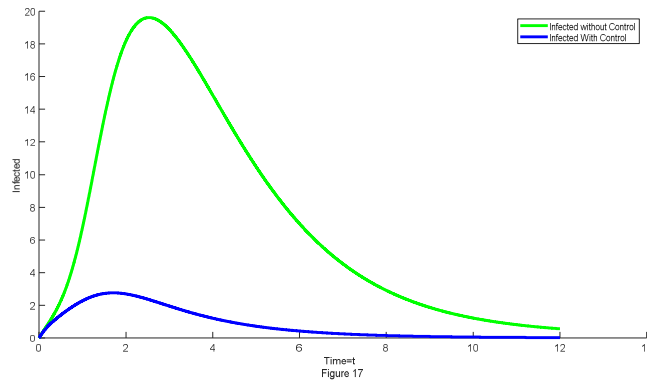


Fig. 17. Simulation of the dynamics of infected computers, when vaccination control (τ_1) and treatment control (τ_2) are applied as optimal control

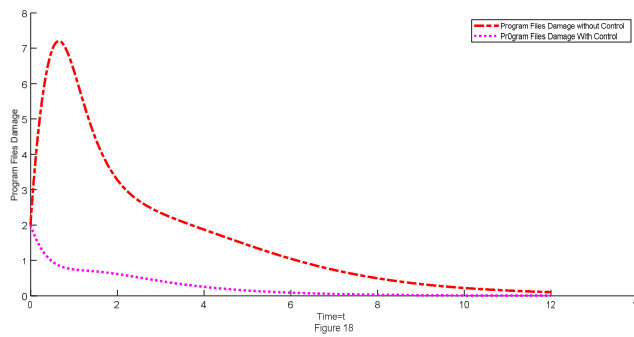


Fig. 18. Simulation of the dynamics of program files damaged computers, when vaccination control (τ_1) and treatment control (τ_2) are applied as optimal control

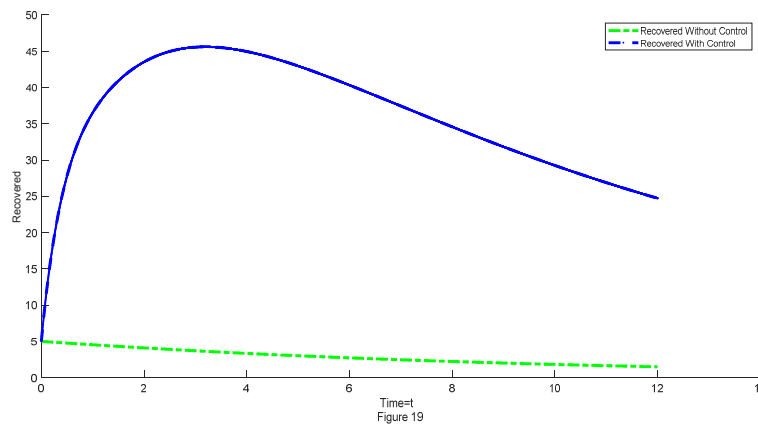


Fig. 19. Simulation of the dynamics recovered computers, when vaccination control (τ_1) and treatment control (τ_2) are applied as optimal control

Table 2. Strategies and their efficiency Indices

Strategy	A ^c	E. I
STR – 1	0.002230	91.23
STR – 2	0.001507	94.07

More so, we shall observe that the number of recovered computer systems increased as the treatment control is applied when compared to the computer systems without optimal control. The effects of vaccination and treatment control measures on the susceptible, exposed, infected, program files damaged and recovered computers over the period of 12 months were shown in Figs. 15 -19. It was seen that the control measure slightly influences the susceptible computer population, but significantly controls the exposed, infected, program files damaged and recovered computers. As anticipated, both infected and program files damaged computers have decreased for the presence of both control measure than the computers without having the optimal control. As a matter of fact, the number of recovered computers increases as we applied the control measure when compared to the computers without optimal control.

For vaccination and treatment controls, we used the maximum level of the control measures on the five compartments of the model for 12 months interval and we observed that the extreme level of the control measure considerably controls the five compartments; as both the infected and program files damaged computers decreases drastically.

5. CONCLUSION

In this paper, we have analyzed the qualitative behavior and optimal control strategy of a $SEID_pR$ model. Two control functions have been used, one for vaccinating the susceptible computer population and the other for controlling the treatment efforts to the program files damage computers. We formulated the optimal control model considering the two control variables by using the most well-known Pontrygin's Maximum Principle. The analysis results were demonstrated using numerical simulation.

Our analysis showed that the application of only one of the control measure have the ability of reducing the exposed, infected and program files damaged computer population, but the combination of optimal vaccination and treatment are much more effective for reducing the exposed, infected and program files damaged computer population, to maximize the recovered computer in the population and also minimize the cost of the two control measures. In view of the fact that there are vaccination strategy available for computer virus infection (which eventually leads to program files damage), therefore from

the simulation, it is shown that the optimal combination of vaccination and treatment is effective to control computer virus infection progression and program files damage.

Consequently to reduce computer virus infection from the population, strong anti-virus software should be installed (vaccination). In cases were some files are infected earlier to anti-virus installation or during a period when the virus signature are not updated, a strong anti-virus must be used to disinfect the infected file, thereby recovering the damaged program files (treatment). Finally, computer virus infection is a significant cause of program file damage and other symptoms worldwide, therefore, it is time to get rid of this serious problem.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Peer-review history:
 The peer review history for this paper can be accessed here:
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