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2-Tuple Prioritized Aggregation Operators and Their Application to Multiple Attribute Group Decision Making

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Short Research Article

Abstract

Aims: The aim of this paper is to present some 2-tuple prioritized aggregation operators for handling the multiple attribute group decision making problems where there exists a prioritization relationship over the attributes and decision makers.
Study Design: Motivated by the idea of the prioritized aggregation (PA) operators, we first develop two 2-tuple prioritized aggregation operators called the 2-tuple prioritized weighted average (2TPWA) operator and the 2-tuple prioritized weighted geometric (2TPWG) operator.
Place and Duration of Study: We examine their desirable properties.
Methodology: The significant feature of these operators is that they not only deal with the linguistic and interval linguistic information but also take the prioritization relationship among the arguments into account.
Results: Then, based on the proposed operators, we propose an approach to multiple attribute group decision making under linguistic environment in which the attributes and decision makers are in different priority level.
Conclusion: Finally, an illustrative example is employed to show the reasonableness and effectiveness of the proposed approach.

Keywords: Multiple attribute group decision making, 2-tuple linguistic information, 2-tuple prioritized weighted average (2TPWA) operator, 2-tuple prioritized weighted geometric (2TPWG) operator.

1 Introduction

Multiple attribute group decision making (MAGDM) consists of finding the most desirable alternative(s) from a given alternative set according to the preferences provided by a group of experts [1-7]. For some MAGDM problems, the decision information about alternatives is usually uncertain or fuzzy due to the increasing complexity of the socio-economic environment and the vagueness of inherent subjective nature of human thinking [8-10]; thus, the decision information cannot be precisely assessed in a quantitative form. However, it may be appropriate and sufficient

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to assess the information in a qualitative form rather than a quantitative form. For example, when evaluating a house's cost, linguistic terms such as ''high'', ''medium'', and ''low'' are usually used, and when evaluating a house's design, linguistic terms like ''good'', ''medium'', and ''bad'' can be frequently used. Up to now, some methods have been developed for coping with linguistic information. These methods can be summarized as follows [11]: (1) The approximative computational model based on the Extension Principle [12]. This model transforms linguistic assessment information into fuzzy numbers and uses fuzzy arithmetic to make computations over these fuzzy numbers. The use of fuzzy arithmetic increases the vagueness. The results obtained by the fuzzy arithmetic are fuzzy numbers that usually do not match any linguistic term in the initial term set. (2) The ordinal linguistic computational model [13]. This model is also called symbolic model which makes direct computations on labels using the ordinal structure of the linguistic term sets. But symbolic method easily results in a loss of information caused by the use of the round operator. (3) The 2-tuple linguistic computational model [14-17]. This model uses the 2-tuple linguistic representation and computational model to make linguistic computations. (4) The model, which computes with words directly [18-26].

The 2-tuple fuzzy linguistic representation model represents the linguistic information by means of a pair of values called 2-tuple, composed by a linguistic term and a number. Meanwhile, the model also gives a computational technique to deal with the 2-tuples without loss of information. Since its introduction, the 2-tuple fuzzy linguistic representation model has received more and more attention. In a MAGDM problem involving the 2-tuple fuzzy linguistic representation model, in order to aggregate the individual linguistic preference information into the overall linguistic preference information, 2-tuple aggregation operators are most widely used. In the past few decades, many scholars have developed a variety of 2-tuple aggregation operators, such as 2-tuple arithmetic mean operator [14,16], 2-tuple weighted averaging operator [14], 2-tuple OWA operator [14], 2-tuple weighted geometric averaging (TWGA) operator [27], 2-tuple ordered weighted geometric averaging (TOWGA) operator [27], 2-tuple hybrid geometric averaging (THGA) operator [27], 2-tuple arithmetic average (TAA) operator [14], 2-tuple weighted average (TWA) operator [14], 2-tuple ordered weighted average (TOWA) operator [14], extended 2-tuple weighted average (ET-WA) operator [14], 2-tuple ordered weighted geometric (TOWG) operator [28], extended 2-tuple weighted geometric (ET-WG) operator [29], extended 2-tuple ordered weighted geometric (ET-OWG) operator [29], generalized 2-tuple weighted average (G-2TWA) operator [11], generalized 2-tuple ordered weighted average (G-2TOWA) operator [11], induced generalized 2-tuple ordered weighted average (IG-2TOWA) operator [11], 2-tuple linguistic power average (2TLPA) operator [30], 2-tuple linguistic weighted power average (2TLWPA) operator [30], and 2-tuple linguistic power ordered weighted average (2TLPOWA) operator [30].
The above MAGDMs are under the assumption that the attribute and the decision makers are at the same priority level respectively. In this situation, we have the ability to trade off between attributes. For instance, if C_i and C_j are two attributes with the weights w_i and w_j respectively, then we can compensate for a decrease of θ in satisfaction to attribute C_i by gain $\frac{w_j}{w_i}\theta$ in satisfaction to attribute C_j. However, in some MAGDM problems, we do not want to allow this kind of compensation between attributes. For example, consider the situation in which we are buying a car based on the safety and cost of cars. We may not allow a benefit with respect to cost to compensate for a loss in safety. In this case we have a kind of prioritization of the attributes, i.e., safety has a higher priority than cost. Additionally, decision making in a company, general manager has a higher priority than vice manager. Yager [31] first investigated criteria aggregation

problems where there is a prioritization relationship over the criteria. Then, Yager [32] and Yager et al. [33] gave much deeper insights into this issue. Motivated by the ideas of Yager [31,32] and Yager et al. [33], Wei [34] generalized prioritized aggregation operators to hesitant fuzzy environment, proposed some hesitant fuzzy prioritized aggregation operators, and applied these operators to develop some models for hesitant fuzzy multiple attribute decision making problems in which the attributes are in different priority level. Yu and Xu [35] investigated the prioritization relationship of attributes in multi-attribute decision making with intuitionistic fuzzy information and developed some prioritized intuitionistic fuzzy aggregation operators by extending the prioritized aggregation operators. Yu et al. [36] proposed some interval-valued intuitionistic fuzzy prioritized aggregation operators and investigated the application of these operators in the group decision making under interval-valued intuitionistic fuzzy environment in which the attributes and experts are in different priority level. However, the existing 2-tuple aggregation operators are difficult to deal with the MAGDM where the attributes and the decision makers are in different priority level respectively. Moreover, we are conscious that there has been rather little work completed for using prioritized aggregation operators to solve the MAGDM with linguistic preference information. Thus, it is necessary to extend prioritized aggregation operators to the linguistic environment. To do it, in the current paper, we develop some 2-tuple prioritized aggregation operators. The prominent characteristic of these proposed operators is that they take prioritization among the attributes and the decision makers into account. Then, we utilize these operators to develop some approaches to the MAGDM where the attributes and the decision makers are in different priority level. Finally, some numerical examples are given to verify the practicality and effectiveness of the proposed operators and approaches.

This paper is organized as follows. Section 2 introduces some basic concepts of the 2-tuple fuzzy linguistic approach and the prioritized average operator. In Section 3, we propose the 2-tuple prioritized weighted average (2TPWA) operator and the 2-tuple prioritized weighted geometric (2TPWG) operator to aggregate the linguistic information. Furthermore, we develop a method for MAGDM based on the proposed operators under the linguistic environment. In Section 4, an example concerning talent introduction is provided to demonstrate the practicality and effectiveness of the developed approach. The final section offers some concluding remarks.

2 Preliminaries

In this section, we will introduce the basic notions of the 2-tuple fuzzy linguistic approach and the prioritized average operator.

2.1 The 2-tuple fuzzy linguistic representation model

Let $S=\{s_i \mid i=0,1,2,\cdots,g\}$ be a finite and totally ordered discrete linguistic term set with odd cardinality, where s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics [14-16,37-39].

(1) The set is ordered: $s_i \geq s_j$ if $i \geq j$;

(2) There is the negation operator: $neg(s_i)=s_j$ such that $j=g-i$;

(3) Max operator: $\max(s_i,s_j)=s_i$ if $s_i \geq s_j$;

(4) Min operator: $\min(s_i, s_j) = s_i$ if $s_i \le s_j$.

For example, a set of seven terms S could be given as follows [40-45]:

$S = \{s_0 = nothing,\ s_1 = very\,low,\ s_2 = low,\ s_3 = medium,\ s_4 = high,\ s_5 = very\,high,\ s_6 = perfect\}$.

To preserve all the given information, Xu [19] extended the discrete linguistic term set S to a continuous linguistic term set $\bar{S} = \{s_\alpha | s_0 \le s_\alpha \le s_g, \alpha \in [0, g]\}$. If $s_\alpha \in S$, then s_α is called an original linguistic term; otherwise, s_α is called a virtual linguistic term. In general, the decision maker uses the original linguistic terms to evaluate alternatives, and the virtual linguistic terms can only appear in operation.

Based on the concept of symbolic translation, Herrera and Martinez [14,15] introduced a 2-tuple fuzzy linguistic representation model for dealing with linguistic information. This model represents the linguistic assessment information by means of a 2-tuple (s_i, α), where $s_i \in S$ represents a linguistic label from the predefined linguistic term set S and $\alpha \in [-0.5, 0.5)$ is the value of symbolic translation.

Definition 2.1 [14,15]. Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g+1$ the cardinality of S. Let $i = \text{round}(\beta)$ and $\alpha = \beta - i$ be two values such that $i \in [0, g]$ and $\alpha \in [-0.5, 0.5)$ then α is called a symbolic translation, where $\text{round}(\bullet)$ is the usual round operation.

Definition 2.2 [14,15]. Let $S = \{s_i | i = 0, 1, 2, \cdots, g\}$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta : [0, g] \to S \times [-0.5, 0.5) \tag{1}$$

$$\Delta(\beta) = (s_i, \alpha), \qquad \text{with } \begin{cases} s_i, & i = \text{round}(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5) \end{cases} \tag{2}$$

where s_i has the closest index label to β and α is the value of the symbolic translation.

Theorem 2.1 [14,15]. Let $S = \{s_i | i = 0, 1, 2, \cdots, g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function such that from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset R$, where

$$\Delta^{-1} : S \times [-0.5, 0.5) \to [0, g] \tag{3}$$

$$\Delta^{-1}(s_i,\alpha)=i+\alpha=\beta . \tag{4}$$

It is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value zero as symbolic translation

$$s_i \in S \Rightarrow (s_i,0).$$

Definition 2.3 [14,15]. The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order. Let (s_k,α_k) and (s_l,α_l) be two 2-tuples, with each one representing a counting of information as follows.

(1) If $k<l$ then (s_k,α_k) is smaller than (s_l,α_l).
(2) If $k=l$ then

- if $\alpha_k=\alpha_l$ then (s_k,α_k), (s_l,α_l) represents the same information;
- if $\alpha_k<\alpha_l$ then (s_k,α_k) is smaller than (s_l,α_l);
- if $\alpha_k>\alpha_l$ then (s_k,α_k) is bigger than (s_l,α_l).

2.2 Prioritized average (PA) operators

The prioritized average (PA) operator was originally introduced by Yager [31,46], which was defined as follows:

Definition 2.4 [31]. Let $C=\{C_1,C_2,\cdots,C_n\}$ be a collection of criteria and that there is a prioritization between the criteria expressed by the linear ordering $C_1 \succ C_2 \succ C_3 \cdots \succ C_n$, indicate criteria C_j has a higher priority than C_k if $j<k$. The value $C_j(x)$ is the performance of any alternative x under criteria C_j, and satisfies $C_j(x)\in[0,1]$. If

$$PA\big(C_j(x)\big)=\sum_{j=1}^{n} w_j C_j(x) \tag{5}$$

where $w_j=\dfrac{T_j}{\sum_{j=1}^{n}T_j}$, $T_j=\prod_{k=1}^{j-1}C_k(x)\ (j=2,\cdots,n)$, $T_1=1$. Then PA is called the prioritized average (PA) operator.

3 2-Tuple Prioritized Aggregation Operators

The prioritized average operators, however, have only been used in situations where the input arguments are the exact values [31,46]. In the following, we extend the PA operator to linguistic environment and develop two 2-tuple prioritized aggregation operators, which can accommodate the situations where the input arguments are linguistic assessment information.

3.1 2-tuple prioritized weighted average (2TPWA) operators

Definition 3.1. Let $\{(r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\}$ $\left(r_j \in S, \alpha_j \in [-0.5, 0.5), j=1,2,\cdots,n\right)$ be a set of 2-tuples, if

$$\mathrm{2TPWA}\left((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\right)=\Delta\left(\frac{T_1}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r_1,\alpha_1)+\frac{T_2}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r_2,\alpha_2)+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r_n,\alpha_n)\right), \quad (6)$$

where $T_1=1$ and $T_j=\prod_{k=1}^{j-1}\left(\frac{\Delta^{-1}(r_k,\alpha_k)}{g+1}\right)$ $(j=2,\cdots,n)$, then 2TPWA is called a 2-tuple prioritized weighted average (2TPWA) operator.

Theorem 3.1 (Boundedness). Let $\{(r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\}$ be a set of 2-tuples, then

$$\min_{1\le i\le n}\{(r_i,\alpha_i)\}\le \mathrm{2TPWA}\left((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\right)\le \max_{1\le i\le n}\{(r_i,\alpha_i)\}. \quad (7)$$

Proof. Because $\min_{1\le i\le n}\{(r_i,\alpha_i)\}\le (r_i,\alpha_i)\le \max_{1\le i\le n}\{(r_i,\alpha_i)\}$, we have

$$\Delta\left(\frac{T_1}{\sum_{j=1}^{n}T_j}\Delta^{-1}\left(\min_{1\le i\le n}\{(r_i,\alpha_i)\}\right)+\frac{T_2}{\sum_{j=1}^{n}T_j}\Delta^{-1}\left(\min_{1\le i\le n}\{(r_i,\alpha_i)\}\right)+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\Delta^{-1}\left(\min_{1\le i\le n}\{(r_i,\alpha_i)\}\right)\right)$$
$$\le\Delta\left(\frac{T_1}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r_1,\alpha_1)+\frac{T_2}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r_2,\alpha_2)+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r_n,\alpha_n)\right)$$
$$\le\Delta\left(\frac{T_1}{\sum_{j=1}^{n}T_j}\Delta^{-1}\left(\max_{1\le i\le n}\{(r_i,\alpha_i)\}\right)+\frac{T_2}{\sum_{j=1}^{n}T_j}\Delta^{-1}\left(\max_{1\le i\le n}\{(r_i,\alpha_i)\}\right)+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\Delta^{-1}\left(\max_{1\le i\le n}\{(r_i,\alpha_i)\}\right)\right).$$

That is, $\min_{1\le i\le n}\{(r_i,\alpha_i)\}\le \mathrm{2TPWA}\left((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\right)\le \max_{1\le i\le n}\{(r_i,\alpha_i)\}$.

Theorem 3.2 (Idempotency). Let $\{(r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\}$ be a set of 2-tuples. If all (r_j,α_j) $(j=1,2,\cdots,n)$ are equal, i.e., $(r_j,\alpha_j)=(r,\alpha)$, for all j, then

$$\mathrm{2TPWA}\left((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\right)=(r,\alpha). \quad (8)$$

Proof. If $(r_j,\alpha_j)=(r,\alpha)$, for all j, then we have

$$\begin{aligned}
\text{2TPWA}\left((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\right) &= \Delta\left(\frac{T_1}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r_1,\alpha_1)+\frac{T_2}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r_2,\alpha_2)+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r_n,\alpha_n)\right)\\
&= \Delta\left(\frac{T_1}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r,\alpha)+\frac{T_2}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r,\alpha)+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r,\alpha)\right)\\
&= \Delta\left(\Delta^{-1}(r,\alpha)\left(\frac{T_1}{\sum_{j=1}^{n}T_j}+\frac{T_2}{\sum_{j=1}^{n}T_j}+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\right)\right)\\
&= \Delta\left(\Delta^{-1}(r,\alpha)\right)\\
&= (r,\alpha).
\end{aligned}$$

The proof is completed.

Theorem 3.3 (Monotonicity). Let $\{(r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\}$ and $\{(r_1',\alpha_1'),(r_2',\alpha_2'),\cdots,(r_n',\alpha_n')\}$ be two set of 2-tuples, if $(r_j,\alpha_j)\le(r_j',\alpha_j')$, for all j, then

$$\text{2TPWA}\left((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\right)\le\text{2TPWA}\left((r_1',\alpha_1'),(r_2',\alpha_2'),\cdots,(r_n',\alpha_n')\right). \tag{9}$$

Proof. This proof is analogous to the proof for monotonicity of the prioritized average operator in Ref. [31].

3.2 2-tuple prioritized weighted geometric (2TPWG) operators

Based on the 2TPWA operator and the geometric mean, here we define a 2-tuple prioritized weighted geometric (2TPWG) operators.

Definition 3.2. Let $\{(r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\}$ $\left(r_j\in S,\alpha_j\in[-0.5,0.5),\, j=1,2,\cdots,n\right)$ be a set of 2-tuples, if

$$\begin{aligned}
&\text{2TPWG}\left((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\right)\\
&=\Delta\left(\left(\Delta^{-1}(r_1,\alpha_1)\right)^{T_1/\sum_{j=1}^{n}T_j}\cdot\left(\Delta^{-1}(r_2,\alpha_2)\right)^{T_2/\sum_{j=1}^{n}T_j}\cdot\cdots\cdot\left(\Delta^{-1}(r_n,\alpha_n)\right)^{T_n/\sum_{j=1}^{n}T_j}\right),
\end{aligned} \tag{10}$$

where $T_1=1$ and $T_j=\prod_{k=1}^{j-1}\left(\frac{\Delta^{-1}(r_k,\alpha_k)}{g+1}\right)$ $(j=2,\cdots,n)$, then 2TPWG is called a 2-tuple prioritized weighted geometric (2TPWG) operator.

Similar to Theorems 3.1-3.3, we have the following theorems.

Theorem 3.4 (Boundedness). Let $\{(r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\}$ be a set of 2-tuples, then

$$\min_{1\le i\le n}\{(r_i,\alpha_i)\} \le 2\text{TPWG}((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)) \le \max_{1\le i\le n}\{(r_i,\alpha_i)\}. \tag{11}$$

Theorem 3.5 (Idempotency). Let $\{(r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\}$ be a set of 2-tuples. If all (r_j,α_j) $(j=1,2,\cdots,n)$ are equal, i.e., $(r_j,\alpha_j)=(r,\alpha)$, for all j, then

$$2\text{TPWG}((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n))=(r,\alpha). \tag{12}$$

Theorem 3.6 (Monotonicity). Let $\{(r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\}$ and $\{(r_1',\alpha_1'),(r_2',\alpha_2'),\cdots,(r_n',\alpha_n')\}$ be two set of 2-tuples, if $(r_j,\alpha_j)\le(r_j',\alpha_j')$, for all j, then

$$2\text{TPWG}((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)) \le 2\text{TPWG}((r_1',\alpha_1'),(r_2',\alpha_2'),\cdots,(r_n',\alpha_n')). \tag{13}$$

Lemma 3.1 [47,48]. Let $x_i>0$, $\lambda_i>0$, $i=1,2,\cdots,n$, and $\sum_{i=1}^{n}\lambda_i=1$, then

$$\prod_{i=1}^{n}(x_i)^{\lambda_i} \le \sum_{i=1}^{n}\lambda_i x_i$$

with equality if and only if $x_1=x_2=\cdots=x_n$.

Theorem 3.7. Let $\{(r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)\}$ be a set of 2-tuples, then we have

$$2\text{TPWG}((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)) \le 2\text{TPWA}((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)).$$

Proof. Because $\sum_{j=1}^{n}\left(\frac{T_j}{\sum_{j=1}^{n}T_j}\right)=\frac{\sum_{j=1}^{n}T_j}{\sum_{j=1}^{n}T_j}=1$, by Definition 3.1, Definition 3.2, and Lemma 3.1, we have

$$\begin{aligned}2\text{TPWG}((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)) &= \Delta\left(\left(\Delta^{-1}(r_1,\alpha_1)\right)^{T_1/\sum_{j=1}^{n}T_j}\cdot\left(\Delta^{-1}(r_2,\alpha_2)\right)^{T_2/\sum_{j=1}^{n}T_j}\cdot\cdots\cdot\left(\Delta^{-1}(r_n,\alpha_n)\right)^{T_n/\sum_{j=1}^{n}T_j}\right)\\ &\le \Delta\left(\frac{T_1}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r_1,\alpha_1)+\frac{T_2}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r_2,\alpha_2)+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\Delta^{-1}(r_n,\alpha_n)\right)\\ &= 2\text{TPWA}((r_1,\alpha_1),(r_2,\alpha_2),\cdots,(r_n,\alpha_n)).\end{aligned}$$

The proof is completed.

Theorem 3.7 shows that the values obtained by the 2TPWG operator are not bigger than the ones obtained by the 2TPWA operator.

3.3 An Approach to Multiple Attribute Group Decision Making with 2-tuple Prioritized Aggregation Operators

In this subsection, we develop an approach to a multiple attribute group decision making problem, where the attribute values are represented by linguistic variables and there exists the prioritization relationships over the attributes and decision makers.

A group decision making problem with linguistic preference information in which the attributes and decision makers are in different priority level can be described as follows: Let $X = \{x_1, x_2, \cdots, x_m\}$ be the set of alternatives. Let $C = \{c_1, c_2, \ldots, c_n\}$ be a collection of attributes and that there is a prioritization between the attributes expressed by the linear ordering $c_1 \succ c_2 \succ c_3 \succ \cdots \succ c_n$, indicate attribute c_j has a higher priority than c_k if $j < k$. Let $D = \{d_1, d_2, \cdots, d_l\}$ is the set of decision makers and that there is a prioritization between the decision makers expressed by the linear ordering $d_1 \succ d_2 \succ d_3 \succ \cdots \succ d_l$, indicate decision maker d_p has a higher priority than d_q if $p < q$. For each alternative $x_i \in X$, the decision maker $d_k \in D$ provided his/her preference value $r_{ij}^{(k)}$ with respect to the attribute $c_j \in C$, where $r_{ij}^{(k)} \in S$ takes the form of linguistic variables, then, all the preference values of the alternatives with respect to the attributes consist the linguistic decision matrix $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m\times n}$ $\left(k = 1, 2, \cdots, l\right)$.

To get the best alternative(s), we next present a method based on 2-tuple prioritized aggregation operators for multiple attribute group decision making with linguistic preference information. The proposed method is depicted as follows:

Step 1. Transform the linguistic decision matrix $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m\times n}$ $\left(k = 1, 2, \cdots, l\right)$ into 2-tuple linguistic decision matrix $\overline{R}^{(k)} = \left(\left(r_{ij}^{(k)}, 0\right)\right)_{m\times n}$ $\left(k = 1, 2, \cdots, l\right)$.

Step 2. Calculate the matrices $T^{(p)} = \left(T_{ij}^{(p)}\right)_{m\times n}$, $\left(p = 1, 2, \cdots, l\right)$ based on the following equations:

$$T_{ij}^{(p)} = \prod_{k=1}^{p-1}\left(\frac{\Delta^{-1}\left(r_{ij}^{(k)}, 0\right)}{g}\right), \ p = 2, \cdots, l, \ i = 1, 2, \cdots, m, \ \ j = 1, 2, \cdots, n, \tag{14}$$

$$T_{ij}^{(1)} = 1, \ i = 1, 2, \cdots, m, \ \ j = 1, 2, \cdots, n. \tag{15}$$

Step 3. Utilize the 2TPWA operator:

$$\text{2TPWA}\left(\left(r_{ij}^{(1)},0\right),\left(r_{ij}^{(2)},0\right),\ldots,\left(r_{ij}^{(l)},0\right)\right)=\Delta\left(\frac{T_{ij}^{(1)}}{\sum_{p=1}^{l}T_{ij}^{(p)}}\Delta^{-1}\left(r_{ij}^{(1)},0\right)+\frac{T_{ij}^{(2)}}{\sum_{p=1}^{l}T_{ij}^{(p)}}\Delta^{-1}\left(r_{ij}^{(2)},0\right)+\cdots+\frac{T_{ij}^{(l)}}{\sum_{p=1}^{l}T_{ij}^{(p)}}\Delta^{-1}\left(r_{ij}^{(l)},0\right)\right) \quad (16)$$

or the 2TPWG operator:

$$\text{2TPWG}\left(\left(r_{ij}^{(1)},0\right),\left(r_{ij}^{(2)},0\right),\ldots,\left(r_{ij}^{(l)},0\right)\right)=\Delta\left(\left(\Delta^{-1}\left(r_{ij}^{(1)},0\right)\right)^{T_{ij}^{(1)}/\sum_{p=1}^{l}T_{ij}^{(p)}}\cdot\left(\Delta^{-1}\left(r_{ij}^{(2)},0\right)\right)^{T_{ij}^{(2)}/\sum_{p=1}^{l}T_{ij}^{(p)}}\cdot\cdots\cdot\left(\Delta^{-1}\left(r_{ij}^{(l)},0\right)\right)^{T_{ij}^{(l)}/\sum_{p=1}^{l}T_{ij}^{(p)}}\right) \quad (17)$$

to aggregate all the individual 2-tuple linguistic decision matrices $\overline{R}^{(k)}=\left(\left(r_{ij}^{(k)},0\right)\right)_{m\times n}$ $(k=1,2,\cdots,l)$ into the collective 2-tuple linguistic decision matrix $\overline{R}=\left(\overline{r}_{ij}\right)_{m\times n}=\left(\left(r_{ij},\alpha_{ij}\right)\right)_{m\times n}$.

Step 4. Calculate the matrix $T=\left(T_{ij}\right)_{m\times n}$ based on following equations:

$$T_{ij}=\prod_{k=1}^{j-1}\left(\frac{\Delta^{-1}\left(r_{ik},\alpha_{ik}\right)}{g}\right)\left(i=1,2,\cdots,m,\ j=2,\cdots,n\right), \quad (18)$$

$$T_{i1}=1\ \left(i=1,2,\cdots,m\right). \quad (19)$$

Step 5. Utilize the 2TPWA operator:

$$\begin{aligned}\overline{r}_i&=\left(r_i,\alpha_i\right)=\text{2TPWA}\left(\left(r_{i1},\alpha_{i1}\right),\left(r_{i2},\alpha_{i2}\right),\ldots,\left(r_{in},\alpha_{in}\right)\right)\\&=\Delta\left(\frac{T_{i1}}{\sum_{j=1}^{n}T_{ij}}\Delta^{-1}\left(r_{i1},\alpha_{i1}\right)+\frac{T_{i2}}{\sum_{j=1}^{n}T_{ij}}\Delta^{-1}\left(r_{i2},\alpha_{i2}\right)+\cdots+\frac{T_{in}}{\sum_{j=1}^{n}T_{ij}}\Delta^{-1}\left(r_{in},\alpha_{in}\right)\right)\end{aligned} \quad (20)$$

or the 2TPWG operator:

$$\begin{aligned}\overline{r}_i&=\left(r_i,\alpha_i\right)=\text{2TPWG}\left(\left(r_{i1},\alpha_{i1}\right),\left(r_{i2},\alpha_{i2}\right),\ldots,\left(r_{in},\alpha_{in}\right)\right)\\&=\Delta\left(\left(\Delta^{-1}\left(r_{i1},\alpha_{i1}\right)\right)^{T_{i1}/\sum_{j=1}^{n}T_{ij}}\cdot\left(\Delta^{-1}\left(r_{i2},\alpha_{i2}\right)\right)^{T_{i2}/\sum_{j=1}^{n}T_{ij}}\cdot\cdots\cdot\left(\Delta^{-1}\left(r_{in},\alpha_{in}\right)\right)^{T_{in}/\sum_{j=1}^{n}T_{ij}}\right)\end{aligned} \quad (21)$$

to derive the collective overall preference value $\overline{r}_i=\left(r_i,\alpha_i\right)$ of the alternative x_i.

Step 6. Rank the collective overall preference values $\overline{r}_i=\left(r_i,\alpha_i\right)$ $\left(i=1,2,\cdots,m\right)$ in descending order. Rank all the alternatives x_i $\left(i=1,2,\cdots,m\right)$ and select the best one(s) in accordance with the collective overall preference values $\overline{r}_i=\left(r_i,\alpha_i\right)$ $\left(i=1,2,\cdots,m\right)$.

Step 7. End.

4 An Illustrative Example

In this section, let us consider a numerical example adapted from Herrera et al. [17], and Herrera and Herrera-Viedma [15].

Example 4.1. Suppose that an investment company wants to invest a sum of money in the best option. There is a panel with four possible alternatives in which to invest the money: (1) x_1 is a car industry; (2) x_2 is a food company; (3) x_3 is a computer company; and (4) x_4 is an arms industry. The investment company must make a decision according to the following four attributes: (1) c_1 is the risk analysis; (2) c_2 is the growth analysis; (3) c_3 is the social-political impact analysis; and (4) c_4 is the environmental impact analysis. The four possible alternatives x_i $(i=1,2,3,4)$ are to be evaluated using the linguistic term set

$$S=\left\{\begin{array}{l} s_0=\text{extremely poor},\ s_1=\text{very poor},\ s_2=\text{poor},\ s_3=\text{slightly poor},\ s_4=\text{fair}, \\ s_5=\text{slightly good},\ s_6=\text{good},\ s_7=\text{very good},\ s_8=\text{extremely good} \end{array}\right\}$$

by three decision makers d_k $(k=1,2,3)$ under the above four attributes, and construct, respectively, the decision matrices $R^{(k)}=\left(r_{ij}^{(k)}\right)_{4\times 4}$ $(k=1,2,3)$ as shown in Tables 1-3. The decision maker d_1 has the absolute priority for decision making, the decision maker d_2 comes next. That is, there is a prioritization between three decision makers expressed by the linear ordering $d_1 \succ d_2 \succ d_3$. In three decision makers' opinion, there exists the prioritization relationship among these attributes, for example, the risk analysis of the candidate is the most important, but the environmental impact analysis of the candidate is not so important comparing with other attributes. Therefore, the prioritization relationship can be denoted by: $c_1 \succ c_2 \succ c_3 \succ c_4$.

Table 1. Decision matrix $R^{(1)}$ provided by d_1

1	c_1	c_2	c_3	c_4
x_1	s_4	s_4	s_1	s_5
x_2	s_3	s_6	s_5	s_8
x_3	s_3	s_2	s_7	s_5
x_4	s_8	s_1	s_3	s_6

Table 2. Decision matrix $R^{(2)}$ provided by d_2

2	c_1	c_2	c_3	c_4
x_1	s_5	s_2	s_7	s_3
x_2	s_7	s_4	s_8	s_8
x_3	s_7	s_8	s_6	s_6
x_4	s_8	s_6	s_5	s_2

Table 3. Decision matrix $R^{(3)}$ provided by d_3

3	c_1	c_2	c_3	c_4
x_1	s_2	s_1	s_2	s_8
x_2	s_7	s_8	s_6	s_8
x_3	s_5	s_6	s_4	s_4
x_4	s_6	s_8	s_5	s_7

Step 1. Transform the linguistic decision matrices $R^{(k)}=\left(r_{ij}^{(k)}\right)_{4\times4}$ $(k=1,2,3)$ given in Tables 1-3 into 2-tuple linguistic decision matrices $\overline{R}^{(k)}=\left(\left(r_{ij}^{(k)},0\right)\right)_{4\times4}$ $(k=1,2,3)$ which are given in Tables 4-6.

Table 4. 2-tuple linguistic decision matrix $\overline{R}^{(1)}$

4	c_1	c_2	c_3	c_4
x_1	$(s_4,0)$	$(s_4,0)$	$(s_1,0)$	$(s_5,0)$
x_2	$(s_3,0)$	$(s_6,0)$	$(s_5,0)$	$(s_8,0)$
x_3	$(s_3,0)$	$(s_2,0)$	$(s_7,0)$	$(s_5,0)$
x_4	$(s_8,0)$	$(s_1,0)$	$(s_3,0)$	$(s_6,0)$

Table 5. 2-tuple linguistic decision matrix $\overline{R}^{(2)}$

5	c_1	c_2	c_3	c_4
x_1	$(s_5,0)$	$(s_2,0)$	$(s_7,0)$	$(s_3,0)$
x_2	$(s_7,0)$	$(s_4,0)$	$(s_8,0)$	$(s_8,0)$
x_3	$(s_7,0)$	$(s_8,0)$	$(s_6,0)$	$(s_6,0)$
x_4	$(s_8,0)$	$(s_6,0)$	$(s_5,0)$	$(s_2,0)$

Table 6. 2-tuple linguistic decision matrix $\overline{R}^{(3)}$

6	c_1	c_2	c_3	c_4
x_1	$(s_2,0)$	$(s_1,0)$	$(s_2,0)$	$(s_8,0)$
x_2	$(s_7,0)$	$(s_8,0)$	$(s_6,0)$	$(s_8,0)$
x_3	$(s_5,0)$	$(s_6,0)$	$(s_4,0)$	$(s_4,0)$
x_4	$(s_6,0)$	$(s_8,0)$	$(s_5,0)$	$(s_7,0)$

Step 2. Utilize Eqs. (14) and (15) to calculate the matrices $T^{(1)}$, $T^{(2)}$, and $T^{(3)}$ as follows:

$$T^{(1)} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}, \quad T^{(2)} = \begin{pmatrix} 0.5000 & 0.5000 & 0.1250 & 0.6250 \\ 0.3750 & 0.7500 & 0.6250 & 1.0000 \\ 0.3750 & 0.2500 & 0.8750 & 0.6250 \\ 1.0000 & 0.1250 & 0.3750 & 0.7500 \end{pmatrix},$$

$$T^{(3)} = \begin{pmatrix} 0.3125 & 0.1250 & 0.1094 & 0.2344 \\ 0.3281 & 0.3750 & 0.6250 & 1.0000 \\ 0.3281 & 0.2500 & 0.6563 & 0.4688 \\ 1.0000 & 0.0938 & 0.2344 & 0.1875 \end{pmatrix}.$$

Step 3. Utilize the 2TPWA operator (Eq. (16)) to aggregate all the individual 2-tuple linguistic decision matrices $\bar{R}^{(k)} = \left(\left(r_{ij}^{(k)}, 0\right)\right)_{4\times4}$ $(k = 1,2,3)$ into the collective 2-tuple linguistic decision matrix $\bar{R} = \left(\bar{r}_{ij}\right)_{4\times4} = \left(\left(r_{ij}, \alpha_{ij}\right)\right)_{4\times4}$ (see Table 7).

Table 7. The collective 2-tuple linguistic decision matrix $\bar{R}$.

7	c_1	c_2	c_3	c_4
x_1	$(s_4, -0.0690)$	$(s_3, 0.1538)$	$(s_2, -0.3038)$	$(s_5, -0.2941)$
x_2	$(s_5, -0.3486)$	$(s_6, -0.3529)$	$(s_6, 0.1111)$	$(s_8, 0)$
x_3	$(s_4, 0.2661)$	$(s_4, -0.3333)$	$(s_6, -0.1235)$	$(s_5, 0.0746)$
x_4	$(s_7, 0.3333)$	$(s_2, 0.0513)$	$(s_4, -0.2427)$	$(s_5, -0.4516)$

Step 4. Calculate the matrix $T = \left(T_{ij}\right)_{4\times4}$ based on Eqs. (18) and (19):

$$T = \begin{pmatrix} 1 & 0.4914 & 0.1937 & 0.0411 \\ 1 & 0.5814 & 0.4104 & 0.3135 \\ 1 & 0.5333 & 0.2444 & 0.1795 \\ 1 & 0.9167 & 0.2350 & 0.1104 \end{pmatrix}$$

Step 5. Utilize the 2TPWA operator (Eq. (20)) to derive the collective overall preference value $\bar{r}_i = \left(r_i, \alpha_i\right)$ of the alternative x_i .

$$\bar{r}_1 = \left(s_3, 0.4774\right),\ \bar{r}_2 = \left(s_6, -0.3822\right),\ \bar{r}_3 = \left(s_4, 0.3780\right),\ \bar{r}_4 = \left(s_5, -0.3146\right).$$

Step 6. Rank the collective overall preference values $\bar{r}_i = \left(r_i, \alpha_i\right)$ $(i = 1,2,3,4)$ in descending order.

$$\bar{r}_2 > \bar{r}_4 > \bar{r}_3 > \bar{r}_1 .$$

Because $\bar{r}_2 > \bar{r}_4 > \bar{r}_3 > \bar{r}_1$, we have $x_2 \succ x_4 \succ x_3 \succ x_1$. Therefore, the best candidate is x_2.

If we deal with Example 3.1 using the 2TPWG operator instead of the 2TPWA operator, then the main steps are shown as follows:

Step 1': See Step 1.
Step 2': See Step 2.
Step 3'. Utilize the 2TPWG operator (Eq. (17)) to aggregate all the individual 2-tuple linguistic decision matrices $\bar{R}^{(k)} = \left(\left(r_{ij}^{(k)}, 0\right)\right)_{4\times 4}$ $(k = 1, 2, 3)$ into the collective 2-tuple linguistic decision matrix $\bar{R}' = \left(\bar{r}_{ij}'\right)_{4\times 4} = \left(\left(r_{ij}', \alpha_{ij}'\right)\right)_{4\times 4}$ (see Table 8).

Table 8. The collective 2-tuple linguistic decision matrix $\bar{R}'$

8	c_1	c_2	c_3	c_4
x_1	$(s_4, -0.2252)$	$(s_3, -0.0952)$	$(s_1, 0.2950)$	$(s_4, 0.4682)$
x_2	$(s_4, 0.2564)$	$(s_5, 0.4708)$	$(s_6, -0.0067)$	$(s_8, -0.0000)$
x_3	$(s_4, -0.0108)$	$(s_3, 0.0262)$	$(s_6, -0.2595)$	$(s_5, 0.0224)$
x_4	$(s_7, 0.2685)$	$(s_1, 0.4102)$	$(s_4, -0.3598)$	$(s_4, -0.0195)$

Step 4'. Calculate the matrix $T' = \left(T_{ij}'\right)_{4\times 4}$ based on Eqs. (18) and (19):

$$T' = \begin{pmatrix} 1 & 0.4718 & 0.1713 & 0.0277 \\ 1 & 0.5320 & 0.3638 & 0.2726 \\ 1 & 0.4986 & 0.1886 & 0.1353 \\ 1 & 0.9086 & 0.1602 & 0.0729 \end{pmatrix}.$$

Step 5'. Utilize the 2TPWG operator (Eq. (21)) to derive the collective overall preference value $\bar{r}_i' = (r_i', \alpha_i')$ of the alternative x_i.

$\bar{r}_1' = (s_3, 0.1502)$, $\bar{r}_2' = (s_5, 0.1900)$, $\bar{r}_3' = (s_4, -0.0930)$, $\bar{r}_4' = (s_3, 0.3725)$.

Step 6'. Rank the collective overall preference values $\bar{r}_i' = (r_i', \alpha_i')$ $(i = 1, 2, 3, 4)$ in descending order.

$$\bar{r}_2' > \bar{r}_3' > \bar{r}_4' > \bar{r}_1'.$$

Because $\bar{r}_2' > \bar{r}_3' > \bar{r}_4' > \bar{r}_1'$, we have $x_2 \succ x_3 \succ x_4 \succ x_1$. Therefore, the best candidate is x_2.

Through Example 4.1, we can see that there are different priority levels among four attributes and three decision makers respectively. For instance, if a candidate owns bad morality, then this candidate is impossible to be selected by three decision makers, no matter how well he/she performs on the other three attributes. If a candidate receives a bad evaluation from university president, then he/she is also impossible to be selected no matter how high evaluations he has received from the other two decision makers. Clearly, the existing 2-tuple linguistic aggregation operators are difficult to deal with such cases due to the fact that these operators are usually used to solve MAGDM where the attributes and the decision makers are at the same priority level. However, the proposed operators in this paper not only accommodate the linguistic preference information but also take the prioritization among the attributes and the decision makers into account; thus, our operators and approaches can effectively cope with the situations in which the attributes and the decision makers are at different priority levels.

Recently, Zhou et al. [49] developed some uncertain linguistic prioritized aggregation operators and their application to multiple attribute group decision making. Peng et al. [50] developed several multigranular uncertain linguistic prioritized aggregation operators and their application to multiple criteria group decision making. The main differences between these two papers and our paper is that these two papers deal with MAGDM problems with uncertain linguistic information, while our paper deal with MAGDM problems with 2-tuple linguistic information. As shown in the introduction section, uncertain linguistic model computes with words directly, while the 2-tuple linguistic computational model uses the 2-tuple linguistic representation and computational model to make linguistic computations, which is more reasonable and reliable than uncertain linguistic model in some practical situations [14-17]. As a consequence, the developed 2-tuple prioritized aggregation operators are more reasonable and effective than uncertain linguistic prioritized aggregation operators and multigranular uncertain linguistic prioritized aggregation operators in some practical MAGDM problems.

5 Conclusion

Considering that there may exist a prioritization relationship over the attributes and decision makers in some multiple attribute group decision making problems with linguistic information, this paper provides some 2-tuple prioritized aggregation operators to handle the multiple attribute group decision making problems where there exists a prioritization relationship over the attributes and decision makers. The significant feature of these operators is that they not only deal with the linguistic and interval linguistic information but also take the prioritization relationship among the arguments into account. Furthermore, we apply the proposed operators to solve some multiple attribute group decision making problems and propose an approach to multiple attribute group decision making under linguistic environment in which the attributes and decision makers are in different priority level. Finally, some illustrative examples are employed to show that the proposed approaches are not only more reasonable but more efficient in practical applications due to the fact that these approaches consider the prioritization relationship among the attributes and decision makers. The limitation of this paper is that we only propose the 2TPWA operator and the 2TPWG operator and do not propose the ordered weighted forms of them, such as the 2TPOWA and 2TPOWG operators and the hybrid forms. In the future, we will focus on addressing this issue and extending the prioritized aggregation (PA) operators to the other domains.

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Author's Contribution

'Zhiming Zhang' designed the study, performed the statistical analysis, wrote the protocol, and wrote the overall draft of the manuscript. The author read and approved the final manuscript.

Competing Interests

Author has declared that no competing interests exist.

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