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Fuzzy Goal Programming Model for Optimal Electricity Allocation

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Abstract

In this paper, an optimal electricity allocation model (**OEAM**) was developed for efficient energy consumption. This was done by constructing a weighted maxi sum fuzzy goal programming model where the membership functions (of the objectives) are considered as fuzzy goals and weights are introduced to the over deviational variables. In the achievement function, the weighted deviational variables are minimized. The model optimizes the amount of energy to be delivered to various business units of the XYZ electricity distribution company based on the objective functions and constraints. The results further strengthen our belief that fuzzy goal programming (FGP) models have considerable promise in terms of control, flexibility and real world applicability.

Keywords: Fuzzy sets; fuzzy goal programming; weighted maxi sum.

1. Introduction

Supply of adequate, reliable and economically priced power supply is vital for the socio-economic growth and development of any nation. It has been observed that the gross domestic product (GDP) growth rate of a nation has a direct relationship with the growth in the per capita electricity consumption.

The development of the various sectors of the economy, such as industry and commerce, agriculture, health, education, information, banking, tourism, etc. depends heavily on reliable, adequate and economically priced

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power. However, in rapidly developing economies where power is in short supply with increasing load demand from consumers, and where the load demand becomes in excess of what the generators can cope with, generator operators have adopted methods to save the power plants from damage which has in turn affected the quality of electricity delivered from the generators [1].

In some cases, with the understanding of the system, operator sequential load distribution and load allocation patterns are adopted, but these do not only negate the purpose of providing good uninterrupted quality electricity to consumers, it is also unpredictable and does not allow the recognition of any pattern of allocation for the consumers to plan adequately [1].

An energy allocation model and electricity distribution algorithm that considers the amount of energy received on daily basis for distribution to the various districts of XYZ would facilitate the effective utilization of the scarce resource (electricity) in the XYZ Electricity Distribution Zone which comprises of thirteen (13) business units [2]. The pattern to optimally allocate the available electricity supply to improve the economy of Nigeria is hereby presented.

2. Problem Description

Nigeria like most developing economies has some common factors that militate against improved efficiency in electricity supply. Amongst these are: Weak and ageing infrastructure; inadequate transmission and transformation capacity at 330KV and 132KV voltage levels; inadequate protection systems; inadequate control and communication system; aged and obsolete switchgear and protection equipment; faulty lines and substations; overloaded distribution transformers; insufficient raw materials for Power plants; insufficiency of power supply; lack of proper maintenance culture; effect of vandalization; unplanned Network etc. [1].

The aforementioned factors amongst others, militating against providing good uninterrupted quality electricity to the consumers can be minimized if consistent load distribution and load allocation patterns are adopted in which voltages that reaches the consumers are utilizable and with no brown outs.

The unpredicted electricity distribution in Nigeria has demonstrated a presence of several distribution objectives and continuous shifts in the priorities of these objectives. The problem of increasing energy demand by consumers has increased urgency in building new power plants by Independent Power Producers (IPPs) and the upgrading of the existing power plants by Power Holding Company of Nigeria (PHCN) Plc because of the socio-economic importance of electricity in the country.

The increase in customer population is also critical to the success of PHCN Plc as it has direct relationship with profit generation and energy demand. Another important objective is the satisfaction of it customers by supplying the required voltage. A realization of the multiple natures of the electricity distribution objectives and the historical experiences of continuous shifts in the priorities of these objectives suggest that evaluation and distribution of electricity supply might be conducted on the basis of a multi-objective analytical framework.

3. Review of Optimization Models

The roots of goal programming lie in [3]. Goal programming in fuzzy environment is further developed in [4-7]. Fuzzy sets are mathematical concept proposed in [8].

[9] defined a fuzzy decision as the fuzzy set of alternatives resulting from the intersection of the goals/objectives and constraints. The use of fuzzy set theory in goal programming was first considered in [10]. [11] extended his fuzzy linear programming approach to the multi objective linear programming problem. In the recent past, [6] studied some fuzzy programming models by using the concept of conventional goal programming.

In this paper, an optimal electricity allocation model (**OEAM**) was developed for efficient energy consumption. This was done by constructing a weighted maxi sum fuzzy goal programming model where the membership functions (of the objectives) are considered as fuzzy goals and weights are introduced to the over deviational variables . In the achievement function, the weighted deviational variables are minimized .The model optimizes the amount of energy to be delivered to various business units of the XYZ electricity distribution company based on the objective functions and constraints.

4. Development of Oeam

The OEAM optimizes and selects the appropriate energy distribution option for the zone based on factors such as energy transmitted, energy received, energy billed, minimum energy requirements and revenue generated. The objectives of the model are maximizing energy transmitted and maximizing revenue generated (maximizing profit). The other factors are used as constraints in the model. The mathematical representation of the optimal electricity allocation model (OEAM) is given by the following equations:

$$
\begin{aligned}
&\max : z_1 = \sum_{i=1}^{m} x_i \left(\sum_{j=1}^{n} D_j \right) \qquad (i) \\
&\max : z_2 = \sum_{i=1}^{m} x_i \left(\sum_{j=1}^{n} R_j \right) \qquad (ii) \\
&subject\ to: \\
&\sum_{i=1}^{m} a_i x_i \le b_j , \ j=1 \ to \ n \qquad (iii) \\
&x_i \ \ge \ 0, \ i = 1, 2, 3, ..., m. \qquad (iv) \\
&\sum = summation \text{ sign} \\
&i = 1, 2, 3, ..., m \\
&j = 1, 2, 3, ..., n .
\end{aligned}
\tag{1}
$$

m represents number of business units and n represents number of months.

where in equation (1) , (i) and (ii) are the multiple objective functions, (iii) are the set of constraints and (iv) is the nonnegative condition, where D_j are the energy transmitted to each district, R_j are the revenues generated from customers billed in each months, a_i are the energy billed monthly by various business unit in the different district, b_j are the available energy for transmission to each district monthly and x_i are the monthly district energy allocation to be determined.

When it is difficult to determine the precise goal value of each objective due to incomplete information, the model is then fuzzified. To incorporate uncertainty and imprecision in the model formulation, the fuzzy set theory, initially proposed in [8] was introduced in the decision making problems, where aspiration levels of objectives are assigned in an imprecise manner. According to the fuzzy set-based theory, the imprecise objectives and limitations are represented by the associated membership functions presented in [9].

Now, let the aspiration levels be g_1 *and* g_2 for energy and revenue maximizations respectively. Let t_1^u *and* t_2^u denote the tolerance ranges (subjectively chosen constants of admissible violations) for the respective aspiration levels.

Assigning aspiration levels g_1 *and* g_2 to energy and revenue objectives respectively, the multi-objective linear programming model of equation 1 transformed into equation 2, which is a goal programming problem.

4.1 Introducing Aspiration Levels to Energy and Revenue Objectives

$$
\begin{aligned}
&Find\ X = (x_1, x_2, x_3, ..., x_m)\\
&so\ as\ to\ satisfy:\\
&\sum_{i=1}^{m} x_i \left(\sum_{j=1}^{n} D_j\right) \le g_1\\
&\sum_{i=1}^{m} x_i \left(\sum_{j=1}^{n} R_j\right) \le g_2\\
&subject\ to:\\
&\sum_{i=1}^{m} a_i x_i \le b_j,\ j=1,2,3,...,n\\
&x_i \ge 0,\ i = 1, 2, 3, ..., m.
\end{aligned}
\tag{2}
$$

Let t_1^u *and* t_2^u be the tolerance limits for the achievements of the desired aspiration levels g_1 *and* g_2 of the goals in (1) then the corresponding membership functions take the form:

$$
\mu(z_k(x)) = \begin{cases} 1 & , if\ z_k(x) \le g_k, \\ \dfrac{(g_k + t_k^u) - z_k(x)}{t_k^u} & , if\ g_k < z_k(x) \le g_k + t_k^u, \\ 0 & , if\ z_k(x) > g_k + t_k^u. \end{cases}
\tag{3}
$$

$$k = 1\ to\ 2.$$

The highest value of a membership function is unity:

$$
\begin{aligned}
&\mu(z_k(x)) + d_k^- - d_k^+ = 1\\
&d_k^-, d_k^+ \ge 0,\ d_k^- . d_k^+ = 0,\ k = 1, 2.
\end{aligned}
\tag{4}
$$

for the membership objective goal of the '$\ge$ *type and* $\le$ *type*',

This transforms the goal program into fuzzy goal programming problem.

where d_k^- are the amounts by which the energy and revenue objectives are underachieved and d_k^+ are the amounts by which the energy and revenue objectives are overachieved , of the kth membership goals. The deviational variables can either be zero or positive in the first condition and in the second condition, they cannot be basic variables simultaneously since they are dependent.

4.2 Weighted Maxi Sum Fuzzy Goal Programming (WMFGP) Model Transformation

The overachieved energy and revenue goals are minimized in the objective function with weights attached. The determination of these weights is subjective, here the respective reciprocal of each tolerance limits for the goals were utilized.

From (1), (2), (3) and (4) the WMFGP model can be expressed as:

Find X so as to Minimize $Z = \sum_{k=1}^{2} w_k d_k^+$

subject to:

$$\frac{(g_k + t_k^u) - z_k(x)}{t_k^u} + d_k^- - d_k^+ = 1$$

$$\sum_{i=1}^{m} a_i x_i \le b_j, \; j = 1, 2, 3, ..., n \tag{5}$$

$$x_i, d_k^-, d_k^+ \ge 0; \; d_k^- . d_k^+ = 0; \; w_k = \frac{1}{t_k^u}; \; k = 1, 2; \; i = 1, 2, 3, ..., m.$$

The goal,

$$\frac{(g_k + t_k^u) - z_k(x)}{t_k^u} + d_k^- - d_k^+ = 1 \tag{6}$$

Can be transformed into

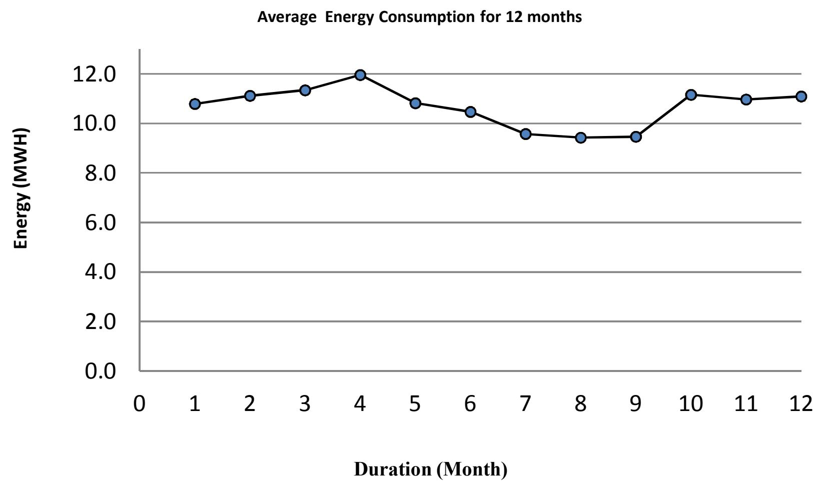
$$z_k(x) = g_k + t_k^u d_k^- - t_k^u d_k^+ \tag{7}$$

The weighted maxi sum fuzzy goal programming model provides more control by assigning weights to variables thereby being more flexible in real life application than simple additive fuzzy goal programming models [12].

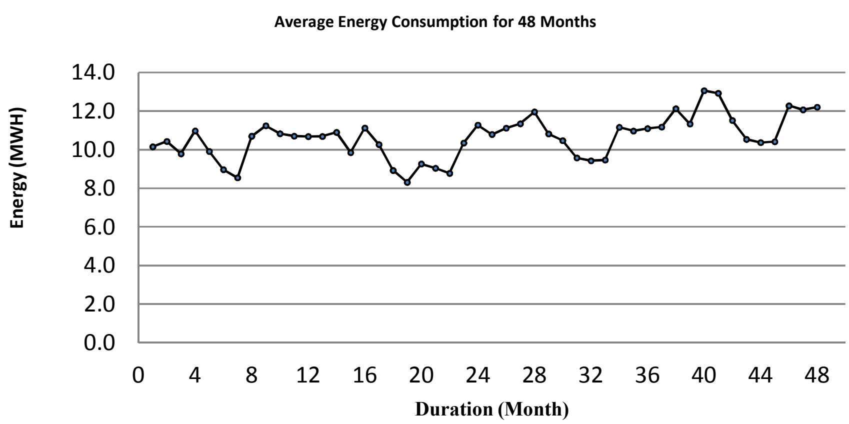
5. Data Analysis

The study in this paper was conducted using data collected from XYZ Electricity Distribution Company. For the purpose of this analysis, we recorded the energies required by all the thirteen business districts of the electricity distribution company, each for the duration of twelve (12) and forty eight (48) months. The analyzed data shows that the monthly average energy requirement of each district varies throughout the durations of both 12 and 48 months. Thus, the data suggests that the average monthly energy requirement for the entire districts is non-stationary for the durations of 12 and 48 months as depicted in Fig. 1(a) and Fig. 1(b) respectively.

Fig. 1(a) shows the monthly average amount of energy consumed per hour by the districts. For the 12 months period, the amount of energy consumed is within the range of 9 mega watts per hour to 12 mega watts per hour. Furthermore, the range of consumption of energy extends from 8 mega watts per hour to 13 mega watts per hour during the period of 48 months, the result of which is shown in Fig. 1(b).



(a) This is the profile of average energy consumed for the period of 12 months



(b) This is the profile of average energy consumed for the period of 48 months

Fig. 1. Average energy consumption for all districts

Tables 1, 2 and 3 contain data for developing fuzzy goals for the proposed model. In the model, the electricity company has approximately 1, 313, 385, 935kwh of electricity in the period of 12 months and 4, 990, 866, 553kwh for 48 months. Tables 4 and 5 show the approximate amount of energy required by each district per month for both 12 and 48 months durations, respectively. In addition, the data show a non-stationary behavior as a result of the different energy requirement of the individual districts, the result of which is also shown in Fig. 2a and 2b for 12 and 48 months durations respectively. Although, there is a little variation between the data collected for 12 and 48 months, it shows a positive linear non-stationary pattern as shown in Fig. 3, and thus a linear non-stationary time series.

Table 1. Data description for the Fuzzy goals for 12 months

Goal	Aspiration/desired level	Tolerance limit	
		Upper	Lower
1.Total energy	1,313,385,935 kwh		1,000,000,000 kwh
2. Revenue generated	₦ 7,550,184,339.25 k		>=₦ 7,000,000,000

These are values of the aspiration levels for the two objectives of energy and revenue, and their respective acceptable minimum values for the period of 12 months

Table 2. Data description for the Fuzzy goals for 48 months

Goal	Aspiration/desired level	Tolerance limit	
		Upper	Lower
1.Total energy	4,990,866,553kwh		4,000,000,000 kwh
2. Revenue generated	₦ 28,690,700,489.15k		>=₦ 28,000,000,000

These are values of the aspiration levels for the two objectives of energy and revenue, and their respective acceptable minimum values for the period of 48 months

Table 3. Objectives data

Districts	Energy in KWH/HR	Rev.₦/HR.
X1	9773	68480
X2	13720	108445
X3	20381	101863
X4	7042	39498
X5	8085	49849
X6	15282	112821
X7	21495	88157
X8	4335	21877
X9	3937	19449
X10	16065	96950
X11	10658	68627
X12	9600	60523
X13	7671	37426

These are the unit returns of energy in kilowatts per hour and revenue generated per hour of energy consumed

The non-stationary nature of all the analyzed data proved the presence of uncertainty and imprecision, and hence the use of fuzzy goal programming model.

6. Application

The study used the analyzed data collected from XYZ electricity distribution company and PQR transmission station. The model developed is applied to the case developed based on the data. A revenue generation of ₦7, 550, 184, 339.25 and ₦28, 690, 700, 489.15 k for 12 months and 24 months respectively was achieved by billing customer populations. It is implied that XYZ electricity distribution company would like to maximize the total energy transmitted/received and the revenue generated. The objective is to develop a model for the optimal allocation of the energy transmitted to various business units such that the energy and revenue generated are maximized as well as the transmission voltage and the customer populations billed are satisfied.

7. Results and Discussion

The allocation model formulated has been demonstrated using weighted maxi sum fuzzy goal programming (WMFGP) model. The LINGO (Version 12) software was used to run the model. The total values of goals for the model are: For 12 months -Goal 1(amount of energy)-1, 313, 385, 935kwh, Goal 2 (revenue generated) -₦7, 550, 184, 339.25k. For 48 months -Goal 1(amount of energy) - 4, 990, 866, 553kwh; Goal 2(revenue generated) - ₦28, 690, 700, 489.15k.

For the period of 12 months, goal 1 was underachieved by 1.311178 kwh of energy while goal 2 was overachieved by ₦0.74k. This implies that goal 1 will be achieved at 1, 313, 385, 933.69 kwh and goal 2 at ₦7, 550, 184, 339.99k. These results are acceptable because the objective function minimized gave a value of 0.0 which is a Global optimum solution and the deviations from the targeted goals are not significant.

For the period of 48 months, goal 1 was underachieved by 1.247200 kwh of energy and goal 2 was underachieved by ₦0.60k. This implies that goal 1 will be achieved at 4, 990, 866, 551.75kwh and goal 2 at ₦28, 690, 700, 488.55k. These results are acceptable because the objective function minimized gave a value of 0.0 which is a Global optimum solution; infeasibility zero and the deviations from the targeted goals are not significant.

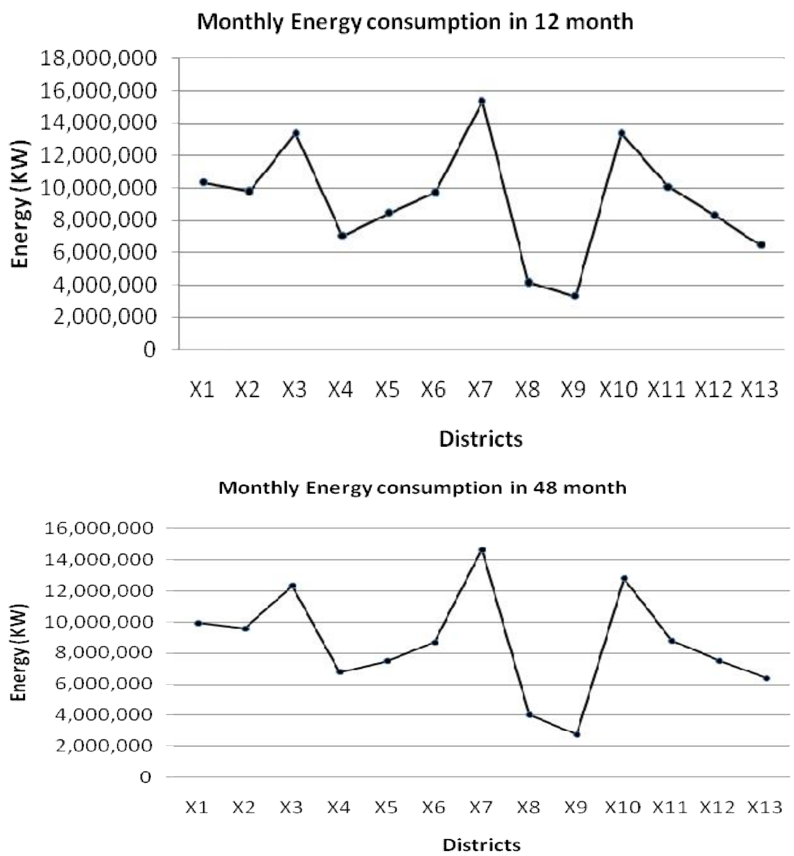
Table 4. Results of model (KWh) for 12 months

Districts	Energy/month
X1	10,354,320
X2	9,819,360
X3	13,381,200
X4	7,053,120
X5	8,488,080
X6	9,740,160
X7	15,390,000
X8	4,178,880
X9	3,307,680
X10	13,386,240
X11	10,087,200
X12	8,346,240
X13	6,467,040

Table 5. Results of model (KWh) for 48 months

Districts	Energy /month
X1	9,954,000
X2	9,583,200
X3	12,350,880
X4	6,797,520
X5	7,498,080
X6	8,699,760
X7	14,675,760
X8	4,047,120
X9	2,736,000
X10	12,823,920
X11	8,826,480
X12	7,522,560
X13	6,395,760

Tables 4 and 5 are the results obtained for the energy values for both 12 months and 48 months respectively.



(a) (b)

(a) and (b) are the profiles of monthly energy consumed for 12 and 48 months respectively

Fig. 2. Monthly energy consumption per district

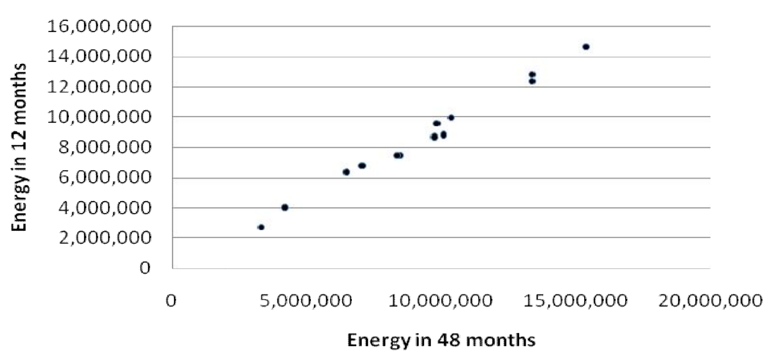


Fig. 3. Positive linear correlation of energy consumption between 12 and 48 months

This depicts the relationship between the values obtained from the model for different set of periods which is a positive correlation.

8. Future Direction

The future direction of the paper is to maintain an equitable distribution of the scarce electric energy supplied for the thirteen districts of the XYZ electricity distribution company.

9. Primary Source

XYZ electricity distribution zone and PQR transmission station.

10. Conclusion

This paper demonstrates how weighted maxi sum fuzzy goal programming (WMFGP) model can be used efficiently and effectively to develop more flexible and realistic model for providing solutions to the electricity company energy allocation problems. The result of energy fall short by $9.98x10^{-8}$% and that of revenue increase by $9.80x10^{-9}$% for the period of 12 months; while in the period of 48 months; the result of energy fall short by $2.50x10^{-8}$% and fall short by $2.23x10^{-9}$%. These results further prove that FGP models have considerable promise in terms of control, flexibility and real world applicability. While XYZ Electricity Distribution Company has been selected for demonstrating the application of this model, the model is flexible enough to be extended to handle larger sizes electricity allocation problems.

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Competing Interests

Author has declared that no competing interests exist.

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