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Distribution Functions and PIC Electrostatic Simulation for Three-Component Dusty Plasma

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Authors' contributions

This work was carried out in collaboration between all authors. Author NAH designed the study, Author AHA and EGS managed the analyses of the study. Author EGS wrote the protocol and wrote the first draft of the manuscript, managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

Due to the complexity of plasma dynamics and kinetics, it is often used in research model plasma through computer simulations. A new approach to the electrostatic simulation of dilute plasma, based on the Particle-in-Cell (PIC) simulation method, is explored. We discuss the distribution functions and simulation for three-component dilute plasma e-p-i (electrons, positrons, and ions) model which is homogeneous and in equilibrium. Finally, our code is successfully against with three-component dilute plasma distribution function.

Keywords: Dusty plasma; PIC simulation; maxwellian distribution; e-p-i model..

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1 INTRODUCTION

A dusty plasma is an ionized gas containing dust particles. The dust particles have sizes ranging from tens of nanometers to hundreds of microns. Ferdousi et al. (2015) proposed three-component unmagnetized plasma system consisting of electrons, positrons, and ions [1]. The history of dusty plasmas is quite old (Mendis 1997 [2]) and the importance of e-p-i or three component plasma study is due to develop an understanding of the behavior of both astrophysical and laboratory plasma. Because of the long lifetime of the positrons, most of the astrophysical and laboratory plasmas become an admixture of electrons, positrons, and ions. Therefore, the study of electron–positron–ion (e-p-i) plasmas is important to understand the behavior of both astrophysical [3], [4] and laboratory plasmas [5].

The three component dusty plasma which is contain non-inertial nonextensive electrons, nonextensive positrons, and inertial ions have been studied by several authors during the past many years [6], [7], [8] and [9]. The study of plasma thermodynamics is one of the attractive problems in the theoretical physics and the thermodynamics in curved spacetime is very important especially in the context of astrophysics and cosmology. Relativistic plasmas are objects encountered in many astrophysical situations. For instance, they occur in the magnetosphere of pulsars where they are strongly magnetized, or in the quasar jets [10]. Such plasmas may be created either by heating a gas to very high temperatures. In the relativistic plasma the relativistic corrections to a particle's mass and velocity are important. Such corrections typically become important when a significant number of electrons reach speeds greater than 0.86c.

Modeling of plasmas is complicated by the presence of external and self-induced electromagnetic fields, inter-particle interactions, the presence of solid objects, and the different characteristic time scales at which ions and electrons propagate.

The particle in cell technique refers to a technique used to simulate the motion of charged particles, or plasma [11]. As the name implies, the particle-in-cell (PIC) technique represents matter as discrete particles (or "macroparticle" ensembles) occupying positions within a lattice of "cells." Time advances stepwise, and during each timestep, the electromagnetic field is interpolated from the cell vertices to the particles' positions, the force on the particles is calculated, their velocity is updated accordingly, and the current produced by moving charges are distributed to the cell vertices in a self-consistent manner.

There are various PIC implementations available, with one of the primary variable elements being the explicit inclusion or omission of binary interactions (particle collisions, such as those mediated by the Coulomb force between point charges). Astrophysical plasmas are typical of such a low density that collisions are exceedingly unlikely, so PIC solutions for these collisionless plasmas can safely assume that only collective effects play a significant role in the equations of motion for individual particles. This paper presents an overview of the PIC technique and discuss the distribution functions and simulations of three-component e-p-i plasma.

2 PHASE SPACE DISTRIBUTION: MAXWELLIAN DISTRIBUTIONS

The basic technique of the PIC model is to represent the phase space distribution as a collection of macroparticles, each representing a given number of physical plasma particles. The distribution function F^N of a macroscopic system of N particles is in a complete though in an intractably complex in $6N$dimensional phase space, must be spanned by the coordinates and velocities of all individual particles $F^N(t, x_A, u_A), A = 1, 2, \ldots, N, x_A$ being the position of particle A, and $u_A = v_A/\sqrt{1-(v_A/c)^2}$ the spatial components of its four-velocity.

At equilibrium, the charge-neutrality condition demands $n_{e0} = Z_i n_{i0} + n_{p0}$, where n_{s0} is the number density of sth species ($s = e$ for electrons, p for positrons, and i for ions) and Z_i is the charge state of ions.

The electrostatic oscillations of the electrons, ions or dust particles, which are due to the internal space charge field are described by the continuity equation

$$\frac{\partial n_s}{\partial t} + \nabla.(n_s v_s) = 0 \tag{2.1}$$

the momentum equation

$$\frac{\partial v_s}{\partial t} + (v_s.\nabla)v_s = -\frac{q_s}{m_s}\nabla\phi \tag{2.2}$$

and Poisson's equation

$$\nabla^2\phi = -4\pi\sum_s q_s n_s \tag{2.3}$$

where v_s is the velocity and ($E = -\nabla\phi$) is the electrostatic field with the induced electrostatic potential ϕ.

We can define the $s-$particle reduced distribution as:

$$F^{(s)}(t, x_A, u_A) = \int F^N \prod_{R=s+1}^{N} d^3x_R d^3u_R \tag{2.4}$$

Of the important equations in statistical mechanics describing time evolution of the distribution function of plasma consisting of charged particles with long-range (for example, Coulomb) interaction is the Vlasov equation[12]:

$$\frac{\partial F^{(s)}}{\partial t} + u_A\frac{\partial F^{(s)}}{\partial x_A} + \frac{q_s E}{m_s}\frac{\partial F^{(s)}}{\partial u_A} = 0, \tag{2.5}$$

where q_s is the charge and m_s is the mass of the species, respectively. The Poisson's equation for the scalar potential is given by:

$$\epsilon_0\frac{\partial^2\phi}{\partial x^2} = -\rho \tag{2.6}$$

where we can compute the net charge density from the distribution functions as:

$$\rho(x,t) = \sum_s q_s \int F^{(s)}(t, x_A, u_A)du_A \tag{2.7}$$

The basis of the statistical description of collective processes in a plasma is the BBGKY hierarchy. Now for N electrons, ions or dust particles moving in the Hamiltonian:

$$H = \sum_{i=1}^{N}\frac{p_i^2}{2m} + \sum_{i=1}^{N}U_i + \sum_{i<j}^{N}v_{ij} \tag{2.8}$$

Swithing back to varibles (r,p)), Then Liouville's equation is given by:

$$\frac{\partial g}{\partial t} = \sum_{i=1}^{N}(-\overrightarrow{F_i} - \sum_{i=1,i\neq j}^{N}\overrightarrow{K_{ij}}).\nabla_{p_i}g - \frac{\overrightarrow{p_i}}{m}.\nabla_{r_i}g \tag{2.9}$$

where $F = -\nabla U$, $K_{ij} = \nabla_{r_i}v_{ij}$ and g is refer to the distribution function for N particle phase space.

And the BBGKY equation can be define as

$$\begin{aligned} &\int(\frac{\partial}{\partial t} + h_n(z_1, z_2, ..., z_n))f_n(z_1, z_2, ..., z_n)dz_n \\ = \quad &-\int\sum_{i=1}^{N}\overrightarrow{K_{i,n+1}}.\nabla_{p_i}f_{n+1}(z_1, z_2, ..., z_n)dz_{n+1} \end{aligned} \tag{2.10}$$

where $z = (r, p)$ and the differential operator h_N is defind as

$$h_N(r_1,p_1,r_2,p_2,...,r_N,p_N) = \sum_{i=1}^{N}[\frac{\overrightarrow{p_i}}{m}.\nabla_{r_i} + \overrightarrow{F_i}.\nabla_{p_i}] + \frac{1}{2}\sum_{i,j=1}^{N}\overrightarrow{K_{ij}}\,.\,(\nabla_{p_i} - \nabla_{pj}) \quad (2.11)$$

The equations (2.10) are known as Bogoliubov, Born, Green, Kirkwood and Yvon (BBGKY) hierarchy [10]. For n=1 the first equation of BBGKY is

$$(\frac{\partial}{\partial t} + \frac{\overrightarrow{p_1}}{m}.\nabla_{r_1} + \overrightarrow{F_1}.\nabla_{p1})f_1(z_1,t) = -\int \overrightarrow{K_{12}}.\nabla_{p_1} f_2(z_1,z_2,t)dz_2 \quad (2.12)$$

The second equation for n=2 has a different structure

$$(\frac{\partial}{\partial t} + \frac{\overrightarrow{p_1}}{m}.\nabla_{r_1} + \overrightarrow{F_1}.\nabla_{p_1} + \frac{\overrightarrow{p_2}}{m}.\nabla_{r_2} + \overrightarrow{F_2}.\nabla_{p_2} + \frac{1}{2}\overrightarrow{K_{12}}.(\nabla_{p_1} - \nabla_{p_2}))f_2(z_1,z_2 t) = -\int(\overrightarrow{K_{13}}.\nabla_{p_1} + \overrightarrow{K_{23}}.\nabla_{p_2})f_3(z_1,z_2,z_3,t)dz_3 \quad (2.13)$$

For equilibrium plasma in homogeneous case the one-particle distribution function f_1 must set for the relativistic Maxwellian distribution [13] as

$$f_1(v_A) = \frac{\mu_A}{4\pi(mc^3)K_2(\mu_A)}\exp(\frac{-\mu_A}{\sqrt{1-(v_A/c)^2}} - q_A\phi(x_A)) \quad (2.14)$$

Where $K_2(\mu)$ denotes the modified Bessel function.

3 FUNDAMENTAL EQUATIONS OF PARTICLE-IN-CELL TECHNIQUE

Particle-In-Cell (PIC) is a technique used to simulate motion of charged particles, or plasma in this paper we use PIC technique for plasma in dilute relativistic case.

In a system in which the collective effects of long-range Coulomb interactions between plasma particles dominate the binary collision process, the evolution of the phase space distribution function $F^{(s)}(t,x_A,u_A)$.The PIC method is obtained by assuming that the distribution function of each species is given by the superposition of several elements (called computational particles or superparticles):

$$F^{(s)}(t,x_A,u_A) = \sum_p F^{(p)}(t,x_A,u_A) \quad (3.1)$$

Each element represents a large number of physical particles that are near each other in the phase space. For simplicity, we'll make the following assignments; $\Delta x = \Delta t$ =1, $q_e/m_e = -1$ for electrons, $q_p/m_p = 1$ for positrons and $q_i/m_i = 1/25$ for ions. (The ions are actually much heavier). Also we will take $\omega_p << 1$, $q_e/\epsilon_0 < 1$ and $v_{the} = 1$. The particle mover can be describe by

$$\frac{x_{k+1} - x_k}{\Delta t} = v_{k+1/2} \tag{3.2}$$

and

$$\frac{v_{k+1/2} - v_{k-1/2}}{\Delta t} = \frac{q}{m} E_k \tag{3.3}$$

The leap-frog scheme is an explicit solver, i.e. it depends on old forces from the previous time step k. Contrary to implicit schemes, when for calculation of particle velocity a new filed (at time step $k+1$) is used, explicit solvers are simpler and faster, but their stability requires a smaller time step Δt. where the subscript k refers to "old" quantities from the previous time step, k+1 to updated quantities from the next time step (i.e. $t_{k+1} = t_k + \Delta t$), and velocities are calculated in-between the usual time steps t_k.

The particle-in-cell simulation was implemented using the Mathematica 7 Program.

While there are numerous models for solving the Vlasov-Maxwell equations (see for example Refs. [4-6]), the most common approach is known as the Particle-in-Cell (PIC) method.

4 RESULTS

In our study, we used a dusty plasma e-p-i model which is homogeneous and in equilibrium.The simulation results obtained are illustrated in Figs (1-5). The results from the simulation show that the behaviors of ion density and plasma potential are in close correlation throughout the simulation process.

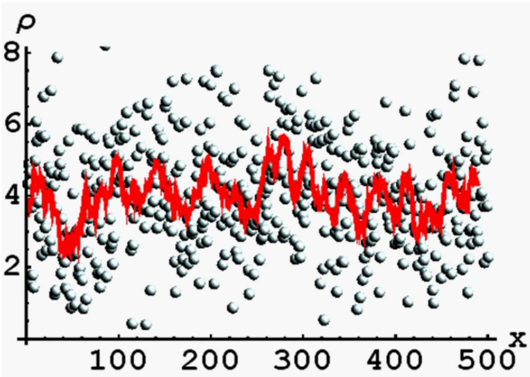

Fig. 1. The electrons density we use moving average to map the electrons's position to the charge density at each point for n$_e = 2000$ electrons

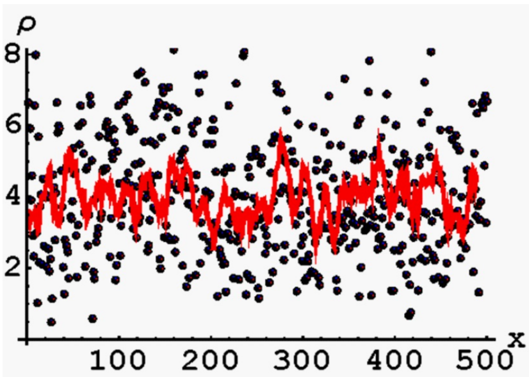

Fig. 2. The positrons density we use moving average to map the positrons's position (in-between the grid points) to the charge density at each point for n$_p = 2000$ positrons

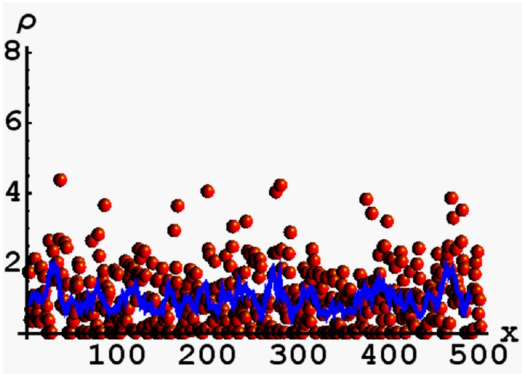

Fig. 3. The ions density we use moving average to map the ions's position to the charge density at each point for n$_i = 500$ ions

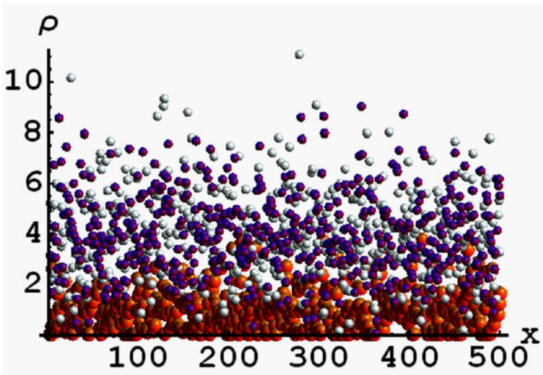

Fig. 4. The three-component dilute plasma density for $n_e = n_p = 2000$ and $n_i = 500$ ions, we chose the silver color of the electrons, the violet color of the positrons and the orange color of the ions

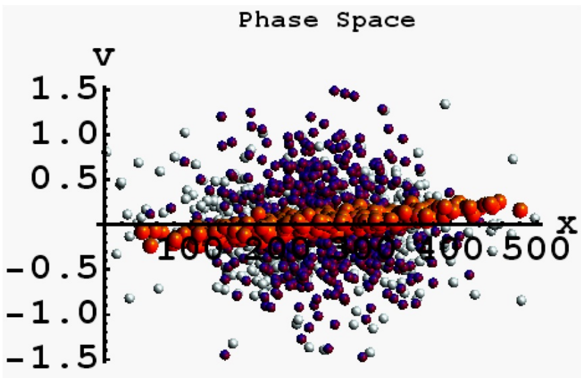

Fig. 5. The phase space distribution of three-component dilute relativistic plasma, we chose the silver color of the electrons, the violet color of the positrons and the orange color of the ions

5 DISCUSSION

We note form Fig. 3 and Fig. 5 that the ions are heavier and slower in motion than the electrons and positrons. In our simulation model, we chose the silver color of the electrons, the violet color of the positrons and the orange color of the ions. It is clear to us from Fig. 5 that the velocities of ions express very slowly compared to the velocities of electrons and positrons.

6 CONCLUSIONS

This study aims to simulate three-component dilute plasma by using PIC Code. There are several techniques in research to the model plasma through computer simulations, but one popular one is Particle-in-cell (PIC) simulations. In this work, we worked to develop the code to allow studying the dusty plasma containing the ions in addition to electrons and positrons. The forces acting on the particles in a classical PIC scheme correspond to macro fields, so that the simulated plasma is assumed to be collisionless.

A computer program has been written to simulate plasmas in the electrostatic limit using a particle in cell (PIC) method. The validity of the program has been tested through the study of three-component dilute plasma. This research work gives an overview of the particle in cell as used for the simulation of electrostatic plasma.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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