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Research Article

Current-Component Independent Transition Form Factors for Semileptonic and Rare $D \longrightarrow \pi(K)$ Decays in the Light-Front Quark Model

Ho-Meoyng Choi

Department of Physics, Teachers College, Kyungpook National University, Daegu, Republic of Korea 702-701

Correspondence should be addressed to Ho-Meoyng Choi; homyoung@knu.ac.kr

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We investigate the exclusive semileptonic and rare $D \longrightarrow \pi(K)$ decays within the standard model together with the light-front quark model (LFQM) constrained by the variational principle for the QCD-motivated effective Hamiltonian. The form factors are obtained in the $q^+ = 0$ frame and then analytically continue to the physical timelike region. Together with our recent analysis of the current-component independent form factors $f_\pm(q^2)$ for the semileptonic decays, we present the current-component independent tensor form factor $f_T(q^2)$ for the rare decays to make the complete set of hadronic matrix elements regulating the semileptonic and rare $D \longrightarrow \pi(K)$ decays in our LFQM. The tensor form factor $f_T(q^2)$ are obtained from two independent sets $(J_T^{+\perp}, J_T^{+-})$ of the tensor current $J_T^{\mu\nu}$. As in our recent analysis of $f_-(q^2)$, we show that $f_T(q^2)$ obtained from the two different sets of the current components gives the identical result in the valence region of the $q^+ = 0$ frame without involving the explicit zero modes and the instantaneous contributions. The implications of the zero modes and the instantaneous contributions are also discussed in comparison between the manifestly covariant model and the standard LFQM. In our numerical calculations, we obtain the q^2-dependent form factors $(f_\pm, f_T)$ for $D \longrightarrow \pi(K)$ and branching ratios for the semileptonic $D \longrightarrow \pi(K)\ell\nu_\ell(\ell = e, \mu)$ decays. Our results show in good agreement with the available experimental data as well as other theoretical model predictions.

1. Introduction

The three flavors of charged leptons, (e, μ, τ), are the same in many respects. In the standard model (SM), the couplings of leptons to gauge bosons are supposed to be independent of lepton flavors, which is known as lepton flavor universality (LFU) [1]. The experimental tests of LFU in various semileptonic B decays have been reported [2–6] by measuring the ratios of branching fractions $\mathscr{R}_{D^{(*)}} = \mathrm{Br}(B \longrightarrow D^{(*)}\tau\nu_\tau)/\mathrm{Br}(B \longrightarrow D^{(*)}\ell\nu_\ell)(\ell = e, \mu)$. Currently, the SM prediction is roughly three standard deviations away from the global average of results from the *BABAR*, Belle, and LHCb experiments. Many theoretical efforts have been made in resolving the issue of $\mathscr{R}_{D^{(*)}}$ anomaly and searching for new physics beyond the SM [7–10]. In view of this, tests of LFU in D decays are also intriguing complementary endeavors.

Exclusive semileptonic and rare D decays provide rigorous tests of the SM in the charm sector including not only the LFU but also the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [11, 12], which describe the mixings among the quark flavors in the weak decays and hold the key to the CP violation issues in the quark sector. Compared to the semileptonic $D \longrightarrow \pi(K)\ell\nu(\ell = e, \mu)$ decays induced by flavor-changing charged current, the rare $D \longrightarrow \pi(K)\ell\ell$ decays are induced by the flavor-changing neutral current (FCNC). Since the rare decays are loop-suppressed in the SM as they proceed through FCNC, they are also pertinent to test the SM and search for physics beyond the SM. Recent BES III measurements [13–20] for many exclusive semileptonic charm decays also allow one to test the SM in the charm sector more precisely.

While the experimental measurements of exclusive decays are much easier than those of inclusive ones, the theoretical knowledge of exclusive decays is sophisticated essentially due to the hadronic form factors entered in the long-distance nonperturbative contributions. Along with new particle effects beyond the SM, which may amend the Wilson coefficients of the effective weak Hamiltonian that describes physics below the electroweak scale, the reliable and precise calculations of the hadronic form factors are very important to constrain the SM and search for new physics effects beyond the SM.

The calculations of hadronic form factors for semileptonic and rare D decays have been made by various theoretical approaches, such as lattice QCD (LQCD) [21–24], QCD sum rules [25, 26], QCD light-cone sum rules [27–29], symmetry-preserving continuum approach to the SM strong-interaction bound-state problem [30], quark potential model [31–33], relativistic quark model(RQM) based on the quasipotential approach [34], covariant confining quark model (CCQM) [35], chiral quark model [36], and constituent quark model [37].

Perhaps, one of the most apt formulations for the analysis of exclusive processes involving hadrons may be provided in the framework of light-front (LF) quantization [38]. The semileptonic and rare D decays have also been analyzed by the light-front quark model (LFQM) [39–47] based on the LF quantization.

In the standard LFQM that we use in this work, the constituent quark and antiquark in a bound state are required to be on-mass shells, and the spin-orbit wave function (WF) is obtained by the interaction-independent Melosh transformation [48] from the ordinary equal-time static spin-orbit WF assigned by the quantum number J^{PC}. For the radial part, we use the phenomenologically accessible Gaussian WF $\phi(x, \mathbf{k}_\perp)$. Since the standard LFQM itself is not amenable to pin down the zero modes, the exactly solvable manifestly covariant Bethe-Salpeter (BS) model with the simple multipole type $q\bar{q}$ vertex was utilized [45, 46, 49, 50] to help identify the zero modes in a systematic way. On the other hand, this BS model is less realistic than the standard LFQM. Thus, as an attempt to apply the zero modes found in the BS model to the standard LFQM, the effective replacement [45, 46, 49] of the LF vertex function $\chi(x, \mathbf{k}_\perp)$ obtained in the BS model with the more realistic Gaussian WF $\phi(x, \mathbf{k}_\perp)$ in the standard LFQM has been made.

However, we found [51–53] that the correspondence relation between χ and ϕ proposed in [45, 46, 49] encounters the self-consistency problem, e.g., the vector meson decay constant obtained in the standard LFQM was found to be different for different sets of the LF current components and polarization states of the vector meson [51]. We also resolved [51–53] this self-consistency problem by imposing the on-mass shell condition of the constituent quark and antiquark in addition to the original correspondence relation between χ and ϕ. Specifically, our new finding for the constraint of the on-mass shell condition corresponds to the replacement of physical meson mass M with the invariant mass M_0 in the calculation of the matrix element. The remarkable feature of our new additional correspondence relation ($M \longrightarrow M_0$) between the two models in the calculations of the two-point functions such as the weak decay constants and the distribution amplitudes of mesons [51–53] was that the LF treacherous points such as the zero modes and the off-mass shell instantaneous contributions appeared in the BS model are absent in the standard LFQM. This prescription ($M \longrightarrow M_0$) can be regarded as an effective inclusion of the zero modes in the valence region of the LF calculations.

As an extension our analysis of the two-point functions [51–53] to the three-point ones, in our very recent LFQM analysis [54] of the semileptonic $B \longrightarrow D\ell\nu_\ell$ decays, we presented the self-consistent descriptions of the weak transition form factors (TFFs) $f_+(q^2)$ and $f_-(q^2)$. Especially, $f_-(q^2)$ should be obtained by using least two components of the weak vector current J^μ_V while $f_+(q^2)$ can be obtained from the single and "good" component (J^+_V) of the current. Because of this, $f_-(q^2)$ has been known to receive the zero mode mainly due to the unavoidable usage of the so-called "bad" components of the current, i.e., $\mathbf{J}_{\perp V} = (J_x, J_y)$ and J^-_V, many efforts have been made to obtain the Lorentz covariant form factors [45, 46, 49] within the standard LFQM by properly handling the zero-mode as well as the instantaneous contributions. Applying the same correspondence relations found in [51–53] to the $B \longrightarrow D\ell\nu_\ell$ decays, we found that the zero modes and instantaneous contributions to $f_-(q^2)$ are made to be absent in the standard LFQM while they exist in the BS model. In other words, we obtained the current-component independent form factor $f_-(q^2)$ in the standard LFQM, i.e., $f_-(q^2)$ obtained from $(J^+, J^-)_V$ is exactly the same as the one obtained from $(J^+, \mathbf{J}_\perp)_V$ numerically, and both are expressed as the convolution of the initial and final state LFWFs in the valence region of the $q^+ = 0$ frame. This verifies that our new correspondence relations found in the two-point functions are also applicable to the three-point functions.

The purpose of this paper is to extend our previous analysis [54] of the form factors $f_\pm(q^2)$ for the semileptonic decays between the two pseudoscalar mesons to obtain the current-component independent tensor form factor $f_T(q^2)$ for the rare decays, which complete the set of hadronic matrix elements regulating the exclusive semileptonic and rare decays between the two pseudoscalar mesons. We then apply our Lorentz covariant form factors $(f_\pm, f_T)$ for the analysis of the semileptonic and rare $D \longrightarrow \pi(K)$ decays within the standard model and the light-front quark model (LFQM) constrained by the variational principle for the QCD-motivated effective Hamiltonian [49, 55–57].

The paper is organized as follows: in Section 2, we introduce three form factors $(f_\pm, f_T)$ for the semileptonic and rare decays between two pseudoscalar mesons. In the $q^+ = 0$ frame, we define the form factors extracted from the various combinations of vector and tensor currents. In Section 3, we set up the current matrix elements for the form factors in an exactly solvable model based on the covariant BS model of (3 + 1) dimensional fermion field theory. We then present our LF calculations of tensor form factor f_T in the BS model using the two different sets ($J^{+\perp}_T$ and J^{+-}_T) of the tensor current $J^{\mu\nu}_T$. For completeness, we also present the results of the current-component independent form factors $f_\pm(q^2)$ found [54]. We note that while $f_T(q^2)$ obtained from $J^{+\perp}_T$

is immune to the zero mode and the instantaneous contribution, $f_T(q^2)$ obtained from J_T^{+-} cannot avoid those contributions in this BS model. Linking the covariant BS model to the standard LFQM with our new correspondence relations between the two models [51–53], however, we find that $f_T(q^2)$ obtained from J_T^{+-} in the standard LFQM is made to be free of the zero mode as well as the instantaneous contribution. In other words, we obtained the current-component independent tensor form factor $f_T(q^2)$ in the standard LFQM regardless of using $J_T^{+\perp}$ or J_T^{+-} as in the case of $f_-(q^2)$ calculation [54]. Finally, we present the current-component independent TFFs $(f_\pm, f_T)$ in the $q^+=0$ frame of the standard LFQM. In Section 4, we present our numerical results of the form factors for the semileptonic and rare $D \longrightarrow \pi(K)$ decays as well as the branching ratios for the semileptonic $D \longrightarrow \pi(K)\ell\nu_\ell(\ell=e,\mu)$. Summary and discussion follow in Section 5.

2. Theoretical Framework

The matrix elements of the vector $J_V^\mu = \bar{q}\gamma^\mu c$ and the tensor $J_T^{\mu\nu} = \bar{q}\sigma^{\mu\nu} c$ currents for the weak $c \longrightarrow q(q=u,d,s)$ transitions between the initial D meson and the final π or K meson can be parametrized by the following set of invariant form factors, (f_+, f_-, s) [33]:

$$\mathcal{M}_V^\mu \equiv \langle P_2 | J_V^\mu | P_1 \rangle = f_+(q^2) P^\mu + f_-(q^2) q^\mu, \quad (1)$$

$$\mathcal{M}_T^{\mu\nu} \equiv \langle P_2 | J_T^{\mu\nu} | P_1 \rangle = i s(q^2) [P^\mu q^\nu - q^\mu P^\nu], \quad (2)$$

where $P = P_1 + P_2$ and $q = P_1 - P_2$ are the four-momentum transfer to the lepton pair $(\ell\nu_\ell)$ with $m_\ell^2 \le q^2 \le (M_1 - M_2)^2$ for the semileptonic decays or to the pair $(\ell^+\ell^-)$ with $4m_\ell^2 \le q^2 \le (M_1 - M_2)^2$ for the rare decays, respectively. The antisymmetric tensor in Eq. (2) is given by $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$.

On many occasions, it is useful to express Eq. (1) in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$, which are related to the transition amplitude with the exchange of a vector (1^-) and a scalar (0^+) boson in the t-channel, respectively, and satisfy

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_1^2 - M_2^2} f_-(q^2). \quad (3)$$

Likewise, the tensor form factor $s(q^2)$ in Eq. (2) can also be redefined by

$$s(q^2) = \frac{f_T(q^2)}{M_1 + M_2}, \quad (4)$$

to make $f_T(q^2)$ dimensionless.

Including the nonzero lepton mass (m_ℓ), the differential decay rate for the semileptonic $P_1 \longrightarrow P_2 \ell\nu_\ell$ process is given by [58, 59].

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{96\pi^3} \left|\vec{p}^{\,*}\right| \left|V_{Q_1\bar{Q}_2}\right|^2 \frac{q^2}{M_1^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right)|H_+|^2 + \frac{3m_\ell^2}{2q^2}|H_0|^2\right], \quad (5)$$

where $G_F = 1.166 \times 10^{-5}$ GeV^{-2} is the Fermi constant, $V_{Q_1\bar{Q}_2}$ is the relevant CKM mixing matrix element, and

$$\left|\vec{p}^{\,*}\right| = \frac{1}{2M_1}\sqrt{(M_1^2 + M_2^2 - q^2)^2 - 4M_1^2 M_2^2}, \quad (6)$$

is the modulus of the three-momentum of the daughter meson in the parent meson rest frame, and the helicity amplitudes H_+ and H_0 are given by

$$H_+ = \frac{2M_1\left|\vec{p}^{\,*}\right|}{\sqrt{q^2}} f_+(q^2), H_0 = \frac{M_1^2 - M_2^2}{\sqrt{q^2}} f_0(q^2). \quad (7)$$

We note that $q^2 = q^2_{max}$ corresponds to the zero-recoil of the final meson in the initial meson rest frame, and the $q^2 = 0$ corresponds to the maximum recoil of the final meson recoiling with the maximum three momentum $|\vec{P}_2| = (M_1^2 - M_2^2)/2M_1$.

In the LF calculation of the form factors, we use the metric convention $a \cdot b = (1/2)(a^+b^- + a^-b^+) - \mathbf{a}_\perp \cdot \mathbf{b}_\perp$. Performing the LF calculation in the $q^+=0$ frame (i.e., $q^2 = -\mathbf{q}_\perp^2 = -Q^2 < 0$) with $P_1 = (P_1^+, P_1^-, \mathbf{P}_{1\perp}) = (P_1^+, M_1^2/P_1^+, \mathbf{0}_\perp)$, we utilize all three components $(\mu, \nu = +, -, \perp)$ of the current J_V^μ and $J_T^{\mu\nu}$ in Eqs. (1) and (2) to obtain $f_+(q^2)$, $f_-(q^2)$ [or $f_0(q^2)$], and $f_T(q^2)$. The form factors obtained in the spacelike region $(q^2 < 0)$ are then analytically continued to the timelike region by changing $\mathbf{q}_\perp^2$ to $-q^2$ in the form factors as we show in our numerical calculations.

While the form factor $f_+(q^2)$ can be obtained from the plus component (J_V^+) of the vector current, one cannot but use two different combinations of the current to obtain $f_-(q^2)$ such as $(J^+, \mathbf{J}_\perp)_V$ or $(J^+, J^-)_V$. That is, using those sets of the current components in the $q^+=0$ frame, one obtains the relations between the weak form factors $f_\pm(q^2)$ and the current matrix elements in Eq. (1) as follows [54]:

$$f_+(q^2) = \frac{\mathcal{M}_V^+}{2P_1^+}, \quad (8)$$

$$f_-^{(+\perp)}(q^2) = \frac{\mathcal{M}_V^+}{2P_1^+} + \frac{\mathcal{M}_V^\perp \cdot \mathbf{q}_\perp}{\mathbf{q}_\perp^2}, \quad (9)$$

$$f_-^{(+-)}(q^2) = -\frac{\mathcal{M}_V^+}{2P_1^+}\left(\frac{\Delta M_+^2 + \mathbf{q}_\perp^2}{M_-^2 - \mathbf{q}_\perp^2}\right) + \frac{P_1^+ \mathcal{M}_V^-}{\Delta M_-^2 - \mathbf{q}_\perp^2}, \quad (10)$$

where $\Delta M_\pm^2 = M_1^2 \pm M_2^2$, and we denote $f_-(q^2)$ obtained from $(J^+, \mathbf{J}_\perp)_V$ and $(J^+, J^-)_V$ as $f_-^{(+\perp)}(q^2)$ and $f_-^{(+-)}(q^2)$, respectively. It is prerequisite to show that $f_-^{(+\perp)}(q^2) = f_-^{(+-)}(q^2)$ to assert the Lorentz invariance of the form factor and the self-consistency of the model.

Likewise, the tensor form factor $s(q^2)$ can be obtained from using either $J_T^{+\perp}$ or J_T^{+-}. In this case, the relations between $s(q^2)$ and the current matrix element in Eq. (2) are given by

$$s^{(+\perp)}(q^2) = -\frac{i\mathcal{M}_T^{+\perp}\cdot\mathbf{q}_\perp}{2\mathbf{q}_\perp^2 P_1^+}, \quad (11)$$

$$s^{(+-)}(q^2) = -\frac{i\mathcal{M}_T^{+-}}{2(\Delta M_-^2 - \mathbf{q}_\perp^2)}, \quad (12)$$

where $s^{(+\perp)}(q^2)$ and $s^{(+-)}(q^2)$ represent the form factor $s(q^2)$ obtained from $J_T^{+\perp}$ and J_T^{+-}, respectively. Of course, $s^{(+\perp)}(q^2) = s^{(+-)}(q^2)$ should be satisfied in the self-consistent model calculation.

Our aim in this work is to show $s^{(+\perp)}(q^2) = s^{(+-)}(q^2)$ in addition to our previous verification of $f_-^{(+\perp)}(q^2) = f_-^{(+-)}(q^2)$ [54] in our LFQM, which completes the analysis of the exclusive semileptonic and rare decays between two pseudoscalar mesons. For this purpose, we start from the exactly solvable manifestly covariant BS model and then connect it to our phenomenologically accessible LFQM. Although we analyzed $f_\pm(q^2)$ in [54], we shall include them again in the next section for the completeness of the analysis.

3. Model Description

3.1. Manifestly Covariant Model. In the solvable model, based on the covariant BS model of $(3+1)$-dimensional fermion field theory [50, 54], the matrix elements $\mathcal{M} = (\mathcal{M}_V^\mu, \mathcal{M}_T^{\mu\nu})$ are given by

$$\mathcal{M} = iN_c \int \frac{d^4k}{(2\pi)^4} \frac{H_{p_1}\mathcal{T} H_{p_2}}{N_{p_1}N_k N_{p_2}}, \quad (13)$$

where the corresponding trace terms $\mathcal{T} = (S^\mu, T^{\mu\nu})$ are given by

$$S^\mu = \mathrm{Tr}\left[\gamma_5(\not{p}_1 + m_1)\gamma^\mu(\not{p}_2 + m_2)\gamma_5(-\not{k} + m_q)\right], \quad (14)$$

for the vector current and

$$T^{\mu\nu} = \mathrm{Tr}\left[\gamma_5(\not{p}_1 + m_1)\sigma^{\mu\nu}(\not{p}_2 + m_2)\gamma_5(-\not{k} + m_q)\right], \quad (15)$$

for the tensor current, respectively. N_c is the number of colors, and $p_j = P_j - k (j = 1, 2)$ and k are the internal momenta carried by the quark and antiquark propagators of mass m_j and m_q, respectively. The corresponding denominators are given by $N_{p_j} = p_j^2 - m_j^2 + i\varepsilon$ and $N_k = k^2 - m_q^2 + i\varepsilon$. We take the $q\bar{q}$ bound-state vertex functions $H_{p_j}(p_j^2, k^2) = g_j/(p_j^2 - \Lambda_j^2 + i\varepsilon)$ of the initial $(j = 1)$ and final $(j = 2)$ state pseudoscalar mesons, where g_j and Λ_j are constant parameters in this manifestly covariant model.

Performing the LF calculation in the $q^+ = 0$ frame, one obtains the following identity $\not{q} = \not{q}_{on} + (1/2)\gamma^+\Delta_q^-$, where $\Delta_q^- = q^- - q_{on}^-$ and the subscript (on) denotes the on-mass shell quark momentum, i.e., $p_{jon}^2 = m_j^2$ and $k_{on}^2 = m_q^2$. Using this identity, one can separate the trace terms into the "on"-mass shell propagating part and the "off"-mass shell instantaneous one, i.e., $S^\mu = S_{\rm on}^\mu + S_{\rm off}^\mu$ for the vector current and $\mathrm{T}^{\mu\nu} = T_{\rm on}^{\mu\nu} + T_{\rm off}^{\mu\nu}$ for the tensor current.

The explicit LF calculation in parallel with the manifestly covariant calculation of Eq. (13) to compute $f_\pm(q^2)$ can be found in [49] where $f_-(q^2)$ was obtained from $f_-^{(+\perp)}(q^2)$. The identical result for $f_-^{(+\perp)}(q^2)$ was also obtained in [45, 46] using the so-called "covariant LFQM" analysis. As shown in Ref. [45, 46, 49], while $f_+(q^2)$ obtained from the plus current was immune to the zero mode, the form factor $f_-(q^2)$ received both instantaneous and zero-mode contributions. The same situation happens for $f_-^{(+-)}(q^2)$ although the zero-mode and the instantaneous contributions may differ quantitatively from $f_-^{(+\perp)}(q^2)$. However, as we have shown in [54], $f_-^{(+\perp)}(q^2)$ and $f_-^{(+-)}(q^2)$ obtained in the standard LFQM by using our new correspondence relations between the BS model and the standard LFQM show identical result in the valence region of the $q^+ = 0$ frame without involving explicit zero modes and instantaneous contributions. In this work, we shall show that the tensor form factor $s(q^2)$ in the standard LFQM is independent of the components of the current, i.e., $s^{(+\perp)}(q^2) = s^{(+-)}(q^2)$. We should note that all of those equalities, i.e., $f_-^{(+\perp)}(q^2) = f_-^{(+-)}(q^2)$ and $s^{(+\perp)}(q^2) = s^{(+-)}(q^2)$ are derived from the constraint of the on-mass shellness of the quark and antiquark propagators together with the zero-binding energy limit (i.e., $M = M_0$) used in the standard LFQM.

Therefore, from now on, we discuss only for the on-mass shell contributions in the valence region of the $q^+ = 0$ frame between the manifestly covariant BS model and the standard LFQM. The on-shell contributions to S^μ and $T^{\mu\nu}$ are given by

$$\begin{aligned} S_{\rm on}^\mu = 4\big[&p_{1on}^\mu(p_{2on}\cdot k_{on} + m_2 m_q) + p_{2on}^\mu(p_{1on}\cdot k_{on} + m_1 m_q) \\ &+ k_{\rm on}^\mu(m_1 m_2 - p_{1on}\cdot p_{2on})\big], \end{aligned} \quad (16)$$

and

$$\begin{aligned} T_{\rm on}^{\mu\nu} = 4i\big[&p_{1on}^\mu(m_2 k_{on}^\nu + m_q p_{2on}^\nu) - p_{2on}^\mu(m_1 k_{on}^\nu + m_q p_{1on}^\nu) \\ &+ k_{\rm on}^\mu(m_1 p_{2on}^\nu - m_2 p_{1on}^\nu)\big], \end{aligned} \quad (17)$$

respectively. The LF four-momenta of the on-mass shell quark and antiquark propagators in the $q^+ = 0$ (i.e. $P_1^+ = P_2^+$) frame are given by

$$p_{1on} = \left[xP_1^+, \frac{m_1^2 + \mathbf{k}_\perp^2}{xP_1^+}, -\mathbf{k}_\perp\right], \tag{18}$$

$$p_{2on} = \left[xP_1^+, \frac{m_2^2 + (\mathbf{k}_\perp + \mathbf{q}_\perp)^2}{xP_1^+}, -\mathbf{k}_\perp - \mathbf{q}_\perp\right], \tag{19}$$

$$k_{on} = \left[(1-x)P_1^+, \frac{m_q^2 + \mathbf{k}_\perp^2}{(1-x)P_1^+}, \mathbf{k}_\perp\right], \tag{20}$$

where $x = p_1^+/P_1^+$ and $\bar{x} = k^+/P_1^+$ are the LF longitudinal momentum fractions of the quark and antiquark, which satisfy $x + \bar{x} = 1$.

By the integration over k^- in Eq. (13) and closing the contour in the lower half of the complex k^- plane, one picks up the residue at $k^- = k^-_{on}$ in the valence region of $0 < k^+ < P_2^+$ (or $0 < x < 1$). We denote the on-mass shell contribution to $\mathcal{M}_V^\mu$ and $\mathcal{M}_T^{\mu\nu}$ in the valence region as $[\mathcal{M}_V^\mu]_{on}^{BS}$ and $[\mathcal{M}_T^{\mu\nu}]_{on}^{BS}$, respectively. The explicit forms of $[\mathcal{M}_V^\mu]_{on}^{BS}$ and $[\mathcal{M}_T^{\mu\nu}]_{on}^{BS}$ are obtained as [54].

$$\mathcal{M}_{on}^{BS} = N_c \int_0^1 \frac{dx}{\bar{x}} \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \chi_1(x, \mathbf{k}_\perp) \chi_2\left(x, \mathbf{k}_\perp'\right) \mathcal{T}_{on}, \tag{21}$$

where $\mathcal{M} = (\mathcal{M}_V^\mu, \mathcal{M}_T^{\mu\nu})$ pairs with $\mathcal{T} = (S^\mu, T^{\mu\nu})$. The LF quark-meson vertex function $\chi_{1(2)}$ of the initial (final) state is given by

$$\chi_{1(2)}\left(x, \mathbf{k}^{(\prime)}{}_\perp\right) = \frac{g_{1(2)}}{x^2 \left(M_{1(2)}^2 - M_0^{(\prime)2}\right)\left(M_{1(2)}^2 - M_{\Lambda_1(\Lambda_2)}^{(\prime)2}\right)}, \tag{22}$$

where $\mathbf{k}_\perp' = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$ and

$$M_0^2 = \frac{\mathbf{k}_\perp^2 + m_1^2}{x} + \frac{\mathbf{k}_\perp^2 + m_q^2}{1-x}, \tag{23}$$

$$M_0'^2 = \frac{\mathbf{k}_\perp'^2 + m_2^2}{x} + \frac{\mathbf{k}_\perp'^2 + m_q^2}{1-x}, \tag{24}$$

are the invariant masses of the initial and final states, respectively. Likewise, $M_{\Lambda_{1(2)}}$ is obtained as $M_{\Lambda_1} = M_0(m_1 \longrightarrow \Lambda_1)$and $M_{\Lambda_2}' = M_0'(m_2 \longrightarrow \Lambda_2)$.

For the trace $\mathcal{T} = (S^\mu, T^{\mu\nu})$ calculations relevant to the form factors, the on-mass shell contributions $S^\mu_{\rm on}$ obtained from all three components $\mu = (+, \perp, -)$ of the vector current J_V^μ are given by [54].

$$S_{on}^+ = \frac{4P_1^+}{\bar{x}}\left(\mathbf{k}_\perp \cdot \mathbf{k}_\perp' + \mathcal{A}_1\mathcal{A}_2\right), \tag{25}$$

$$\begin{aligned}\mathbf{S}_{\perp on} = \frac{-2\mathbf{k}_\perp}{x\bar{x}}\Big[&2\mathbf{k}_\perp \cdot \mathbf{k}_\perp' + \bar{x}\left(\mathbf{q}_\perp^2 + m_1^2 + m_2^2\right) + 2x^2 m_q^2 \\ &+ 2x\bar{x}\left(m_1 m_q + m_2 m_q - m_1 m_2\right)\Big] \\ &- \frac{2\mathbf{q}_\perp}{x\bar{x}}\left(\mathbf{k}_\perp^2 + \mathcal{A}_1^2\right),\end{aligned} \tag{26}$$

$$\begin{aligned}S_{on}^- = \frac{4}{x^2\bar{x}P_1^+}\Big[&\bar{x}\left(m_1\mathcal{A}_1 + \mathbf{k}_\perp^2\right)\left[m_2^2 + (\mathbf{k}_\perp + \mathbf{q}_\perp)^2\right] \\ &+ x^2\bar{x}M_0^2\left(\mathbf{k}_\perp^2 + \mathbf{k}_\perp \cdot \mathbf{q}_\perp\right) + x^2 m_1 m_2\left(m_q^2 + \mathbf{k}_\perp^2\right) \\ &+ x\bar{x}m_2 m_q\left(m_1^2 + \mathbf{k}_\perp^2\right)\Big],\end{aligned} \tag{27}$$

where $\mathcal{A}_j = (1-x)m_j + xm_q (j = 1, 2)$. Likewise, the on-shell contributions $T_{on}^{\mu\nu}$ obtained from the two sets of the tensor current $J_T^{\mu\nu}$, i.e., $(\mu, \nu) = (+,\perp)$ and $(+, -)$, are given by

$$T_{on}^{+\perp} = -4iP_1^+[(m_1 - m_2)\mathbf{k}_\perp + \mathcal{A}_1\mathbf{q}_\perp], \tag{28}$$

$$\begin{aligned}T_{\rm on}^{+-} = \frac{4i}{x\bar{x}}\big[&(1-2x)(m_1 - m_2)\mathbf{k}_\perp^2 + 2\bar{x}\mathcal{A}_1\mathbf{k}_\perp \cdot \mathbf{q}_\perp \\ &+ \bar{x}\mathcal{A}_1\mathbf{q}_\perp^2 + (m_2 - m_1)\mathcal{A}_1\mathcal{A}_2\big].\end{aligned} \tag{29}$$

Using Eqs. (8), (9), (10), (11), (12), and (21), one obtains the on-mass shell contributions to the weak form factors (f_+, f_-, s) as follows

$$\mathcal{F}_{\rm on}^{\rm BS} = N_c \int_0^1 \frac{dx}{\bar{x}} \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \chi_1(x, \mathbf{k}_\perp) \langle\mathcal{O}\rangle_{on}^{\rm BS} \chi_2\left(x, \mathbf{k}_\perp'\right), \tag{30}$$

where the form factors $\mathcal{F} = \{f_+, f_-^{(+\perp)}, f_-^{(+-)}, s^{(+\perp)}, s^{(+-)}\}$ obtained from different combinations of the vector and tensor currents pair with the following corresponding operators $\langle\mathcal{O}\rangle_{on}^{\rm BS} = \{\mathcal{O}_+, \mathcal{O}_-^{(+\perp)}, \mathcal{O}_-^{(+-)}, \mathcal{O}_s^{(+\perp)}, \mathcal{O}_s^{(+-)}\}$ including the spin and external momenta factors:

$$\mathcal{O}_+ = \frac{S_{on}^+}{2P_1^+}, \tag{31}$$

$$\mathcal{O}_-^{(+\perp)} = \frac{S_{on}^+}{2P_1^+} + \frac{\mathbf{S}_{\perp on} \cdot \boldsymbol{q}_\perp}{\boldsymbol{q}_\perp^2}, \tag{32}$$

$$\mathcal{O}_-^{(+-)} = -\frac{S_{on}^+}{2P_1^+}\left(\frac{\Delta M_+^2 + \boldsymbol{q}_\perp^2}{\Delta M_-^2 - \boldsymbol{q}_\perp^2}\right) + \frac{P_1^+ S_{on}^-}{\Delta M_-^2 - \boldsymbol{q}_\perp^2}, \tag{33}$$

$$\mathcal{O}_s^{(+\perp)} = -\frac{{\rm i}T_{\rm on}^{+\perp} \cdot \mathbf{q}_\perp}{2\mathbf{q}_\perp^2 P_1^+}, \tag{34}$$

$$\mathcal{O}_s^{(+-)} = -\frac{iT_{on}^{+-}}{2\left(\Delta M_-^2 - \boldsymbol{q}_\perp^2\right)}. \tag{35}$$

In the manifestly covariant BS model given by Eq. (13), we note that only the two form factors $f_+(q^2)$ and $s^{(+\perp)}(q^2)$ defined by Eqs. (31) and (34), respectively, are immune to the zero modes as well as the instantaneous contributions and thus exactly equal to the full exact solution (i.e., manifestly covariant solution), i.e., $[f_+]_{on}^{BS} = f_+^{Cov}$

and $[s^{(+\perp)}]_{on}^{BS} = s^{Cov}$, without involving such LF treacherous points. However, since the other three form factors $f_-^{(+\perp)}$, $f_-^{(+-)}$, and $s^{(+-)}$ are contaminated by the zero modes as well as the instantaneous contributions, the on-mass shell contributions $[f_-^{(+\perp)}]_{on}^{BS}$, $[f_-^{(+-)}]_{on}^{BS}$, and $[s^{(+\perp)}]_{on}^{BS}$ themselves can never be the same as the exact solutions unless the zero modes and the instantaneous contributions are taken into account. Furthermore, one can easily check that $[f_-^{(+\perp)}]_{on}^{BS} \neq [f_-^{(+-)}]_{on}^{BS}$ and $[s^{(+-)}]_{on}^{BS} \neq [s^{(+\perp)}]_{on}^{BS}$.

However, in the following subsection, we shall show in the standard LFQM (denoted by SLF) that $f_-^{SLF} = [f_-^{(+\perp)}]_{on}^{SLF} = [f_-^{(+-)}]_{on}^{SLF}$ and $s^{SLF} = [s^{(+-)}]_{on}^{SLF} = [s^{(+\perp)}]_{on}^{SLF}$ without involving explicit zero-mode and instantaneous contributions, which comes after using our new correspondence relations between the BS model and the standard LFQM.

3.2. The Standard LFQM. In the standard LFQM [39, 41, 42, 55–57, 60, 61], the LF wave function (LFWF) of a ground state pseudoscalar meson as a $q\bar{q}$ bound state is given by

$$\Psi_{\lambda\bar{\lambda}}(x, \mathbf{k}_\perp) = \phi(x, \mathbf{k}_\perp)\mathcal{R}_{\lambda\bar{\lambda}}(x, \mathbf{k}_\perp), \tag{36}$$

where $\mathcal{R}_{\lambda\bar{\lambda}}(x, \mathbf{k}_\perp)$ is the spin-orbit WF that is obtained by the interaction-independent Melosh transformation from the ordinary spin-orbit WF assigned by the quantum number J^{PC}. The covariant form of $\mathcal{R}_{\lambda\bar{\lambda}}$ with the definite spin $(S, S_z) = (0, 0)$ constructed out of the LF helicity $\lambda(\bar{\lambda})$ of a quark (antiquark) is given by

$$\mathcal{R}_{\lambda\bar{\lambda}} = \frac{\bar{u}_\lambda\left(p_q\right)\gamma_5 v_{\bar{\lambda}}\left(p_{\bar{q}}\right)}{\sqrt{2}\left[M_0^2 - \left(m_1 - m_q\right)^2\right]^{1/2}}, \tag{37}$$

which satisfies the unitarity condition, $\sum_{\lambda\bar{\lambda}}\mathcal{R}_{\lambda\bar{\lambda}}^\dagger\mathcal{R}_{\lambda\bar{\lambda}} = 1$. Its explicit matrix form is given by

$$\mathcal{R}_{\lambda\bar{\lambda}} = \frac{1}{\sqrt{2}\sqrt{\mathbf{k}_\perp^2 + \mathcal{A}_1^2}}\begin{pmatrix} -k^L & \mathcal{A}_1 \\ -\mathcal{A}_1 & -k^R \end{pmatrix}, \tag{38}$$

where $k^R = k_x + ik_y$ and $k^L = k_x - ik_y$.

For the radial WF $\phi(x, \mathbf{k}_\perp)$ in Eq. (36), we use the Gaussian WF

$$\phi(x, \mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\beta^{3/2}}\sqrt{\frac{\partial k_z}{\partial x}}\exp\left(-\vec{k}^2/2\beta^2\right), \tag{39}$$

where $\vec{k}^2 = \mathbf{k}_\perp^2 + k_z^2$ and β is the variational parameter. The longitudinal component k_z is defined by $k_z = (x - 1/2)M_0 + (m_q^2 - m_1^2)/2M_0$, and the Jacobian of the variable transformation $\{x, \mathbf{k}_\perp\} \longrightarrow \vec{k} = (\mathbf{k}_\perp, k_z)$ is given by $\partial k_z/\partial x = (M_0/4x(1-x))[1 - (m_1^2 - m_q^2)^2/M_0^4]$. The normalization of our Gaussian radial WF is then given by

$$\int_0^1 dx \int \frac{d^2\boldsymbol{k}_\perp}{16\pi^3}|\phi(x, \boldsymbol{k}_\perp)|^2 = 1. \tag{40}$$

In particular, the key idea in our LFQM [49, 55–57] for mesons is to treat $\phi(x, \mathbf{k}_\perp)$ as a trial function for the variational principle to the QCD-motivated effective Hamiltonian saturating the Fock state expansion by the constituent quark and antiquark. Using this Hamiltonian, we analyze the meson mass spectra and various wave-function-related observables, such as decay constants, electromagnetic form factors of mesons in a spacelike region, and the weak form factors for the exclusive semileptonic and rare decays of pseudoscalar mesons in the timelike region [49, 51–57].

In this standard LFQM, the matrix elements of the vector and tensor currents in Eqs. (1) and (2) are obtained by the convolution formula of the initial and final state LFWFs in the $q^+ = 0$ frame as follows:

$$\begin{aligned}\mathcal{M}_{on}^{SLF} = \sum_{\lambda' s}\int_0^1 dx \int \frac{d^2\boldsymbol{k}_\perp}{16\pi^3}\phi_1(x, \boldsymbol{k}_\perp)\phi_2\left(x, \boldsymbol{k}_\perp'\right) \\ \times \mathcal{R}_{\lambda_2\bar{\lambda}}^\dagger\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{p_2^+}}\Gamma\frac{u_{\lambda_1}(p_1)}{\sqrt{p_1^+}}\mathcal{R}_{\lambda_1\bar{\lambda}},\end{aligned} \tag{41}$$

where $\mathcal{M} = (\mathcal{M}_V^\mu, \mathcal{M}_T^{\mu\nu})$ pairs with $\Gamma = (\gamma^\mu, \sigma^{\mu\nu})$.

Then, we first compute the zero-mode free form factors, i.e. $[f_+]_{on}^{SLF}$ and $[s^{(+\perp)}]_{on}^{SLF}$, in the SLF formulation as follows

$$\mathcal{F}_{on}^{SLF} = \int_0^1 \bar{x}dx \int \frac{d^2\mathbf{k}_\perp}{32\pi^3}\frac{\phi_1(x, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}_1^2 + \mathbf{k}_\perp^2}}\langle\mathcal{O}\rangle_{on}^{SLF}\frac{\phi_2\left(x, \mathbf{k}_\perp'\right)}{\sqrt{\mathcal{A}_2^2 + \mathbf{k}_\perp'^2}}, \tag{42}$$

where the form factors $\mathcal{F} = \{f_+, s^{(+\perp)}\}$ pair with the following corresponding operators $\langle\mathcal{O}\rangle_{on}^{SLF} = \langle\mathcal{O}\rangle_{on}^{BS} = \{\mathcal{O}_+, \mathcal{O}_s^{(+\perp)}\}$ given by Eqs. (31) and (34).

We should note that the main differences between the covariant BS model and the standard LFQM are attributed to the different spin structures of the $q\bar{q}$ system (i.e., off-shellness in the BS model vs. on-shellness in the standard LFQM) and the different meson-quark vertex functions (χ vs. ϕ). In other words, while the results of the covariant BS model allow the nonzero binding energy $E_{B.E} = M^2 - M_0^2$, the SLF result is obtained from the condition of on-mass shell quark and antiquark (i.e., $M \longrightarrow M_0$).

Comparing these two form factors $\mathcal{F} = \{f_+, s^{(+\perp)}\}$ defined in Eq. (30) in the BS model and Eq. (42) in the standard LFQM, one can easily find the correspondence relation between the two models as follows:

$$\sqrt{2N_c}\frac{\chi_{1(2)}\left(x, \mathbf{k}_\perp^{(\prime)}\right)}{1-x} \longrightarrow \frac{\phi_{1(2)}\left(x, \mathbf{k}_\perp^{(\prime)}\right)}{\sqrt{\mathcal{A}_{1(2)}^2 + \mathbf{k}_\perp^{(\prime)2}}}. \tag{43}$$

In many previous LFQM analyses [40, 45, 46, 49], the correspondence in Eq. (43) has also been used for the mapping of other physical observables contaminated by the zero modes and/or the instantaneous contributions. However, applying Eq. (43) together with the same operators given by Eqs. (32), (33), and (35) to the other form factors $\mathcal{F} = \{f_-^{(+\perp)}, f_-^{(+-)}, s^{(+-)}\}$ obtained from the only on-mass shell contributions, one encounters the same problems as the BS model, i.e., $[f_-^{(+\perp)}]_{on}^{SLF} \neq [f_-^{(+-)}]_{on}^{SLF}$ and $[s^{(+-)}]_{on}^{SLF} \neq [s^{(+\perp)}]_{on}^{SLF}$ implying that the same physical quantities obtained from different components of the current yield different results.

In our previous analysis [51–53], however, we found that the correspondence relation including only LF vertex functions given by Eq. (43) brings about the self-consistency problem, i.e., the same physical quantity obtained from different components of the current and/or the polarization vectors yields different results in the standard LFQM. Furthermore, we also discovered the additional requirement for the correct correspondence relation between the two models to obtain the current-component independent physical observables in the standard LFQM.

Our new correspondence relation (denoted by "CJ-scheme" for convenience) to restore the self-consistency in the standard LFQM is given by [51–54]:

$$\sqrt{2N_c}\frac{\chi_{1(2)}\left(x, \mathbf{k}_\perp^{(\prime)}\right)}{1-x} \longrightarrow \frac{\phi_{1(2)}\left(x, \mathbf{k}_\perp^{(\prime)}\right)}{\sqrt{\mathscr{A}_{1(2)}^2 + \mathbf{k}_\perp^{(\prime)2}}}, M_{1(2)} \longrightarrow M^{(\prime)}{}_0, \tag{44}$$

that is, the physical mass $M_{1(2)}$ included in the integrand of the BS amplitude, e.g., the operators $\langle\mathcal{O}\rangle_{on}^{BS}$ in Eqs. (31) (32), (33), (34), and (35) should be replaced by the invariant mass $M_0^{(\prime)}$ as all constituent quark and antiquarks are required to be on their respective mass shell in the standard LFQM. We should note that this "CJ-scheme" has been verified through our previous analyses for the decay constants and the twist-2 and-3 DAs of pseudoscalar and vector mesons [51–53] and the form factor $f_-(q^2)$ for the semileptonic B decays [54].

We now show in this work that the "CJ-scheme" is also valid to obtain the current-component independent tensor form factor $s(q^2)$ in addition to $f_-(q^2)$ [54]. That is, applying Eq. (44) to the form factors $\mathscr{F} = \{f_-^{(+-)}, s^{(+-)}\}$ defined in Eq. (30) implies the replacements of the current operators $\langle\mathcal{O}\rangle_{on}^{BS} = \{\mathcal{O}_-^{(+-)}, \mathcal{O}_s^{(+-)}\}$ in the BS model with $\langle\mathcal{O}\rangle_{on}^{SLF} = \{\tilde{\mathcal{O}}_-^{(+-)}, \tilde{\mathcal{O}}_s^{(+-)}\}$ in the standard LFQM, i.e.,

$$\tilde{\mathcal{O}}_-^{(+-)} = -\frac{S_{on}^+}{2P_1^+}\left(\frac{\Delta M_{0+}^2 + \boldsymbol{q}_\perp^2}{\Delta M_{0-}^2 - \boldsymbol{q}_\perp^2}\right) + \frac{P_1^+ S_{on}^-}{\Delta M_{0-}^2 - \boldsymbol{q}_\perp^2}, \tag{45}$$

$$\tilde{\mathcal{O}}_s^{(+-)} = -\frac{iT_{on}^{+-}}{2\left(\Delta M_{0-}^2 - \boldsymbol{q}_\perp^2\right)}, \tag{46}$$

where $\Delta M_{0\pm}^2 = M_0^2 \pm M_0'^2$. Then, we obtain from Eqs. (42), (45), and (46) the current-component independent form factors, i.e., $[f_-^{(+\perp)}]_{on}^{SLF} \doteq [f_-^{(+-)}]_{on}^{SLF}$ and $[s^{(+-)}]_{on}^{SLF} \doteq [s^{(+\perp)}]_{on}^{SLF}$ in the standard LFQM, where "$\doteq$" represents the equality of both sides numerically. The additional requirement in the "CJ-scheme", i.e., $M_{1(2)} \longrightarrow M_0^{(\prime)}$, can therefore be regarded as the effective inclusion of the zero modes in the valence region of the $q^+ = 0$ frame in the standard LFQM. This replacement $M_{1(2)} \longrightarrow M_0^{(\prime)}$ is not possible in the BS model due to the form of the LF vertex function χ given by Eq. (22).

The final results for f_+, $f_- = f_-^{(+\perp)} \doteq f_-^{(+-)}$, and $s = s^{(+\perp)} \doteq s^{(+-)}$ in the standard LFQM are given by

$$f_+(q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi_1(x, \mathbf{k}_\perp)}{\sqrt{\mathscr{A}_1^2 + \mathbf{k}_\perp^2}} \frac{\phi_2\left(x, \mathbf{k}_\perp'\right)}{\sqrt{\mathscr{A}_2^2 + \mathbf{k}_\perp'^2}} \left(\mathscr{A}_1\mathscr{A}_2 + \mathbf{k}_\perp \cdot \mathbf{k}_\perp'\right), \tag{47}$$

$$f_-^{(+\perp)}(q^2) = \int_0^1 \bar{x}dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi_1(x, \mathbf{k}_\perp)}{\sqrt{\mathscr{A}_1^2 + \mathbf{k}_\perp^2}} \frac{\phi_2\left(x, \mathbf{k}_\perp'\right)}{\sqrt{\mathscr{A}_2^2 + \mathbf{k}_\perp'^2}} \cdot \left\{-\bar{x}M_0^2 + (m_2 - m_q)\mathscr{A}_1 - m_q(m_1 - m_q) + \frac{\mathbf{k}_\perp \cdot \mathbf{q}_\perp}{q^2} \cdot \left[M_0^2 + M'^2_0 - 2(m_1 - m_q)(m_2 - m_q)\right]\right\}, \tag{48}$$

$$f_-^{(+-)}(q^2) = \int_0^1 \frac{dx}{x^2} \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi_1(x, \mathbf{k}_\perp)}{\sqrt{\mathscr{A}_1^2 + \mathbf{k}_\perp^2}} \frac{\phi_2\left(x, \mathbf{k}_\perp'\right)}{\sqrt{\mathscr{A}_2^2 + \mathbf{k}_\perp'^2}} \left\{a_0\left[x^2\bar{x}M_0^2(\mathbf{k}_\perp^2 + \mathbf{k}_\perp \cdot \mathbf{q}_\perp) + \bar{x}(m_1\mathscr{A}_1 + \mathbf{k}_\perp^2)\left[m_2^2 + (\mathbf{k}_\perp + \mathbf{q}_\perp)^2\right] + x^2 m_1 m_2\left(m_q^2 + \mathbf{k}_\perp^2\right) + x\bar{x}m_2 m_q(m_1^2 + \mathbf{k}_\perp^2)\right] - x^2 b_0\left(\mathbf{k}_\perp \cdot \mathbf{k}_\perp' + \mathscr{A}_1\mathscr{A}_2\right)\right\}, \tag{49}$$

for the vector current, and

$$s^{(+\perp)}(q^2) = -\int_0^1 (1-x)dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi_1(x, \mathbf{k}_\perp)}{\sqrt{\mathscr{A}_1^2 + \mathbf{k}_\perp^2}} \frac{\phi_2\left(x, \mathbf{k}_\perp'\right)}{\sqrt{\mathscr{A}_2^2 + \mathbf{k}_\perp'^2}} \cdot \left[(m_1 - m_2)\frac{\mathbf{k}_\perp \cdot \mathbf{q}_\perp}{\mathbf{q}_\perp^2} + \mathscr{A}_1\right], \tag{50}$$

$$s^{(+-)}(q^2) = \int_0^1 \frac{dx}{x} \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi_1(x, \mathbf{k}_\perp)}{\sqrt{\mathscr{A}_1^2 + \mathbf{k}_\perp^2}} \frac{\phi_2\left(x, \mathbf{k}_\perp'\right)}{\sqrt{\mathscr{A}_2^2 + \mathbf{k}_\perp'^2}} \frac{a_0}{2}\left[(1-2x)(m_1 - m_2)\mathbf{k}_\perp^2 + 2(1-x)\mathscr{A}_1\mathbf{k}_\perp \cdot \mathbf{q}_\perp + (1-x)\mathscr{A}_1\mathbf{q}_\perp^2 + (m_2 - m_1)\mathscr{A}_1\mathscr{A}_2\right], \tag{51}$$

for the tensor current, where $a_0 = 2/(M_0^2 - M_0'^2 - \boldsymbol{q}_\perp^2)$ and $b_0 = (M_0^2 + M_0'^2 + \boldsymbol{q}_\perp^2)/(M_0^2 - M_0'^2 - \boldsymbol{q}_\perp^2)$. Indeed, our prescription $M_{1(2)} \longrightarrow M_0^{(\prime)}$ is applied through the two terms (a_0, b_0) in $f_-^{(+-)}$ and $s^{(+-)}$. Finally, we confirm from the numerical calculations the current independencies of the form factors, i.e., $f_-(q^2) = f_-^{(+\perp)} \doteq f_-^{(+-)}$ and $s(q^2) = s^{(+\perp)} \doteq s^{(+-)}$, which supports the universality of the "CJ-scheme" given by Eq. (44) and the self-consistency of our standard LFQM.

For our numerical calculations in the following section, we use the tensor form factor $f_T(q^2) = s(q^2)(M_1 + M_2)$ as defined in Eq. (4). We should emphasize that the physical masses $M_{1(2)}$ used in defining f_T is nothing to do with our correspondence relations. Only the physical masses $M_{1(2)}$ appeared as a result from the choice of minus component $(\mu, \nu = -)$ of the vector and tensor currents given by Eqs. (1) and (2) are eligible for the transformation into the corresponding invariant masses $M_0^{(\prime)}$ as shown in Eq. (10).

4. Numerical Results

In our numerical calculations for the semileptonic and rare $D \longrightarrow (\pi, K)$ decays, we use the model parameters $(m_{q\bar{q}}, \beta_{q\bar{q}})$ for the harmonic oscillator (HO) confining potential given in Table 1 obtained from the calculation of the ground state meson mass spectra [49, 57]. The decay constants of (π, K, D) mesons obtained from the HO parameters are given by $(f_\pi, f_K, f_D) = (131{,}155{,}197)$ MeV compared to the experimental data [62], $(f_\pi^{\text{exp.}}, f_K^{\text{exp.}}, f_D^{\text{exp.}}) = (130.2(1.2), 155.7(3), 212.6(7))$ MeV. While the decay constant of D meson is not quite sensitive to the quark mass variation, e.g., $f_D = 199^{-2}_{+1}$ MeV for $m_c = 1.7^{+0.1}_{-0.1}$ GeV, we find that the form factors are somewhat sensitive to m_c. Thus, as a sensitivity check of our LFQM, we use this charm quark mass variation for the calculations of the form factors and the branching ratios. For the physical (D, K, π) meson masses, we use the central values quoted by the Particle Data Group (PDG) [62].

In principle, it is possible to use the $q^+ \neq 0$ frame satisfying $q^2 = q^+ q^- - \mathbf{q}_\perp^2 > 0$ for this timelike semileptonic and rare decays. However, in this $q^+ \neq 0$ frame, it is inevitable to confront the particle-number-nonconserving Fock state (or nonvalence) contribution [63, 64]. The main source of difficulty in the LFQM phenomenology is the paucity of information on the nonwave-function vertex [50] in the nonvalence diagram arising from the quark-antiquark pair creation/annihilation. This should contrast with the usual LFWF used in the valence region. Contrary to the $q^+ \neq 0$ frame, the $q^+ = 0$ frame does not suffer from the nonvalence contribution although one needs to be cautious about the zero-mode problem as we discussed already. Once the zero-mode issue is resolved as we proved in this work, it is straightforward to analytically continue the form factors given by Eqs. (47), (48), (49), (50), and (51) obtained in the spacelike region to the timelike physical region.

Our results of the form factors $(f_\pm, f_0, f_T)$ obtained from Eqs. (47), (48), (49), (50), and (51) can also be compared with several parametric forms. Among several forms, a more systematic and model-independent parametrization of semileptonic form factors, often referred to as "z-expansion" or "z-parametrization" [65, 66], has been developed based on general properties of analyticity, unitarity, and crossing symmetries. Especially, this z-parametrization provides better control of theoretical uncertainties in LQCD calculations [23, 24].

Table 1: The constituent quark mass (GeV) and the Gaussian parameters β (GeV) for the HO potential obtained by the variational principle [49, 57]. $q = u$ and d.

m_q	m_s	m_c	β_{qq}	β_{qc}	β_{sc}
0.25	0.48	1.8	0.3194	0.4216	0.4686

Our direct LFQM results for the form factors $f_i(q^2)$ $(i = \pm, 0, T)$ are also well described by the "z-parametrization," which takes the form [23, 24]

$$f_j(q^2) = \frac{f_j(0) + c_j(z - z_0)(1 + (z + z_0)/2)}{1 - b_j q^2}, \quad (52)$$

where

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad (53)$$

and $z_0 = z(q^2 = 0)$ with $t_\pm = (M_1 \pm M_2)^2$ and $t_0 = t_+ (1 - \sqrt{1 - t_-/t_+})$.

The fitted parameters $(b_j, c_j)(j = +, 0, T)$ for the $D \longrightarrow \pi$ and $D \longrightarrow K$ TFFs (f_+, f_0, f_T) are summarized in Tables 2 and 3, respectively, where the errors occur due to the choice of $m_c = 1.7^{+0.1}_{-0.1}$ GeV. In Table 4, we also compare the form factors $f_+(0)$ and $|f_T(0)|$ for $D \longrightarrow (\pi, K)$ transitions at $q^2 = 0$ with those obtained from various theoretical model predictions and experimental data [13, 67, 68].

In Figure 1, we show the q^2 dependences of $f_+^{D\pi}(q^2)$ (black lines), $f_0^{D\pi}(q^2)$ (blue lines), and $f_T^{D\pi}(q^2)$ (red lines) for $D \longrightarrow \pi$ decay, where the solid and dashed lines represent the results obtained from $m_c = 1.8$ GeV and 1.6 GeV, respectively. That is, the bands correspond to the sensitivity coming from the charm quark mass variation, $m_c = 1.7^{+0.1}_{-0.1}$ GeV in our LFQM. We should note that the form factors are displayed not only for the whole timelike kinematic region $[0 \le q^2 \le (M_D - M_\pi)^2]$ (in unit of GeV^2) but also for the spacelike region $(-0.5 \le q^2 \le 0)$ (in unit of GeV^2) to demonstrate the validity of our analytic continuation from spacelike region to the timelike one by changing $\mathbf{q}_\perp^2$ to $-q^2$ in the form factors. For comparison, the data (circles) of the form factors (f_+, f_0, f_T) from the LQCD (for the ETM Collaboration) [23, 24] and the data of f_+ (squares) extracted from the BABAR [67] are shown. Our results for $f_+^{D\pi}(0) = 0.613^{-(21)}_{+(22)}$ and $|f_T^{D\pi}(0)| = 0.501^{+(36)}_{-(39)}$ are in good agreement with $f_+^{D\pi}(0) = 0.610(25)$ from the BABAR [67] and $f_+^{D\pi}(0) = 0.637(24)$ from the BES III [13], as well as $f_+^{D\pi}(0) = 0.612(35)$ and $|f_T^{D\pi}(0)| = 0.506(79)$ from the LQCD [23, 24]. As one can see from Figure 1, the sensitivity to the charm quark mass is more pronounced at the zero-recoil $(q^2 = q^2_{\max})$ of the final meson than the maximum recoil $(q^2 = 0)$. Especially, the q^2-dependent behaviors of

Table 2: Fitted parameters (b_j, c_j) in Eq. (52) for the $D \longrightarrow \pi$ TFFs with $m_c = 1.7^{+0.1}_{-0.1}$ GeV. b_j is in unit of (GeV^{-2}).

$f_{(+,0)}(0)$	b_+	c_+	b_0	c_0	$f_T(0)$	b_T	c_T
$0.613^{-(21)}_{+(22)}$	$0.1899^{-(208)}_{+(233)}$	$-0.8200^{+(275)}_{-(317)}$	$0.2986^{-0.0882}_{+11.8745}$	$1.9051^{-0.5986}_{+107.077}$	$-0.501^{+(36)}_{-(39)}$	$0.1957^{-(216)}_{+(242)}$	$0.6290^{-(417)}_{+(481)}$

Table 3: Fitted parameters (b_j, c_j) in Eq. (52) for the $D \longrightarrow K$ decay with $m_c = 1.7^{+0.1}_{-0.1}$ GeV. b_j is in unit of (GeV^{-2}).

$f_{(+,0)}(0)$	b_+	c_+	b_0	c_0	$f_T(0)$	b_T	c_T
$0.744^{-(22)}_{+(23)}$	$0.1787^{-(157)}_{+(176)}$	$-1.1711^{+(571)}_{-(618)}$	$-0.0563^{+5.0348}_{+0.1426}$	$-2.3039^{+78.3509}_{+1.8289}$	$-0.660^{+(42)}_{-(45)}$	$0.1826^{-(163)}_{+(182)}$	$0.9893^{-(794)}_{+(897)}$

Table 4: Form factors $f_+(0)$ and $|f_T(0)|$ for $D \longrightarrow (\pi, K)$ transitions at $q^2 = 0$ compared with various model predictions and experimental data.

$F(0)$	This work	[23, 24]	[27]	[34]	[35]	[44]	[47]	BES III [13]	BABAR [67, 68]
$f_+^{D\pi}(0)$	$0.613^{-(21)}_{+(22)}$	0.612 (35)	0.63 (11)	0.640	0.63 (9)	–	0.66 (1)	0.637 (24)	0.610 (25)
$\lvert f_T^{D\pi}(0)\rvert$	$0.501^{+(36)}_{-(39)}$	0.506 (79)	–	–	–	$0.84^{+(16)}_{-(13)}$	–	–	–
$f_+^{DK}(0)$	$0.744^{-(22)}_{+(23)}$	0.765 (31)	0.75 (12)	0.716	0.77 (11)	–	0.79 (1)	0.737 (4)	0.727 (11)
$\lvert f_T^{DK}(0)\rvert$	$0.660^{+(42)}_{-(45)}$	0.687 (54)	–	–	–	$0.96^{+(17)}_{-(15)}$	–	–	–

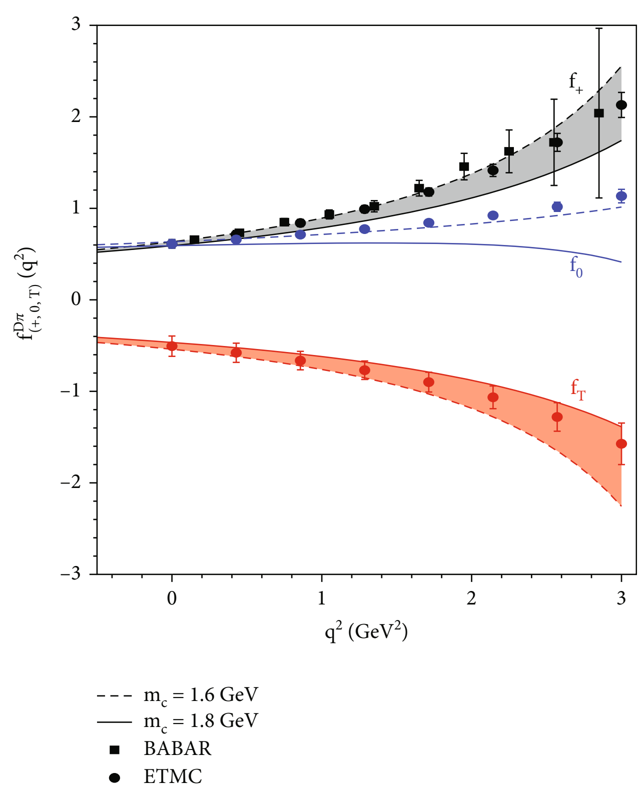

Figure 1: (Color online): the q^2 dependent form factors (f_+, f_0, f_T) of the $D \longrightarrow \pi$ decay for both spacelike and the kinematic timelike regions, $-0.5 \le q^2 \le (M_D - M_\pi)^2$ GeV^2. For comparison, the data taken from the LQCD (circles) [23, 24] and BABAR [67] (squares) are shown.

our results show better agreement with the data from the BABAR [67] and LQCD [23, 24] when we use $m_c \simeq 1.6$ GeV rather than 1.8 GeV.

In Figure 2, we show the q^2 dependences of $(f_+^{DK}, f_0^{DK}, f_T^{DK})$ for $D \longrightarrow K$ decay, compared with the results from the LQCD [23, 24]. The same line codes are used as in Figure 1. Our predictions of $f_+^{DK}(0) = 0.744^{-(22)}_{+(23)}$ and $|f_T^{DK}(0)| = 0.660^{+(42)}_{-(45)}$ agree with $f_+^{DK}(0) = 0.737(4)$ from the BES III [13] and $f_+^{DK}(0) = 0.727(11)$ from the BABAR [68], as well as $f_+^{DK}(0) = 0.764(31)$ and $|f_T^{DK}(0)| = 0.687(54)$ from the LQCD [23, 24] within the error bars. As in the case of $D \longrightarrow \pi$ decay, the q^2-dependent behaviors of our results show better agreement with the data from the LQCD [23, 24] when we use $m_c \simeq 1.6$ GeV rather than 1.8 GeV.

Figures 3 and 4 show our predictions for the differential decay rates of $D \longrightarrow \pi e \nu_e$ and $D \longrightarrow K e \nu_e$ decays, respectively, compared with the experimental data from the BABAR [67, 68] (black circles), CLEO [69] (blue squares), and BES III [13, 14] for neutral D^0 (red diamonds) and charged D^+ with the account of isospin factor (green triangles). In our numerical calculations of the branching ratios, we use the CKM matrix elements $|V_{cd}| = 0.221 \pm 0.004$ and $|V_{cs}| = 0.987 \pm 0.011$ quoted by the PDG [62]. Considering uncertainties coming from the CKM elements and the constituent charm quark mass $m_c = 1.7^{+0.1}_{-0.1}$ GeV, we made band plots, i.e., the solid (dashed) lines represent the results obtained from $m_c = 1.8(1.6)$ GeV with lower (upper) limits of the CKM elements. Our results are shown to be consistent with the current available experimental data within those uncertainties.

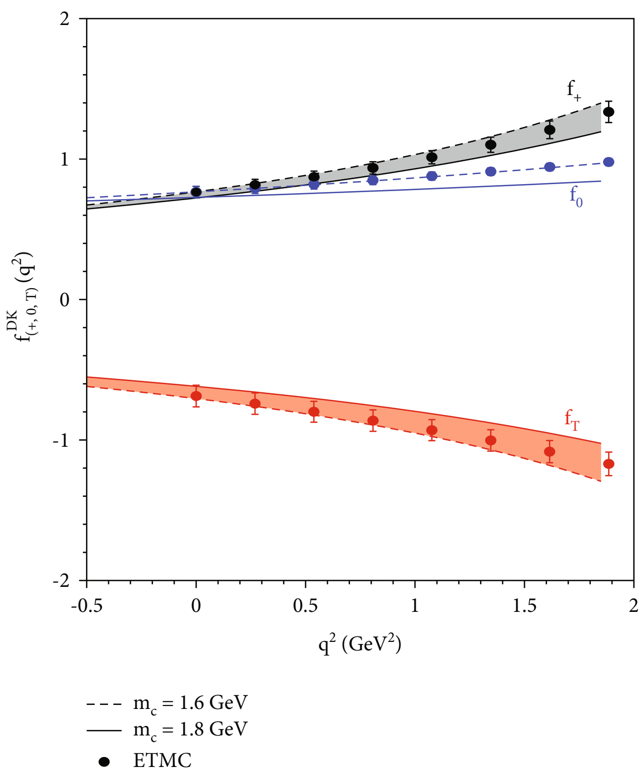

Figure 2: (Color online): the q^2 dependent form factors (f_+, f_0, f_T) of the $D \longrightarrow K$ decay for both spacelike and the kinematic timelike regions, $-0.5 \le q^2 \le (M_D - M_K)^2$ GeV^2. For comparison, the data taken from the LQCD (circles) [23, 24] are shown.

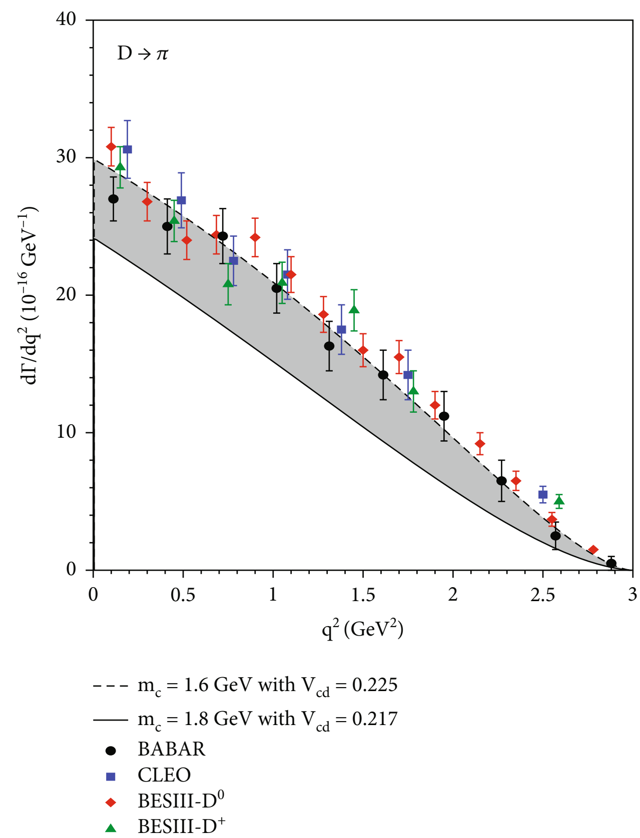

Figure 3: Differential decay rate for the $D \longrightarrow \pi e \nu_e$ decay compared with the experimental data from the BABAR [67, 68] (black circles), CLEO [69] (blue squares), and BES III [13, 14] for neutral D^0 (red diamonds) and charged D^+ with the account of isospin factor (green triangles).

In Tables 5 and 6, we summarize our results for the branching ratios for $D \longrightarrow \pi \ell \nu_\ell$ and $D \longrightarrow K \ell \nu_\ell$ $(\ell = e, \mu)$, respectively, and compare with the experimental data from PDG [62]. Our results for $\mathrm{Br}(D \longrightarrow \pi)$ are best fit to the data with $m_c = 1.6$ GeV but those for $\mathrm{Br}(D \longrightarrow K)$ prefers $m_c =$ 1.7 GeV.

Finally, as a test for the LFU, the R ratios of the semileptonic $D \longrightarrow (\pi, K)$ decays are defined by

$$R_P = \frac{\mathrm{Br}(D \longrightarrow P \mu \nu_\mu)}{\mathrm{Br}(D \longrightarrow P e \nu_e)}, \quad (54)$$

where $P = (\pi, K)$. Our predictions for R_P obtained from using $m_c = 1.7^{+0.1}_{-0.1}$ GeV are as follows: $(R_{\pi^0}, R_{\pi^-}) = (0.980^{+0.165}_{-0.003}, 0.983^{-0.001}_{-0.003})$, $(R_{K^-}, R_{\bar{K}^0}) = (0.974^{-0.002}_{+0.001}, 0.973^{-0.001}_{-0.001})$. Our results are consistent with the recent measurements from the BES III, $(R_{\pi^0}, R_{\pi^-}) = (0.942 \pm 0.046, 0.905 \pm 0.035)$ [15], and $R_{K^-} = 0.974 \pm 0.014$ [18], as well as other theoretical predictions such as $R_\pi = 0.985(2)$ and $R_K = 0.975(1)$ from the LQCD [70], $R_\pi = 0.985$ and $R_K = 0.980$ from the RQM [34], and $R_\pi = 0.98$ and $R_K = 0.97$ from the CCQM [35].

5. Summary and Discussion

In this work, we discussed the self-consistence description on the weak form factors f_+, f_- (or f_0), and f_T for the exclusive semileptonic $D \longrightarrow (\pi, K)\ell\nu_\ell(\ell = e, \mu, \tau)$ and rare $D \longrightarrow (\pi, K)\ell\ell$ decays in the standard LFQM. It has been known in the LF formulation that while the plus component (J^+) of the LF current J^μ in the matrix element can be regarded as the "good" current, the perpendicular $(\mathbf{J}_\perp)$ and the minus (J^-) components of the current were known as the "bad" currents since $(\mathbf{J}_\perp, J^-)$ is easily contaminated by the treacherous points such as the LF zero mode and the off-mass shell instantaneous contributions.

For a systematic analysis of such treacherous points in case one cannot avoid the use of $\mathbf{J}_\perp$ or J^-, we utilized the exactly solvable manifestly covariant BS model to carry out the LF calculations for three form factors, (f_+, f_-, f_T). In particular, we obtained f_- from two sets of the vector current, $(J^+, \mathbf{J}_\perp)_V$ and $(J^+, J^-)_V$, and f_T from two sets of tensor current, $J_T^{+\perp}$ and J_T^{+-}. In this BS model, we found that while f_+ obtained from J^+ and f_T obtained from $J_T^{+\perp}$ are free from the zero modes, f_- obtained from both $(J^+, \mathbf{J}_\perp)_V$ and $(J^+, J^-)_V$ sets and f_T obtained from J_T^{+-} receive the zero-mode contributions as well as the instantaneous ones. We then linked the BS model to the standard LFQM using the "CJ-scheme" [51–54] given by Eq. (44) for the correspondences between the two models and replaced the LF vertex function in the BS model with the more phenomenologically

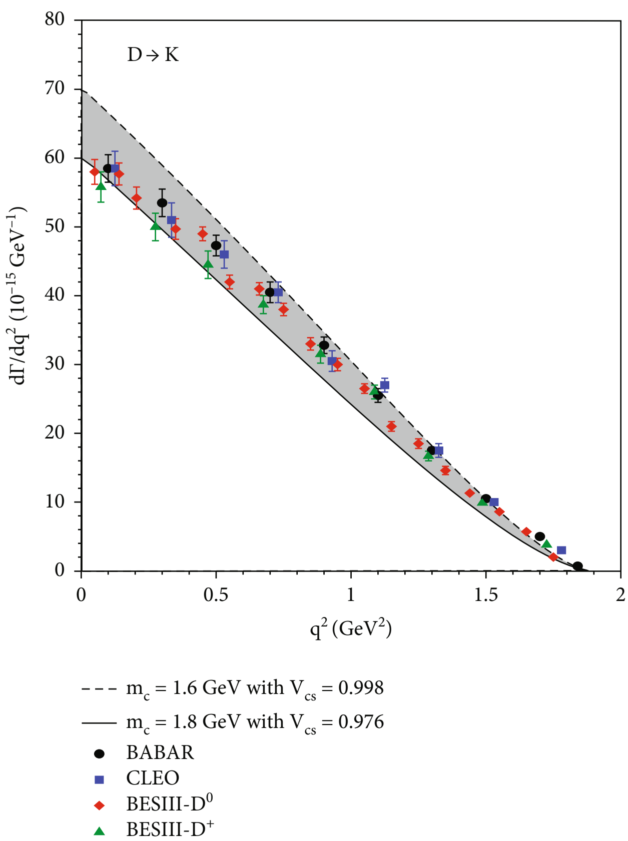

FIGURE 4: Differential decay rate for the $D \longrightarrow K e \nu_e$ decay compared with the experimental data from the BABAR [67, 68] (black circles), CLEO [69] (blue squares), and BES III [13, 14] for neutral D^0 (red diamonds) and charged D^+ with the account of isospin factor (green triangles).

TABLE 5: Branching ratios (in 10^{-3}) for $D \longrightarrow \pi \ell \nu_\ell$ ($\ell = e, \mu$) obtained from using $m_c = 1.7^{+0.1}_{-0.1}$ GeV together with $|V_{cd}| = 0.221 \pm 0.004$ [62].

Channel	Ours	PDG [62]
$D^+ \longrightarrow \pi^0 e^+ \nu_e$	$3.03^{+0.57}_{-0.44}$	3.72 ± 0.17
$D^+ \longrightarrow \pi^0 \mu^+ \nu_\mu$	$2.97^{+0.58}_{-0.44}$	3.50 ± 0.15
$D^0 \longrightarrow \pi^- e^+ \nu_e$	$2.37^{+0.45}_{-0.34}$	2.91 ± 0.04
$D^0 \longrightarrow \pi^- \mu^+ \nu_\mu$	$2.33^{+0.44}_{-0.34}$	2.67 ± 0.12

TABLE 6: Branching ratios (in %) for $D \longrightarrow K \ell \nu_\ell (\ell = e, \mu)$ decays obtained from using $m_c = 1.7^{+0.1}_{-0.1}$ GeV together with $|V_{cs}| = 0.987 \pm 0.011$ [62].

Channel	Ours	PDG [62]
$D^+ \longrightarrow \bar{K}^0 e^+ \nu_e$	$8.88^{+1.10}_{-0.80}$	8.73 ± 0.10
$D^+ \longrightarrow \bar{K}^0 \mu^+ \nu_\mu$	$8.64^{+1.07}_{-0.79}$	8.76 ± 0.19
$D^0 \longrightarrow K^- e^+ \nu_e$	$3.50^{+0.43}_{-0.31}$	3.54 ± 0.034
$D^0 \longrightarrow K^- \mu^+ \nu_\mu$	$3.41^{+0.41}_{-0.30}$	3.41 ± 0.004

accessible Gaussian wave function provided by the LFQM analysis of meson mass spectra [55, 56]. As in the case of previous analysis [51–54], it is astonishing to discover that the zero modes and the instantaneous contributions present in the BS model become absent in the standard LFQM. In other words, our LFQM results of (f_-, f_T) are shown to be independent of the components of the current without involving any of those treacherous contributions. Since the absence of the zero mode found in the standard LFQM is mainly due to the replacement of the physical mass $M_{1(2)}$ with the invariant mass $M_0^{(\prime)}$ in the course of linking the two models, this replacement could be regarded as an effective treatment of the zero mode in the standard LFQM.

In the standard LFQM, the constituent quark and antiquark in a bound state are required to be on-mass shell, which is different from the covariant formalism, in which the constituents are off-mass shell. The common feature of the standard LFQM is thus to use the sum of the LF energy of the constituent quark and antiquark for the meson mass in the spin-orbit wave function, which is obtained by the interaction-independent Melosh transformation from the ordinary equal-time static spin-orbit wave function assigned by the quantum number J^{PC}. Under these circumstances, it is natural to apply the replacement $M_{1(2)} \longrightarrow M_0^{(\prime)}$ in the calculation of the physical observables in the standard LFQM. Indeed, we have shown explicitly that this correspondence relation for the calculations of the decay constants and weak transition form factors between two pseudoscalar mesons provide the current-component independent predictions in the standard LFQM.

We then apply our current-component independent form factors $(f_\pm, f_T)$ for the self-consistent analysis of semileptonic and rare $D \longrightarrow (\pi, K)$ decays using our LFQM constrained by the variational principle for the QCD-motivated effective Hamiltonian with the HO plus Coulomb interaction [49, 57]. The form factors $(f_\pm, f_T)$ obtained in the $q^+ = 0$ frame $(q^2 = -\mathbf{q}_\perp^2 < 0)$ are then analytically continued to the timelike region by changing $\mathbf{q}_\perp^2$ to $-q^2$ in the form factors. In our numerical calculations, we also checked the sensitivity of the constituent charm quark mass $m_c = 1.7^{+0.1}_{-0.1}$ GeV through the analysis of the form factors $(f_\pm, f_T)$ for $D \longrightarrow (\pi, K)$ decays. Our results for the form factors and branching ratios for $D \longrightarrow (\pi, K)$ decays show in good agreement with the available experimental data as well as other theoretical predictions. Especially, the smaller charm quark mass $m_c \simeq 1.6$ GeV seems preferable to larger $m_c = 1.8$ GeV for $D \longrightarrow (\pi, K)$ decays while they are not much different for the analysis of the decay constant of the D meson. Finally, we obtained the the R ratios of the semileptonic $D \longrightarrow (\pi, K)$ decays as a test for the LFU, and our results are consistent with the recent measurements from the BES III [15, 18] as well as other theoretical results [34, 35, 70].

While the rare decay analyses including the tensor form factor can in principle be made, I just focused on the extraction of the current-component independent weak transition form factors as well as the comparison with the available experimental data in the present work. More complete phenomenological analyses regarding on the rare decays of heavy D and B mesons are also under consideration.

Data Availability

The data used to support the findings of this study are available from the author upon request.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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References

[1] S. Fajfer, J. F. Kamenik, I. Nišandžić, and J. Zupan, "Implications of lepton flavor universality violations in B decays," *Physical Review Letters*, vol. 109, no. 16, article 161801, 2012.

[2] J. P. Lee et al. (BABAR Collaboration), "Evidence for an excess of $\bar{B} \longrightarrow D^{(*)}\tau^-\bar{\nu}_\tau$ decays," *Physical Review Letters*, vol. 109, no. 10, article 101802, 2012.

[3] J. P. Lee et al. (BABAR Collaboration), "Measurement of an excess of $\bar{B} \longrightarrow D^{(*)}\tau^-\bar{\nu}_\tau$ decays and implications for charged Higgs bosons," *Physical Review D*, vol. 88, no. 7, article 072012, 2013.

[4] M. Huschle, T. Kuhr, M. Heck et al., "Measurement of the branching ratio of $\bar{B} \longrightarrow D^{(*)}\tau^-\bar{\nu}_\tau$ relative to $\bar{B} \longrightarrow D^{(*)}\ell^-\bar{\nu}_\ell$ decays with hadronic tagging at Belle," *Physical Review D*, vol. 92, no. 7, article 072014, 2015.

[5] Y. Sato, T. Iijima, K. Adamczyk et al., "Measurement of the branching ratio of $\bar{B}^0 \longrightarrow D^{*+}\tau^-\bar{\nu}_\tau$ relative to $\bar{B}^0 \longrightarrow D^{*+}\ell - \bar{\nu}_\ell$ decays with a semileptonic tagging method," *Physical Review D*, vol. 94, no. 7, article 072007, 2016.

[6] R. Aaij, B. Adeva, M. Adinolfi et al., "Measurement of the ratio of branching fractions $\beta(\bar{B}^0 \longrightarrow D^{*+}\tau^-\bar{\nu}_\tau)/\beta(\bar{B}^0 \longrightarrow D^{*+}\mu^-\bar{\nu}_\mu)$," *Physical Review Letters*, vol. 115, no. 11, article 111803, 2015.

[7] H. Na, C. M. Bouchard, G. P. Lepage, C. Monahan, J. Shigemitsu, and HPQCD Collaboration, "$B \longrightarrow Dl\nu$ form factors at nonzero recoil and extraction of $|V_{cb}|$," *Physical Review D*, vol. 92, no. 5, article 054510, 2015.

[8] J. A. Bailey, A. Bazavov, C. Bernard et al., "$B \longrightarrow D\ell\nu$ form factors at nonzero recoil and $|V_{cb}|$ from 2 + 1-flavor lattice QCD," *Physical Review D*, vol. 92, no. 3, article 034506, 2015.

[9] D. Bigi and P. Gambino, "Revisiting $B \longrightarrow D\ell\nu$," *Physical Review D*, vol. 94, no. 9, article 094008, 2016.

[10] T. Zhong, Y. Zhang, X. G. Wu, H. B. Fu, and T. Huang, "The ratio $\mathscr{R}(D)$ and the D-meson distribution amplitude," *The European Physical Journal C*, vol. 78, no. 11, article 937, 2018.

[11] N. Cabibbo, "Unitary symmetry and leptonic decays," *Physical Review Letters*, vol. 10, no. 12, pp. 531–533, 1963.

[12] M. Kobayashi and T. Maskawa, "*CP*-violation in the renormalizable theory of weak interaction," *Progress of Theoretical Physics*, vol. 49, no. 2, pp. 652–657, 1973.

[13] M. Ablikim, M. N. Achasov, X. C. Ai et al., "Study of dynamics of $D^0 \longrightarrow K^-e^+\nu_e$ and $D^0 \longrightarrow \pi^-e^+\nu_e$ decays," *Physical Review D*, vol. 92, no. 7, article 072012, 2015.

[14] M. Ablikim, M. N. Achasov, S. Ahmed et al., "Analysis of $D^+ \longrightarrow \bar{K}^0e^+\nu_e$ and $D^+ \longrightarrow \pi^0e^+\nu_e$ semileptonic decays," *Physical Review D*, vol. 96, no. 1, article 012002, 2017.

[15] M. Ablikim, M. N. Achasov, S. Ahmed et al., "Measurement of the branching fraction for the semileptonic decay $D^{0(+)} \longrightarrow \pi^{-(0)}\mu^+\nu_\mu$ and test of lepton flavor universality," *Physical Review Letters*, vol. 121, no. 17, article 171803, 2018.

[16] M. Ablikim, M. N. Achasov, S. Ahmed et al., "Measurements of the branching fractions for the semileptonic decays $D_s^+ \longrightarrow \phi e^+\nu_e, \phi\mu^+\nu\mu, \eta\mu^+\nu\mu$ and $\eta'\mu^+\nu_\mu$," *Physical Review D*, vol. 97, no. 1, article 012006, 2018.

[17] M. Ablikim, M. N. Achasov, S. Ahmed et al., "Study of the decays $D^+ \longrightarrow \eta^{(\prime)}e^+\nu_e$," *Physical Review D*, vol. 97, no. 9, article 092009, 2018.

[18] M. Ablikim, M. N. Achasov, S. Ahmed et al., "Measurement of the dynamics of the decays $D_s^+ \longrightarrow \eta^{(\prime)}e^+\nu_e$," *Physical Review Letters*, vol. 122, no. 12, article 121801, 2019.

[19] M. Ablikim, M. N. Achasov, S. Ahmed et al., "Study of the $D^0 \longrightarrow K^-\mu^+\nu_\mu$ dynamics and test of lepton flavor universality with $D^0 \longrightarrow K^-\ell^+\nu_\ell$ decays," *Physical Review Letters*, vol. 122, no. 1, article 011804, 2019.

[20] M. Ablikim, M. N. Achasov, S. Ahmed et al., "First measurement of the form factors in $D_s^+ \longrightarrow K^0e^+\nu_e$ and $D_s^+ \longrightarrow K^{*0}e^+\nu_e$ decays," *Physical Review Letters*, vol. 122, no. 6, article 061801, 2019.

[21] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage, and J. Shigemitsu, "$D \longrightarrow K, l\nu$ semileptonic decay scalar form factor and $|V_{cs}|$ from lattice QCD," *Physical Review D*, vol. 82, no. 11, article 114506, 2010.

[22] H. Na, C. T. H. Davies, E. Follana et al., "$D \longrightarrow \pi, l\nu$ semileptonic decays, $|V_{cd}|$ and second row unitarity from lattice QCD," *Physical Review D*, vol. 84, no. 11, article 114505, 2011.

[23] V. Lubicz, L. Riggio, G. Salerno, S. Simula, C. Tarantino, and for the ETM Collaboration, "Scalar and vector form factors of $D \longrightarrow \pi(K)\ell\nu$ decays with $N_f = 2 + 1 + 1$ twisted fermions," *Physical Review D*, vol. 96, no. 5, article 054514, 2017.

[24] V. Lubicz, L. Riggio, G. Salerno, S. Simula, C. Tarantino, and for the ETM Collaboration, "Tensor form factor of $D \longrightarrow \pi(K)\ell\nu$ and $D \longrightarrow \pi(K)\ell\ell$ decays with $N_f = 2 + 1 + 1$ twisted-mass fermions," *Physical Review D*, vol. 98, no. 1, article 014516, 2018.

[25] P. Ball, V. M. Braun, and H. G. Dosch, "Form factors of semileptonic D decays from QCD sum rules," *Physical Review D*, vol. 44, no. 11, pp. 3567–3581, 1991.

[26] P. Colangelo and F. de Fazio, "D_s decays to η and η' final states: a phenomenological analysis," *Physics Letters B*, vol. 520, no. 1-2, pp. 78–86, 2001.

[27] P. Ball, "Testing QCD sum rules on the light-cone in $D \longrightarrow (\pi, K)\ell\nu$ decays," *Physics Letters B*, vol. 641, no. 1, pp. 50–56, 2006.

[28] A. Khodjamirian, C. Klein, T. Mannel, and N. Offen, "Semileptonic charm decays $D \longrightarrow \pi l\nu_l$ and $D \longrightarrow Kl\nu_l$ from QCD light-cone sum rules," *Physical Review D*, vol. 80, no. 11, article 114005, 2009.

[29] H.-B. Fu, L. Zeng, R. Lü, W. Cheng, and X. G. Wu, "The $D \longrightarrow \rho$ semileptonic and radiative decays within the light-cone sum rules," *The European Physical Journal C*, vol. 80, no. 3, article 194, 2020.

[30] Z.-Q. Yao, D. Binosi, Z. F. Cui, C. D. Roberts, S. S. Xu, and H. S. Zong, "Semileptonic decays of $D_{(s)}$ mesons," *Physical Review D*, vol. 102, no. 1, article 014007, 2020.

[31] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, "Semileptonic B and D decays in the quark model," *Physical Review D*, vol. 39, no. 3, pp. 799–818, 1989.

[32] D. Scora and N. Isgur, "Semileptonic meson decays in the quark model: an update," *Physical Review D*, vol. 52, no. 5, pp. 2783–2812, 1995.

[33] N. Isgur and M. B. Wise, "Relationship between form factors in semileptonic $\bar{B}$ and D decays and exclusive rare $\bar{B}$-meson decays," *Physical Review D*, vol. 42, no. 7, pp. 2388–2391, 1990.

[34] R. N. Faustov, V. O. Galkin, and X. W. Kang, "Semileptonic decays of D and D_s mesons in the relativistic quark model," *Physical Review D*, vol. 101, no. 1, article 013004, 2020.

[35] M. A. Ivanov, J. G. Körner, J. N. Pandya, P. Santorelli, N. R. Soni, and C. T. Tran, "Exclusive semileptonic decays of D and D_s mesons in the covariant confining quark model," *Frontiers of Physics*, vol. 14, no. 6, article 64401, 2019.

[36] T. Palmer and J. O. Eeg, "Form factors for semileptonic D decays," *Physical Review D*, vol. 89, no. 3, article 034013, 2014.

[37] D. Melikhov and B. Stech, "Weak form factors for heavy meson decays: an update," *Physical Review D*, vol. 62, no. 1, article 014006, 2000.

[38] S. J. Brodsky, H. C. Pauli, and S. S. Pinsky, "Quantum chromodynamics and other field theories on the light cone," *Physics Reports*, vol. 301, no. 4-6, pp. 299–486, 1998.

[39] W. Jaus, "Semileptonic decays of B and D mesons in the light-front formalism," *Physical Review D*, vol. 41, no. 11, pp. 3394–3404, 1990.

[40] H. Y. Cheng and X. W. Kang, "Branching fractions of semileptonic D and D_s decays from the covariant light-front quark model," *The European Physical Journal C*, vol. 77, no. 9, article 587, 2017.

[41] H.-Y. Cheng, C. Y. Cheung, and C. W. Hwang, "Mesonic form factors and the Isgur-Wise function on the light front," *Physical Review D*, vol. 55, no. 3, pp. 1559–1577, 1997.

[42] L. A. Kondratyuk and D. V. Tchekin, "Transition form factors and probabilities of the semileptonic decays of B and D mesons within covariant light-front dynamics," *Physics of Atomic Nuclei*, vol. 64, no. 4, pp. 727–752, 2001.

[43] Q. Chang, X. N. Li, X. Q. Li, F. Su, and Y. D. Yang, "Self-consistency and covariance of light-front quark models: testing via P, V, and A meson decay constants, and $P \longrightarrow P$ weak transition form factors," *Physical Review D*, vol. 98, no. 11, article 114018, 2011.

[44] Q. Chang, X. L. Wang, and L. T. Wang, "Tensor form factors of $P \longrightarrow P, S, V$ and A transitions within standard and covariant light-front approaches," *Chinese Physics C*, vol. 44, no. 8, article 083105, 2020.

[45] W. Jaus, "Covariant analysis of the light-front quark model," *Physical Review D*, vol. 60, no. 5, article 054026, 1999.

[46] H.-Y. Cheng, C. K. Chua, and C. W. Hwang, "Covariant light-front approach for s-wave and p-wave mesons: its application to decay constants and form factors," *Physical Review D*, vol. 69, no. 7, article 074025, 2004.

[47] R. C. Verma, "Decay constants and form factors of s-wave and p-wave mesons in the covariant light-front quark model," *Journal of Physics G: Nuclear and Particle Physics*, vol. 39, no. 2, article 025005, 2012.

[48] H. J. Melosh, "Quarks: currents and constituents," *Physical Review D*, vol. 9, no. 4, pp. 1095–1112, 1974.

[49] H.-M. Choi and C. R. Ji, "Semileptonic and radiative decays of the B_c meson in the light-front quark model," *Physical Review D*, vol. 80, no. 5, article 054016, 2009.

[50] B. L. G. Bakker, H. M. Choi, and C. R. Ji, "Regularizing the divergent structure of light-front currents," *Physical Review D*, vol. 63, no. 7, article 074014, 2001.

[51] H.-M. Choi and C. R. Ji, "Self-consistent covariant description of vector meson decay constants and chirality-even quark-antiquark distribution amplitudes up to twist 3 in the light-front quark model," *Physical Review D*, vol. 89, no. 3, article 033011, 2014.

[52] H.-M. Choi and C. R. Ji, "Consistency of the light-front quark model with chiral symmetry in the pseudoscalar meson analysis," *Physical Review D*, vol. 91, no. 1, article 014018, 2015.

[53] H.-M. Choi and C. R. Ji, "Two-particle twist-3 distribution amplitudes of the pion and kaon in the light-front quark model," *Physical Review D*, vol. 95, no. 5, article 056002, 2017.

[54] H.-M. Choi, "Self-consistent light-front quark model analysis of $B \longrightarrow D\ell\nu_\ell$ transition form factors," *Physical Review D*, vol. 103, no. 7, article 073004, 2021.

[55] H.-M. Choi and C. R. Ji, "Mixing angles and electromagnetic properties of ground state pseudoscalar and vector meson nonets in the light-cone quark model," *Physical Review D*, vol. 59, no. 7, article 074015, 1999.

[56] H.-M. Choi and C. R. Ji, "Light-front quark model analysis of exclusive $0^- \longrightarrow 0^-$ semileptonic heavy meson decays," *Physics Letters B*, vol. 460, no. 3-4, pp. 461–466, 1999.

[57] H.-M. Choi, "Decay constants and radiative decays of heavy mesons in light-front quark model," *Physical Review D*, vol. 75, no. 7, article 073016, 2007.

[58] J. G. Körner and G. A. Schuler, "Exclusive semileptonic heavy meson decays including lepton mass effects," *Zeitschrift für Physik C: Particles and Fields*, vol. 46, no. 1, pp. 93–109, 1990.

[59] D.-L. Yao, P. Fernandez-Soler, F. K. Guo, and J. Nieves, "New parametrization of the form factors in $\bar{B} \longrightarrow D\ell\bar{\nu}_\ell$ decays," *Physical Review D*, vol. 101, no. 3, article 034014, 2020.

[60] H.-M. Choi and C. R. Ji, "Kaon electroweak form factors in the light-front quark model," *Physical Review D*, vol. 59, no. 3, article 034001, 1998.

[61] H.-M. Choi and C. R. Ji, "Distribution amplitudes and decay constants for(π, K, ρ, K^*) mesons in the light-front quark model," *Physical Review D*, vol. 75, no. 3, article 0340196, 2007.

[62] P. A. Zyla et al. (Particle Data Group), "Review of particle physics," *Progress of Theoretical and Experimental Physics*, vol. 2020, no. 8, article 083C01, 2020.

[63] C. R. Ji and H. M. Choi, "New effective treatment of the light-front nonvalence contribution in timelike exclusive processes," *Physics Letters B*, vol. 513, no. 3-4, pp. 330–336, 2001.

[64] S. J. Brodsky and D. Sung Hwang, "Exact light-cone wavefunction representation of matrix elements of electroweak currents," *Nuclear Physics B*, vol. 543, no. 1-2, pp. 239–252, 1998.

[65] C. Bourrely, B. Machet, and E. de Rafael, "Semileptonic decays of pseudoscalar particles $(M \longrightarrow M' + \ell + \nu_\ell)$ and short-distance behaviour of quantum chromodynamics," *Nuclear Physics B*, vol. 189, no. 1, pp. 157–181, 1981.

[66] C. G. Boyd, B. Grinstein, and R. F. Lebed, "Constraints on form factors for exclusive semileptonic heavy to light meson decays," *Physical Review Letters*, vol. 74, no. 23, pp. 4603–4606, 1995.

[67] J. P. Lees, V. Poireau, V. Tisserand et al., “Measurement of the $D^0 \longrightarrow \pi^- e^+ \nu_e$ differential decay branching fraction as a function of q^2 and study of form factor parametrizations,” *Physical Review D*, vol. 91, no. 5, article 052022, 2015.

[68] B. Aubert, M. Bona, D. Boutigny et al., “Measurement of the hadronic form factor in $D^0 \longrightarrow K^- e^+ \nu_e$ decays,” *Physical Review D*, vol. 76, no. 5, article 052005, 2007.

[69] D. Besson, T. K. Pedlar, J. Xavier et al., “Improved measurements of D meson semileptonic decays to π and K mesons,” *Physical Review D*, vol. 80, no. 3, article 032005, 2009.

[70] L. Riggio, G. Salerno, and S. Simula, “Extraction of $|V_{cd}|$ and $|V_{cs}|$ from experimental decay rates using lattice QCD $D \longrightarrow \pi(K)\ell\nu$ form factors,” *The European Physical Journal C*, vol. 78, no. 6, article 501, 2018.

[71] H.-M. Choi, “Current-component independent transition form factors for semileptonic and rare $D \longrightarrow \pi(K)$ decays in the light-front quark model,” https://arxiv.org/abs/2108.10544.